Water Quality Management Practices
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Hello everyone, welcome to this NPTEL online certification course on Water Quality Management Practices. My name is Gourav, Professor Gourav Dhar bhowmick from the Department of Agriculture and Food Engineering of Indian Institute of Technology Kharagpur. So, in continuation of my discussion in lecture 12, today I will be discussing about the concept of mass balance which is very essential for all of you to understand whenever we will be designing any kind of reactors whether it be batch reactor, continuous reactor or say like a plug flow reactor or fluidized bed reactor. So, this mass balance is something that we need to understand in order to have a fair idea about how the mass is actually moving from one part to another and like how the influent effluent characteristics and the accumulation and the rejection of masses take place in the inside the reactor. So, I will mainly be covering this concepts in this particular lecture first the concept of mass balance, the analysis in case of batch reactor, in case of plug flow reactor and in case of CSTR reactor in series. So, to start with as you know in general it is very important for us to understand the reactor dynamics and in order to understand the reactor dynamics whenever we will be designing a particular reactor for our treatment plant we need to understand the mass balance.

In order to design it optimally so, to not to over design it or so, to not to overcrowded or you know make it less optimized less optimum I would say for a specific flow rate or for a specific waste water discharge rate that we are expecting and all. So, in general we have to follow a fundamental mass balance approach and which is you know the principle that the mass is conserved according to the conservation of mass that. However, in case of non nuclear reactions it is obviously, been conserved and in this conservation of mass is definitely been something that will be considering in designing or you know whenever we will be solving the problems related to this mass balance approaches and all. So, in general we know that the mass which is accumulated inside the reactor which obviously, the equals to the there is a subtraction of the influent mass minus the obviously, the and the effluent mass.

So, that is the amount of mass that is getting accumulated inside the reactor. And the amount of mass which is actually flowing through the reactor this accumulation of reactor accumulation of mass sorry inside the reactor is only possible when there is some additional reaction is taking place and some amount of mass which is coming along with the influent is actually retained inside the system. So, this kind of systems which normally comes under the continuous reactor in this kind of continuous reactor how the how we should do the mass balance we will be discussing in next couple of slides. So, in general in any unit reactions or unit operations without any reactions the mass of pollutant accumulated is the difference between the inflow and outflow right. And for any reactor with the within a system boundary the influent flow rate is defined by Q you can see the influent the flow rate is Q the C 0 is the reactant concentration or the pollutant concentration in the in the in flow rate in the inflow water waste water.

Then the reactor volume is defined by V and the pollutant concentration is C V inside in the in the in the in the reactor or some in at any particular point of time. The leaving concentration is defined by C T for a completely mixed reactor and it is influenced by the reaction rate R ok and it is expressed as mass per volume per time. So, if you see the if we go gradually further the accumulation as I was discussing accumulation is nothing, but inflow minus the decrease due to the reaction minus outflow ok. So, that is the amount of accumulation of mass that is there inside the reactor. How we can define it in terms of the those ah those like you know just nomenclatures that I have already just mentioned.

So, in case of accumulation is nothing, but the change in concentration. So, here the DC. So, here that is changing leaving concentration is DC multiplied by the V, V is what here the V is the volume of the reactor ok equal to the inflow, inflow is what Q is the inflow rate C 0 is the inflow concentration multiplied dt is the time dt is the time that is denoting like you know the the total time that actually we are considering the change in change in time outside minus decrease due to the reaction which is V capital V which is like the reaction the volume of the reactor multiplied by the reaction rate multiplied by the change in time. And also the outflow can be ah represented by C t is this a concentration of the concentration of the ah the leaving concentration for the ah pollutant multiplied by the Q is the flow rate multiplied by the change in time you understand. So, with this ah equation 1 we can easily represent the ah concept of mass balance in a particular particular type of reactor.

