Fundamentals of Food Process Engineering
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Lecture – 37
Size Reduction (contd.)

Hello everyone, welcome to the NPTEL online certification course on Fundamentals of Food Process Engineering. We will continue today with the topic of Size Reduction, and we have started this in our last class.

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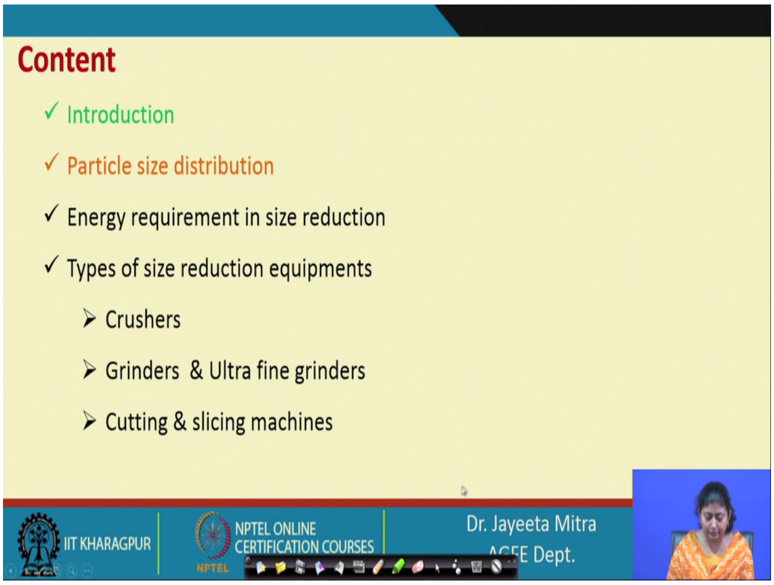

Where we have discussed about the introduction related to the requirement of size reduction the you know the objective of size reduction in the food processing and industrial scale of food processing operation. And we have also learn that how to define a particulate solid, and how to define a single particle in a in a mixture of a bulk solid, what are the different equivalent diameter by that we can express the particles in a bulk mixture.

And also we have learn this concept that there is a need to identify the distribution of particles in the bulk solid mixture, because not a single size is available mostly in case of this, you know this size reduction operation or this bulk solid. So, there is a distribution of different size is available most of the time. And we need to find that what is the predominant size fraction in that mixture. So, we will today discuss in detail the particle size distribution, and also there are other topics in the list of this particular chapter that will cover with due course of time.

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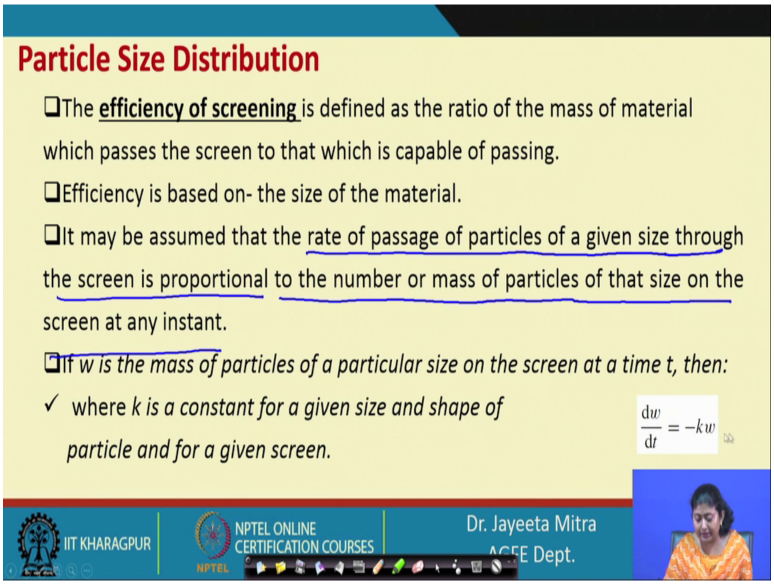

Now, we will start particle size distribution today. So, as in the last class if you remember that there are different kind of, equivalent diameter when we have expressed; that is based on the sieve size opening through which the particle through which the particular diameter of the particle should pass or equivalent diameter particle should pass, or we have seen the based on the settling velocity, or based on the size fraction we can we can differentiate.

So, there are many methods to identify different particle sizes or to find the distribution of particle sizes. The very common one is the microscopic analysis by which the different sizes can be easily separated. There are other methods one is elutriation, settling or sedimentation and there is a laser diffraction; based on that also we define the equivalent diameter. And the most common one and easiest one is the sieve analysis.

And in the sieve analysis also there are many standard sieves are available; one of which is the Taylor series which is the US standard based and in that if we considered the you know distribution based on the sieve analysis. What we do is; we keep the product on the perforated screens or sieves and then there are certain sequence of steps are there that we put a stack of the sieves and then we vibrate it for a certain amount of time.

So, what will happen that the particle sizes which is lower than the equivalent diameter of that perforation of the opening of the perforation will come down to the next lower fraction of the sieve, and which is the bigger one will retain on the upper sieve.

Now, what happened that in most of the time 100 percent efficiency may not be reached. So, some fraction of the particle may retain on the sieves on the upper sieves and some will pass through. So, those will retain on the sieves may have fraction of the particle which is having the size lower than the opening of the screen, but even though they will not pass through the screen. So, what happened that the efficiency of the screening is defined as the ratio of the mass of material, which passes the screen to that which is capable of passing right.

So, some fraction which is capable of passing, but the ratio of the mass which is actually pass through so, that define the efficiency of screening so this is an important parameter in you know understanding the particle size distribution.

So, efficiency is based on the size of the material, what will be the size fraction based on that the efficiency is defined. It may be assume that the rate of passage of the particle of a given size to the screen is proportional to the number or mass of the particle size on the screen at any instant. So, this is important because this will form our basis of analysing the different particle size. So, what it says that it is assumed that the rate of passage of particle of a given size through the screen is proportional to the number or mass of particles of that size on the screen at any instant.

So, if we want to you know express this in terms of a mathematical expression let us consider that w is the mass of particle mass of a particular size. On the screen at any time t then we can write we can write dw by dt that is equal to minus k into; that means, dw by dt will be proportional to the w, that is the mass retained on a sieve on a particular sieve at any instant. And since which time this will reduce we have kept a minus sign here, k is the constant for a given size and shape of the particle and for a given screen. So, this is how the rate of passage of a particle we have we can define.

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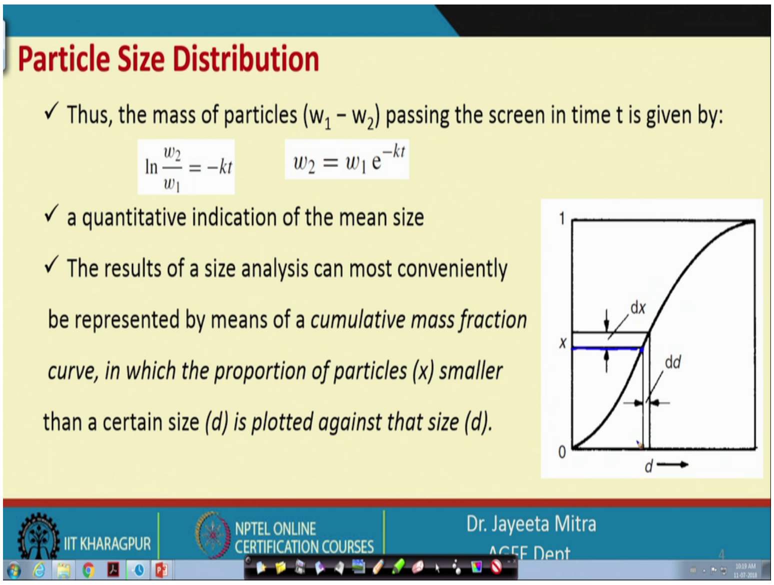

So, then mass of a particle if we considered in this session that is w 1 is the mass retained on a particular size of screen or sieve, at an instant when t equal to 0. And after time t if the mass is a reduced to w 2. So, the mass which is already passed through will be w 1 minus w 2, which is passing the screen at any time. So, if this is happen, this is happening then we can write that ln of w 2 by w 1 that is equal to minus k into t. So that means, it is exponentially reducing the number of the mass fraction of the mass of the material on a particular sieves is reducing in a fashion of exponential decay curve on a particular size of the screen.

So, w 2 that is remaining at any time will be w 1 into e to the power minus k t well k will depend on a particular size of the sieve and the material. Now, quantitative indication of the mean size is important, and quantitative integration is a in the sense that what are the different fraction and what is the mean size I mean the size which is dominating a fraction. So, we need to define the mixture in terms of that particular size and the result of a size analysis can most conveniently be represented by means of cumulative mass fraction. So, cumulative mass fraction curve, in which the proportion of the particle x smaller than a size d is plotted against the size d. So, this is a curve, so this is cumulative mass fraction x is showing the cumulative mass fraction.