So, if you rearrange this equation if you see dC dC t divided by dt if you do it the dt is common in the right hand side just take it out ok just and now dC t by dt into V it will become Q into C 0 minus V into R minus Q into C t ok. Just what we are doing we are taking common we take the dt as common and we are actually dividing it in the in the put it in the left hand side ok. Now, this R this R is the reaction rate like it is it is a if it is a first order reaction which you remember most of the biological ah the degradation processes which we discussed in the over stabilization processes inside the or the substrate ah degradation process inside the reactor ah goes along with the first order reactions. So, in that case R equal to K into C t ok. So, this K is what K is the reaction rate constant and C t is the one the living concentration we already discussed.

Now the dC t by dt into V is equal to Q into C 0 into V into R instead of R you can just simply ah replace it in the equation number 4 with K into C t K is what reaction rate constant ok. In in in ah the the unit of reaction rate constant is what in time inverse ok. So, minus Q into C t that is the equation number 4 that we derived. Now in the equation number 4 if you see so, the most of the unit processes like you know when we design this this unit processes are designed for steady state condition. For steady state condition what will happen the rate of change of pollutant concentration will be 0 we if after a sufficient start up time is not it.

So, what will happen ah because of that that means, the dC t by dt that we were discussing in the equation number 4 will become 0. If the dC t by dt will become 0 so, what will happen now you can simply ah do the further calculations and you can do like in case of steady state design we also know that V by Q is what is the hydraulic retention time which is theta equal ok. So, what is this hydraulic retention time? It means that for the time for which your water is actually get the time to clean itself I mean like you know in order to make it more scientifically if I say the time it requires for the waste water to treat itself inside a reactor ok. That time for which actually waste water particular molecule of waste water stays inside a reactor is the hydraulic retention time theoretically. So, it means suppose your volume of a reactor is 100 litre ok.

So, volume of a reactor is 100 litre that means, V is 100 litre. Now your flow rate is say 50 litre per hour. So, if your flow rate which is like Q, Q is 50 litre per hour and volume is 100 litre. So, what will be the hydraulic retention time for what time like for like what is the total amount of time for which your water can stay inside the reactor? V by Q 100 divided by 50 2 hour for 2 hour your water the waste water that your the influent can actually stays inside the reactor for getting proper treatment then the water will theoretically get out of the system. So, this is called the hydraulic retention time ok.

So, this hydraulic retention time can be can be represented by the Greek letter theta ok. So, we if we substitute this V by Q with the rate of change of and substitute the value of V by Q with theta and also the rate of change of pollutant concentration dC t by dt equals to 0 in the equation number 5 if you remember in the last I mean like the in the equation number 4 I would say and it gives us the equation number 5. How? If you see let us go to the equation number 4 here dC by dt is 0. Now what happens to the ah V by Q? V by Q will become if you take the V in Q ah values in one side Q C 0 minus Q C 0 will be on a one side in the left hand side and Q C t will also be in a left hand side what will it will become? Q into C t minus Q into C 0 take the Q common Q common C t minus C 0 equal to minus V into K C t ok. So, now, we can change the ah notations you can have like ah Q into C 0 minus C t ok equal to K by K V into K into C t ok.

What you can do now? You can simply V by Q V by Q is what? Theta now you replace this V by Q with theta and you change it for the change it for the ah C t value you will get this equation C t is equal to C 0 divided by 1 plus K into theta ok. I would request you to whenever we discussing all these things please write it down this equations. So, it will be easier for you to understand. So, ah what I was saying key C t is equal to we come out to this conclusion that C t is equal to C 0 divided by 1 plus K into theta. So, now, let us do a numerical.

So, what I am asked? I am asked that a completely mixed biological wastewater treatment process that influent BOD ah to the reactor is 180 milligram per liter. So, what is the influent BOD? What how we can we can identify it with which ah like like nomenclature we remember we discussed that C 0 represents the influent BOD is not it. So, C 0 is equal to 180 milligram per liter it is given in the numerical. If the reaction rate the value of K the reaction rate constant is 0.5 per hour and the desired effluent BOD concentration is also given which is 20 milligram per liter.