So, if we considered that at any instant the x fraction which is smaller than the size d. So, as we move from 0 to the higher size if any instant we reach. So, the fraction which is the left side of that will indicate that the size smaller than that particular size d. So, here this representation is a result of the sieve analysis which is cumulative one. So, we have added the fraction which is larger than this particular size d, so then further we can say that.

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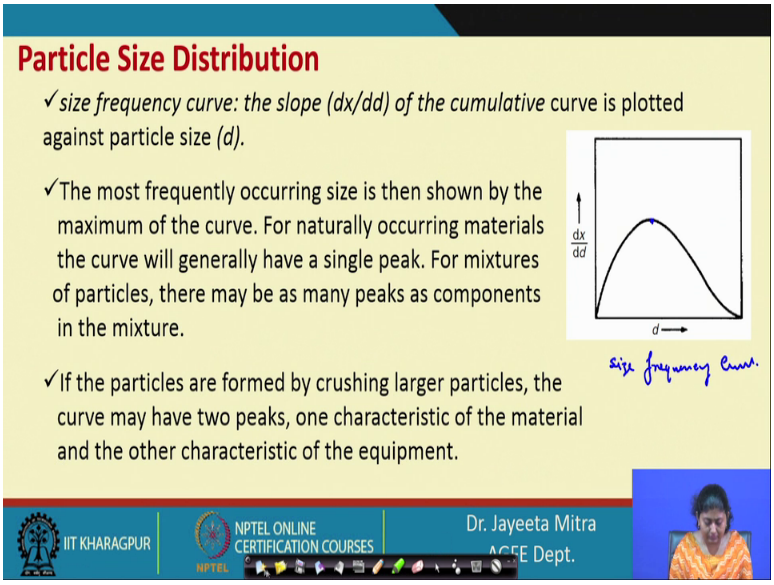

If we take the size frequency curve size frequency in the sense the fraction of having a particular size dd. So, if we take the size frequency curve and then plot with respect to the size d so this is a curve. The slope of dx by did in the slope of the previous curves that we have drawn with respect to the fraction cumulative fraction with the particular size d. So, the slope of that curve when we have plotted with respect to the particular size we are getting this curve which is called the size frequency curve, this is call the size frequency curve.

So, there are many benefit of this curve, this can tell us many thing that; one is the most frequently occurring size is then can be shown by the maximum of this curve. So, the fraction we are getting at maximum points here the frequency is higher; that means, the particle size of this particular fraction will be higher mass fraction in the whole distribution.

So, for naturally occurring material that curve will generally have a single peak and for mixture of the particle there may be as many peak as components in the mixture. So, what happened that when there are different mixtures are therefore, different compositions are there. So, those have different size fraction so it may happen that in one lot one particular size is having dominated. So, we may get different or more than one peak in the frequency curve size frequency distribution curve.

Now, if the particles are formed by crushing larger particles. The curve may have 2 peaks one is the characteristic of the material, because the material has a certain orientation and upon getting a similar kind of food they have a tendency to break in a it in a definite size. So, other is the characteristic of the equipment; so the way the force is coming it will have a tendency to make the particle in a definite kind of shape. So, because of these 2 you have we may get to definite high frequency plot in this frequency curve.

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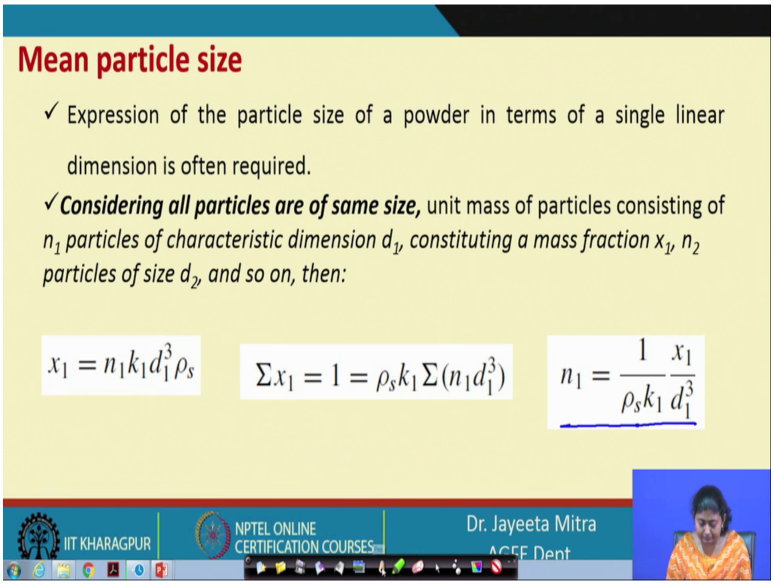

Now, mean particle size so, now we will see that you know we need to we need to express one particular size to express the lot of a distributed bulk particulate solid material. And this expression of the particle size of a powder or granular material, in terms of a single linear dimension is often required.

So, considering that because we have taken the single linear dimension is equivalent diameter most of the time. So, considering all particles are of same size; unit mass of particle consisting of n 1 particle of characteristic dimension d 1. Constituting a mass fraction x 1 similarly n 2 particle of size d 2, constituting the mass fraction of x 2 and so on; is the basis of determining the particular size, because we will assume that the each fraction will have which is passed through letters a particular sieve so; that means, those having lesser size than this particular opening of the screen. So, similarly a particular size linear dimension has been fixed which is dimension d 1, d 2 etcetera. For each section x 1 x 2 etcetera and the number of those particle also is define as n 1 n 2 like that.

So, then we can express that suppose x 1 that one fraction now we want to express which is having the size of d 1. So, x 1 will be n 1 k 1 d 1 cube into rho s. What is this, d 1 is a dimension. So, when we want to express in terms in terms of the volume. So, linear dimension cube we need to express.

So, k 1 into d 1 cube is defining the volume of one particular material, one particular particle. Now n 1 number of particle is there so multiplied with n 1 volume of all the particle n 1 particle into rho s which is the density. So, we are getting the mass fraction of the particle in that. Now, suppose all summation if we do I mean all fraction so summation of x 1.

So, all the fraction will constitute 1 which is the whole mart of the material. And there the rho s and k 1 will be similar rho s is the density k 1 will be the constant parameter which is based on the particular sieve and the material. And summation of n 1 d 1 cube in the sense; there may be different fraction and of different number. So, those are we are summing them here so; that means, n 1 if you want to calculate from this equation n 1 which is the number of particle that will be 1 by rho s k 1 into, rho s k 1 into x 1 refraction divided by d 1 cube. So, this is important that n 1 number of particle in of d 1 dimension in the fraction x 1 will be 1 upon rho s k 1 into x 1 by d 1 cube.

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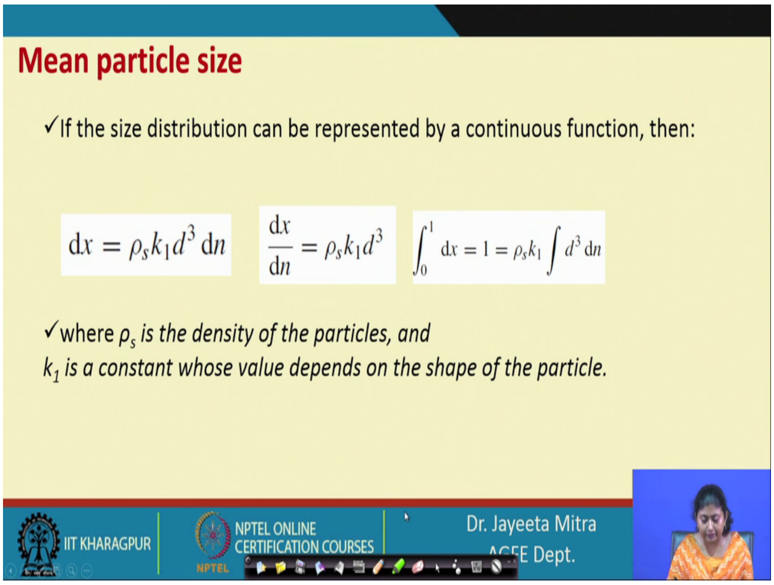

So, now if the size distribution can be represented by a continuous function, then instead of x now we write dx. So, dx the continuous fraction if we take very very small amount of increment in that. So, ds fraction because of the dn number of you know particle. If that fraction increased by ds amount so, that is because of number of dn increase in that size. So, that will be dx equal to rho s into k 1 d cube into dn. So, dx by dn we can write rho s k 1 d cube, and if this dx now we can integrate from fraction 0 to 1. So, when all the fraction will become of that particular size. So, we can write 0 to one dx equal to rho s k 1 into integration d cube dn.