How we can find out this desired ah a BOD concentration? Generally by the the the standards given by the regulatory bodies. If they say it like your effluent concentration has to be say 20 milligram per liter. So, that is what you have to take. So, now, if you have to take if you have to make sure that your effluent BOD concentration should be at least 20 milligram per liter and you know that influent BOD is 180 milligram per liter and you also know the reaction rate constant is ah 0.

5 per hour. How we can get the value of ah the retention time or the theta? It is very easy using the equation if you remember in the equation number 5 C t equal to C 0 divided by 1 plus K into theta. C t is known to us the final concentration which is 20 milligram per liter C 0 is known to us 180 milliliter milligram per liter divided by 1 plus K. K K is known to us 0.5 per hour and we need to find out the theta. If you solve this equation you can easily find out the value of theta which will be coming as 16 hour.

What does that mean? That the retention time of 16 hour is required to get 20 milligram per liter of effluent BOD concentration. I hope you understand ok. So, this is for the say content completely mixed biological wastewater treatment processes. What will happen in case of batch reactor? In case of batch reactor it is not always in I mean like what is happening here in batch reactor you remember it is like a fill and draw type. You fill it and wait for some time and then take the take the water out like when the treatment is done this is called the batch reactor.

In case of batch reactor the we normally fill the batch reactor with some wastewater influent then we wait for some over time and then we ah what we do we take that as what wastewater out once the treatment is done. So, it causes a change in the concentration from the initial value. So, what what what we do need to do? We need to remember that in case of batch reactor no inflow and no outflow is there ok. So, mass balance equation when we are going to do we need to consider only 2 factor. First is the accumulation and which can be easily represented by the decrease due to the reaction no inflow no outflow nothing right.

So, in case of batch reactor it is as easy as that the accumulation is only defined by the negative value to the decrease decrement in the decrement due to the reaction that is being taken place inside the reactor. So, again considering the first order rate of reactions with constant rate constant with a reactor rate reaction rate constant of k the equation 1 ah which we discussed here if you see accumulation equal to minus decrease due to reaction can be represented as ah equation 2 ah the accumulation is nothing, but volume multiplied by the rate of change of the I mean like the change of pollutant concentration over time equal to minus k k is the reactant rate constant into C 3 into V ok. So, that represents the amount of there is a decrement over the because of the reaction that is taking place. Now, if you rearrange this equation 2 for integration the equation 3 can be obtained this C t you will take it in the left side. So, it will be integration from C 0 to C t C 0 is the initial time.

So, like initial concentration to the final concentration and minus k into ah integration from 0 to theta theta is the hydro retention that is the maximum time right. So, of dt what will happen? So, dc by dt dct by ct what what what does that mean? If you integrated it will become log of log of C t in bracket C 0 to C t ok. So, that means, log of C t minus log of C 0 in the left hand side I am talking about if log of C t minus log of C 0 means log of C t by C 0 remember from the logarithmic function. So, in the in the if you log of C t by C 0 is equal to what will happen integration of ah dt is what it is only t only. So, t 0 to t means like only theta.

So, k into theta so that means, log of C t by C 0 is equal to minus k t k theta. Now, if it is a log log means like with the function ah with the base e. So, now, it will if you remove the log it will become C t by C 0 equal to e to the power k k k into theta is not it. Now, C t is equal to C 0 into e to the power minus k theta ok. If you need further ah elaboration of this things I think it is very easy it is a very basic map.

So, I think you can do that. So, if you, but still if you are still having any confusion definitely we can discuss it over in the forum or when we will be I will be coming it for live sessions and all if you have any questions there you can ask me I will show you how actually it is to be done ok. Let us do one numerical. So, in case of first order reactor reactant constant of a batch reactor is given as 0.4 per hour and what will be the percentage removal of the pollutant ah that will occur during the reaction time of 6 hour ok. So, within a 6 hour and with a with a value of k is given as 0.

4 per hour. So, what will be the C t by ah C 0 first we need to find out find out right then only we can easily find out the percentage removal. So, we know the equation C t equal to C 0 e to the power minus k theta. So, C t by C 0 is equal to e to the power minus k theta e to the power minus k k value is 0.4 and theta is 6 6 days if you remember so that means, e to the power minus ah 0.