So, rho s is the density of the particle k 1 is depend on it is it is a constant value depends on the shape of the particle. So, this is how we can represent the size distribution in a continuous function.

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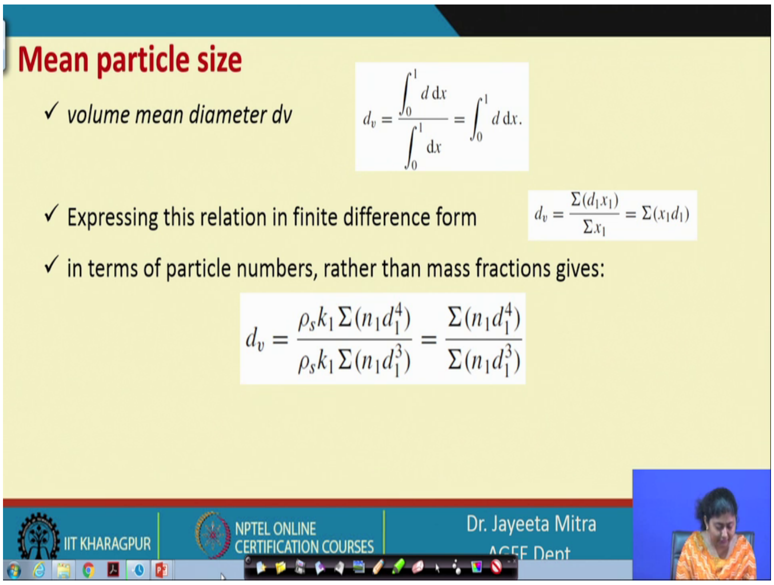

Now different mean particle size we will explain now, first one is the volume mean diameter dv. So, volume mean diameter is expressed as dv equal to integration 0 to 1 d dx divided by 0 to 1 dx that is equal to integration 0 to 1 d into dx. So, d is a particular you know diameter or the dimension and dx is the mass fraction that is increasing that is of particular size d. Expressing this relation in finite difference form we can write dv equal to summation of d 1 x 1 by x 1 summation x 1, that is summation x 1 d 1. So, d 1 is a particular dimension x 1 is the mass fraction of that particular dimension.

Now in terms of particle numbers if you want to express rather than the mass fraction. So, dv will be volume in diameter, so it will be dimension d 1 into that dx fraction that we had. So, dx fraction we remember that x 1 when we try to express the mass fraction x 1. We represented in a way where there was rho s k 1 into n 1 into d 1 cube. So now, with that it will be d 1 to the power 4 because the concept is d into dx by the by integration dx so that will be rho s into k 1 n 1 into d 1 cube. So, finally, we are getting summation of n 1 d 1 to the power 4 by summation n 1 d 1 cube. So, this will sure give you the volume mean diameter dv.

So, all whenever we want to express the particle numbers in the form of the particle number, will recall the previous, you know previous expression and try to relate with that. So, the previous expression was; so for dx you remember this one or for x 1, n 1 k 1 d 1 cube into rho s now when we converted it to dx so that time will convert this to d 1.

And the, this concept we will use every time whenever we try to express in terms of number distribution of the particle right. So, we were in the volume mean diameter. So, this is the expression dv summation n 1 d 1 4 divided by summation n 1 d 1 cube.

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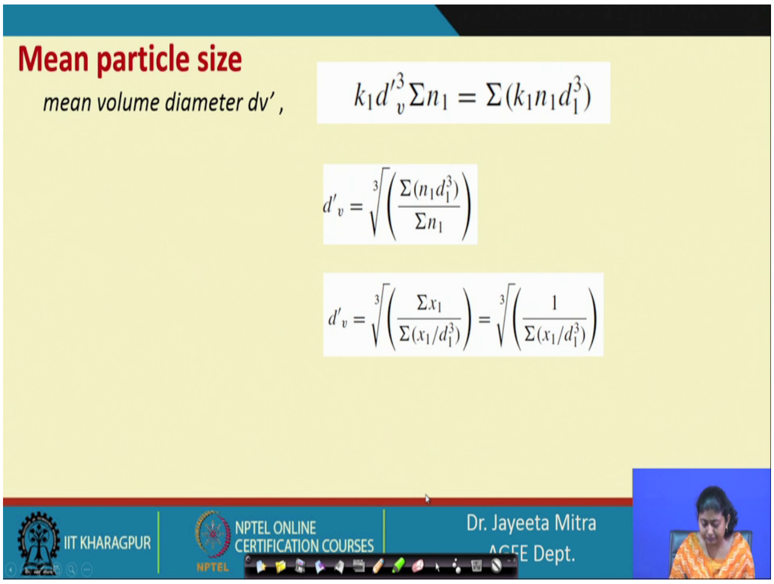