4 into 6 that means, 0.0907. So, C t by C 0 equal to 0.0907. So, what can how we can find out the removal efficiency as easy as that C 0 minus C t by C 0 will give us the removal efficiency is not it C 0 minus C t by C 0 will give you give us the removal efficiency to further simplifying it 1 minus C t by C 0 into 100 it will give us the percentage removal efficiency in percentage. So, C t by C 0 is known to us 0.0907 so that means, the removal efficiency is 90.

93 that means, in 6 day how much pollutant is removed from your batch reactor 90 percentage you understand. So that means, if you have our initial ah concentration of say like ah 100 it will become how much what will be the final ah ah like you know I mean like the final ah BOD I mean like say like initial BOD is 100 final BOD will become 9.07 ok. Same as the if the equation says that see if the initial concentration is 120 what will be the effluent concentration.

So, 120 multiplied by 0.0907 which is 10.88 that means, final effluent concentration is 10.88 milligram per liter which is very good. So, I mean like your BOD your removal that means, your reactor is working really nice more than 90 percent removal is happening. However, that also depends upon the initial concentration and other factors also ok. Initial concentration is say like 1000 milligram per liter 1000 milligram per liter in that case even if it is like removing 90 percent still your final BOD will become somewhere around ah ah 9.

09 almost ah 90.09. So, 90 is quite huge you understand even if the initial BOD is constantly removal efficiency is 90 percent more than 90 percent still it does not give us a very approximation exact approximation of the final ah how better the solution, but better the ah you know the reaction rate is and how better the I mean like the your plant is performing because it depends upon the initial ah ah the effluent concentration also. It reminds me of a very funny situation with the you see in some ah ah advertisement and all you will see people say like you know it will remove 99.99 percentage of blah blah blah ok I am not saying it which company what is it. So, this is a 99.

99 percentage or 99 percent is it will remove. So, it does not give you a exact idea like what is the initial concentration what will happen if it is like initial concentration is too high still 99 percentage removal does not give us any assurance that your final your product is good enough for us to use ok. So, we have to be ah cautious about this ah the advertisement gimmicks like we have to understand that what actually it means ok. So,

initial concentration is also something that we need to understand ok. For plug flow reactor if you remember for plug flow reactor we remember we discussed that how it looks like. So, so, influent comes from one side of the reactor and it goes from the other side of the reactor ok.

So, it stays for a certain period of time and it is a in case of steady state condition with constant flow and influent ah influent ah pollutant concentration if we consider it is it will never overtake or fall back any fluid elements and with the minimal dispersion happens in the inside the reactor based on the definition of plug flow reactor ok. And pollution ah pollutant concentration that C t it changes over the waste water the through like throughout the reactor right. Initially it has a certain ah concentration with time what will happen in the plug flow the concentration of the pollutant will reduce because what the removal will take place right in the reactor. So, at the end the moment it will come out of the effluent zone I mean come out of this outlet point it will be having a minimum concentration the pollutant concentration will be minimum isn't it. So, with time when it will go from say like in for this particular picture left hand side to the right hand side the pollutant concentration will go down ok.

So, say so, in order to do the mass balance equation for a system like this we need to take a small volume of the reactor. Say in the middle you see the disk of ah one small disk we have chosen. So, we can say take a the volume of that small disk it is say like d v and considering a concentration gradient along the it is length it is one side it is C t and the other side it is C t minus d c. d c is the one d c is what is the change of concentration in this tiny fraction in this from one side of it is ah the disk to the other side of the disk. So, what is the change in concentration that can be ah represented by d c ok.

So, initially it is C t now it is become C t minus d c flow is same Q and the volume is d v ok. Now we know all this things let us come to the come to the ah numerical now. If we do the material balance proper mass balance. So, from initially we know that the initially the volume is Q into C t ok. Now we know that the change in volume inside the ah in the in the reactor is Q into C t minus d C t plus R into d v ok.

So, this R into d v is the decrease because of reaction ok. So, we know that decrease because of because of reaction is R into d v and the the the Q into C t minus d c is also something that we need to and here the C t minus d c is what the change in concentration it is right next to the d v that volume in the other side of the disk ok. Anyway I hope you understand now what is this R? R we know it is a reaction rate we can easily ah replaced by minus K into C t K is what reaction rate constant multiplied by the concentration of the pollutant. So, now, if you replace it if you see Q into C t if you see the equation number 1 Q into C t equal to Q into C t minus Q into d into C t Q into d C t right sorry. So, Q C t if you take the right hand side Q C t in the left side it will nullify each other.

So, it will become only now if you take this ah Q d C t in the left side. So, it will become Q d C t equal to R into d v you understand. So, Q d C t equal to R, R is what minus K into C t into d v. So, this ah if you now ah same d C t you take the right hand side the C t from the left hand side it will become d C t by C 1 C t equal to K by Q multiplied into integration of 0 to V of d v and the left hand side integration of C 0 to C e which is like the effluent concentration right. If you remember the the what is the C e C at the end effluent concentration ok the pollutant concentration ok.

So, now, if you integrate it it will become logarithm of C e by C 0 if you remember is equal to minus K into V by Q V by Q means what theta. So, it is minus K theta. So, C e by C 0 it is a logarithm of with the base e. So, that will that becomes the C e become C 0 multiplied by e to the power minus K theta ok. You understand the problem you understand how we come up come to this ah this numbers this this this equation that C e becomes C 0 into e to the power minus K theta please write it down this thing.

So, so that it will be easier for you to you know understand I mean like ah when you will be doing the numerical in the follow up slides. So, let us do the one numerical for that also a plug flow reactor aeration tank of activator sludge process is treating the wastewater having a BOD of 160 milligram per liter. That means, the C 0 the initial concentration is given as 160 milligram per liter at a hydraulic retention time of 5 hour. That means, theta is given hydraulic retention time V by Q which is theta is given as 5 hour. Now, rate of reaction ah is considered to be first order and with the value of reaction rate constant as 0.

4 per hour. That means, K is given 0.4 right 0.4 per hour we have to find out the final effluent concentration is very easy if you know the equation C is C e equal to C 0 into the into e to the power minus K into theta. So, C 0 is known to us 160 milligram per liter K is known to us 0.4 and theta is also known to us 5. So, you can easily find out the C e as 21.65 milligram per liter or say almost equal to 22 milligram per liter.

That means, what will be the final effluent BOD concentration of your plug flow reactor 22 milligram per liter you understand these things. So, with this kind of different kind of reactors how you can ah if you know certain factors how we can actually easily identify easily find out or forecast the rest of the rest of the parameters and all ok. So, this way you can do that ok. Now, let us analyze the ah CSTR reactors ok.

So, continuous tear tank reactors if you remember. So, what happened when it is in series a couple of reactors in series. So, there are it when it becomes ah you know couple of reactors with the different HRT hydraulic retention time it becomes quite off quite quite complex. So, what is the best way of doing it if suppose a you have a group of reactor which is having a same HRT the solution can be very easy like as mentioned in the equation 1. You have to only take care of the initial C 0 like and then and then you just simply do the total ah total HRT and that is it. You put it in the C t equal to C 0 1 plus k into theta that is the total HRT that is it if it is having the same HRT.

However, if it has a 2 if for instance if you have a 2 CSTRs operated in series with the same HRT then equation 2 and equation 3 can predict the effluent concentration effluent pollutant concentration based on the initial concentration and the concentration in the effluent of each CSTR. That means, suppose C 1 is the effluent concentration I mean effluent pollutant concentration for your ah first reactor ok. So, which is equal to C 0 by 1 plus k into theta. Now for C 2 for the next reactor this C 1 becomes the inlet effluent right I mean like the inlet concentration you understand sorry the inlet concentration the C 1 becomes the inlet concentration for reactor 2. So, now, for reactor 2 it becomes C 2 equal to C 1 divided by 1 plus k theta.