Now, next will be mean volume diameter dv dash which is represented as, this way; suppose we have the dimension dv dash which is the mean volume diameters. So, k 1 into dv dash cube that will be of the you know representing the volume of one particle, multiplied with the summation n 1 that is the all number of particle in that. So, that volume will be equated to all the fractions of the particle, so summation k 1 n 1 d 1 cube.

And finally, we are getting dv dash as cube root over summation n 1 d 1 cube divided by summation n 1. Now, also we can write it in this form of mass fraction basis that is cube root over summation x 1 because x 1 relates the number of particle in to dimension cube, x 1 is the mass fraction and all other thing density and k will be cancelled. So, we can represent this as summation of x 1.

Now, n 1 will be done x 1 by d 1 cube. So, then we can write in cube root over 1 by summation x 1 divided by d 1 cube, because summation x 1 is the combination of all fraction that leads to 1. So, finally, the mean volume diameter dv dash can we represented by this formula; cube root over 1 by summation x 1 by d 1 cube.

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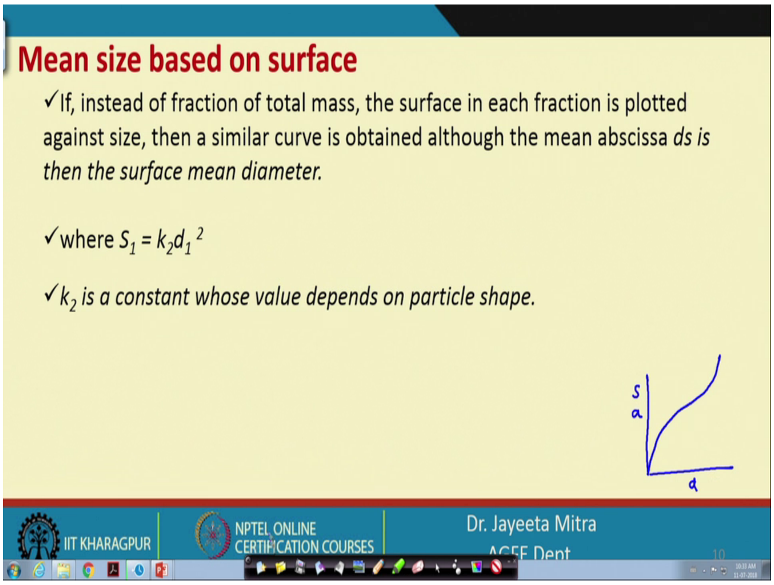

Now, if instead of the fraction of the total mass the surface area in each fraction is plotted against the size. If you remember we have plotted this in you know the particular size d, and the cumulative mass fraction we have taken as you know x and then we have plotted. Now, instead of x we can plot the surface area also, we can plot the surface area a or we can represent it by s etcetera.

So, if we do so, that instead of a fraction of the total mass, if the surface in each fraction is plotted again size. Then similar curve is obtained although the mean abscissa ds is then the surface mean diameter. So, the size d that we have obtained that will be here the surface mean diameter right. So, if we plot the surface area with respect to the diameter of the different particle. So, the mean abscissa ds that will get that will represent the surface mean diameter. Now, what will be the expression for that?

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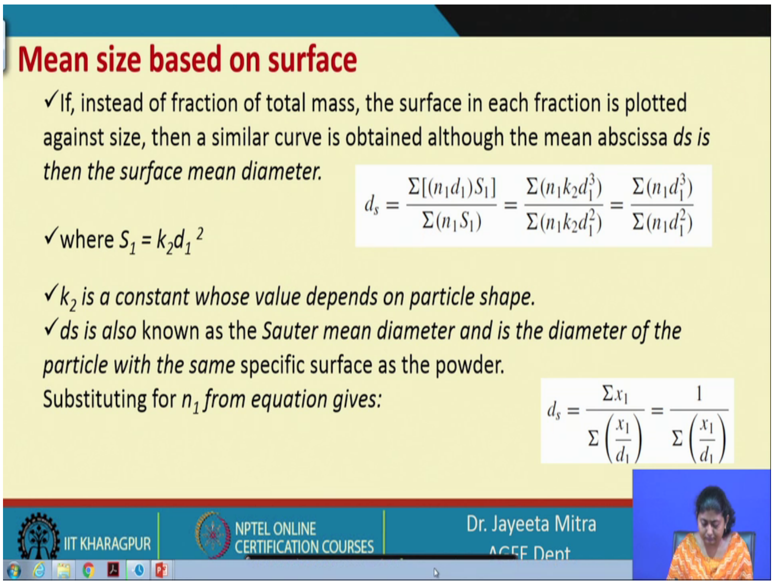

That will be ds equal to summation n 1 d 1 s 1 divided by n 1 into s 1. So, that is equal to n 1 k 2 into d 1 cube divided by n 1 k 2 into d 1 square, because s 1 is what surface area we know surface area will be proportional to square of the linear dimension. So, we can write k 2 into d 1 square and multiplied with d 1 square that will be d 1 cube divided by n 1 into s 1. So, surface area of all the particles at that we are getting.

So, that is why n 1 k 2 into d 1 square that is equal to summation n 1 d 1 cube divided by n 1 d 1 square. So, ideally is that if the surface area is surface area is s 1 a number of particle is n 1 and the uniform surface area is ds. So, that will be equivalent with all the different fraction that is we that we are having n 1 d 1 with s 1.

So, k 2 is a constant whose value depends on the particle shape as it is same for all other cases. And ds is also known as the Sauter mean diameter, and it is the diameter of the particle with the same specific surface as that of the powder, specific surface means the surface area to the volume. So, then it is also represented in the mass fraction basis. So, ds will be summation x 1 which is divided by summation x 1 by d 1, that is summation of x 1 by d 1. So, 1 by summation x 1 by d 1 will be the representation of the Sauter mean diameter.

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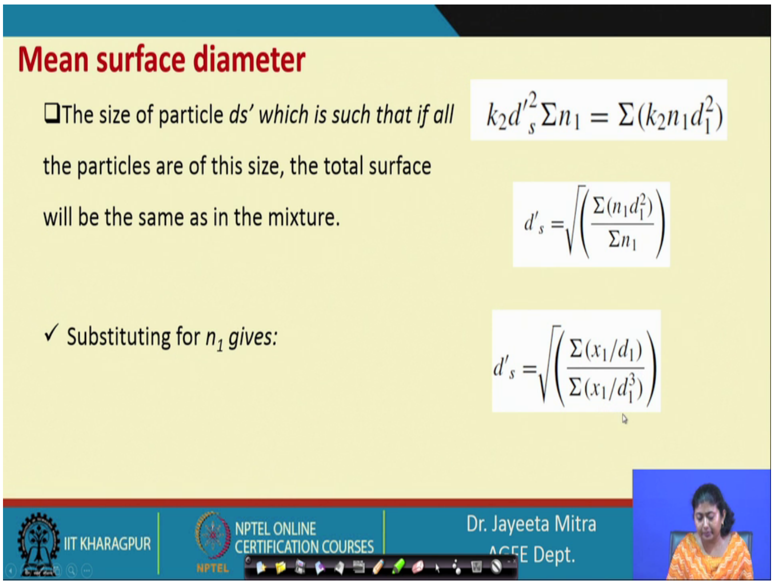

Then the size of the particle ds dash which is; such that if all the particle are of this size the total surface area will be the same as of the mixture. So, in that case k 2 into ds dash square into summation n 1, will be equal to summation of k 2 n 1 d 1 square. So, in that case the ds dash will be root over summation n 1 d 1 square divided by summation n 1, and also if you substitute the n 1 with the value of x 1 there is a mass fraction basis. So, ds dash will be root over summation x 1 by d 1 divided by summation x 1 by d 1 cube.