Likewise if you continue for C 3 it will become C 2 divided by 1 plus k theta you understand likewise you continue. So, how we can say? So, C 2 if you if you replace this C 1 in C 2 reaction in the third reaction. So, instead of C 1 you can write C 0 by 1 plus k theta. So, C 2 becomes C 0 divided by 1 plus k theta square. Likewise for nth number of n number of CSTR's it will become C n equal to C 0 divided by 1 plus k theta to the power n ok.

So, if you have a 5 number of reactor CSTR reactor what will be the equation C n the fifth the final effluent will be initial effluent concentration divided by 1 plus k theta to the power 5 you understand. So, this is very important ah for us to understand how we can find out the CSTR. Another is when we do it in case of continuous this kind of CSTR type of ah reactor in in series when we do the steady state in steady state condition when we do the material balance ah. So, we have to understand the inflow is we already know it is equal to the outflow plus decrease due to the reactions which can be easily identified by Q into C 0 equal to Q into C 1 plus R 1 into V which is a decrease due to the reaction. Now this minus R 1 you understand it very clearly this minus R 1 could be represented by k into C T to the power n ok.

This n is the order of the reactions it fits the first order reactions it will be k into C T if it is like 2 order reactions second order reaction it will be k into C T square ok like that it will be it will be like that. So, for now if we divide it this equation the last equation equation number 6 ok. If you remember the equation 6 if you write it down Q into C 0 equal to Q into C 1 plus R 1 into V. Now divide all those things all the values with the Q what will happen numerator denominator if you sorry the left hand side and right hand side if you divided by Q what will happen C 0 is equal to C 1 plus R 1 into V by Q V by Q means what theta right very good.

So, now, C 0 is equal to C 1 plus R 1 into theta is not it. So, that you can easily find out the equation number 7 ok and here the how we can find out the slope with this graph you can plot with minus R to the to C T and from there you can easily get the this minus 1 by theta which is nothing, but you know y 1 minus y 0 divided by say like you know x 0 minus x 1. So, like that also we can do that or the either one or like C 0 minus say y 0 minus y 1 divided by x x 1 minus x 0. So, from that you can get the slope and this slopes in we can easily find out from this equation as mentioned in the equation number 9 ok. So, I hope you understand this equations and this the mass balance basics it is very important for you to understand it because it will be helpful for us to you know do it with this knowledge will be very much helpful in for us in the coming lectures ok. So, in general the effluent concentration of the pollutant in a series of CSTR can be calculated using the mass balance equation and the rate constant we have already find out we also find out that graphical methods can be used to estimate the effluent concentration in CSTR in series and the number of CSTR required to meet a specific effluent limit can be determined by solving the mass balance equation for the number of reactors.

Also this effluent concentration of the pollutant in each stage of CSTR can also be estimated using the value of the number of reactors and the volume of reactor volume required for the CSTR to achieve a specific pollutant removal can also be calculated using the mass balance equation and desired removal efficiency can also be reach and all these things not only for CSTR we have done it for batch reactor and as well as for plug flow reactor ok. So, I hope you understand and these are the references I hope it will be very useful for you to go through these references and I really think like you know this particular lecture gives you a very important basics about the mass balance and it is necessary necessity in designing a reactor and to forecast the effluent characteristics or the accumulation scenario inside the reactor for us to understand and design the system well in before just by understanding a very few input parameters ok. So, being an engineer like you know whoever like you know whatever profession you have you it be it will be very fascinating for us I hope for this kind of lectures and you also get to know some new informations. Please go through this lecture again and write it down those equation in a notebook and it will become much easier for you to understand and please try to solve those equations time to time and do the numerical ok. This numericals will be asked definitely during your assignment and also in the final examination.

So, please go through those numericals also which will also make your basics very solid ok. I hope you understand the necessity of mass balance in any kind of reactors that will be using in this in the wastewater treatment plant design and we will see you in the next video. Thank you so much.