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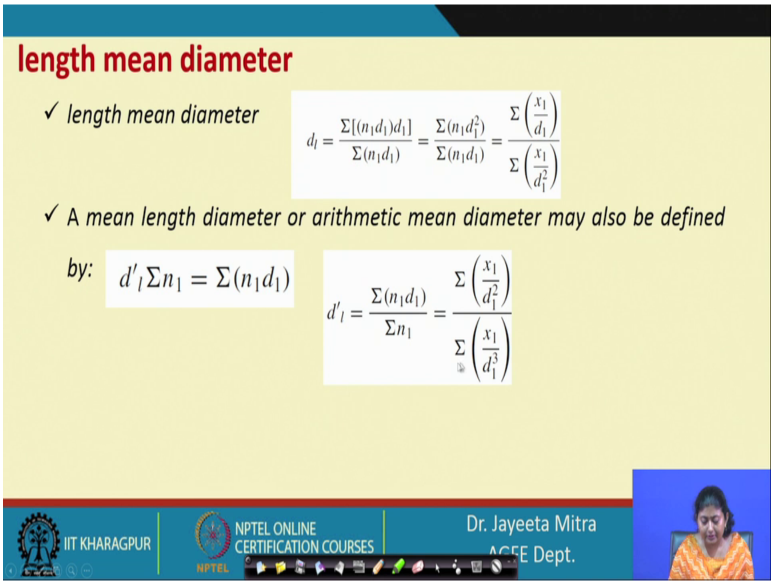

So, there are many representation of this another is the length mean diameter, that is dl represented by x 1 by d 1 divided by summation of x 1 by d 1 square. So, a mean length diameter or arithmetic mean diameter may also be defined by d l dash into summation n 1. So, all the particle having this size will be equal equivalent to summation of all the individual particle n 1 with the size d 1.

So, this may have a different size and different length with equivalent with the size of d l dash which is the arithmetic mean length. In terms of in terms of mass fraction if you want to express them so, that will become summation of n 1 d 1 by summation n 1 or summation x 1 by d 1 square divided by summation x 1 by d 1 cube.

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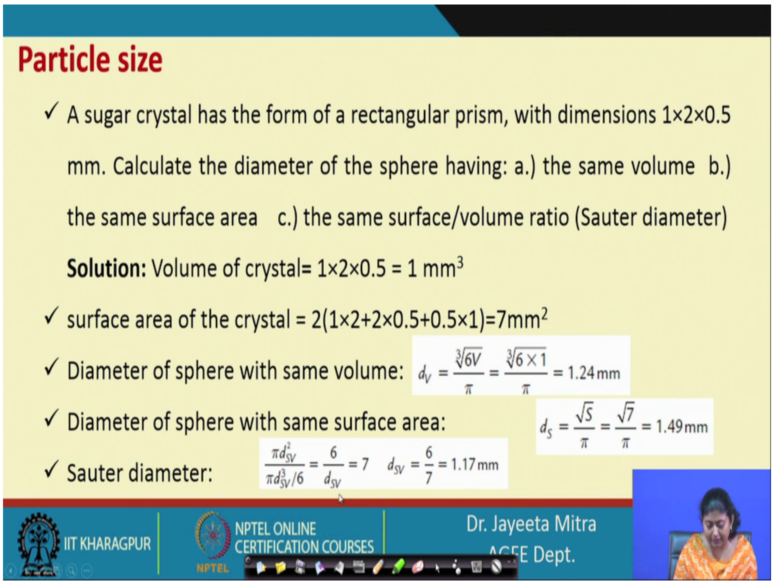

Now, we solve a problem on that a sugar crystal has the form of a rectangular prism with dimension of one into 2 into 0.5 mm, calculate the diameter of the sphere having the same volume, the same surface area, and the same surface to volume ratio which is called the Sauter diameter. So, we have we need to calculate the diameter of the sphere having all such characteristics. So, what we need to do; first volume of crystal that is, we need to calculate 1 into 2 into 0.5 because this is one rectangular one, so we have calculated 1 mm cube.

Now, surface area will be 2 into this dimension 1 into 2 plus 2 into 0.5 plus 0.5 into 1. So, all 3 sides we have taken multiplied with 2 getting 7 mm square this is the surface area. So now, you got the surface area and also got the volume. So, first is that this with this same volume, so same volume if you want to calculate volume 4 by 3 pi r cube. We can convert this to you know pi d cube by 6 in that form. And put the volume here because it should equivalent be given with the volume of this crystal. So, this is 1 so we are getting 1.24 mm cube root over 6 into v by pi so we are getting this.

Second is diameter of a sphere with same surface area. So, surface area is 7 mm square. So, pi d square so, here we can get 1.49 mm. And the last one is the Sauter diameter that is ratio of the surface area to volume. So, we know that surface area we have got 7 volume is 1, so 7 by 1 is the 6 by dsv, so Sauter diameter will be 1.17 mm.

So, we can also try to identify that which area is coming minimum, which dimension is coming minimum if we take the different specific equivalent diameter. So, what is the minimum diameter in which case it is coming that we can calculate.

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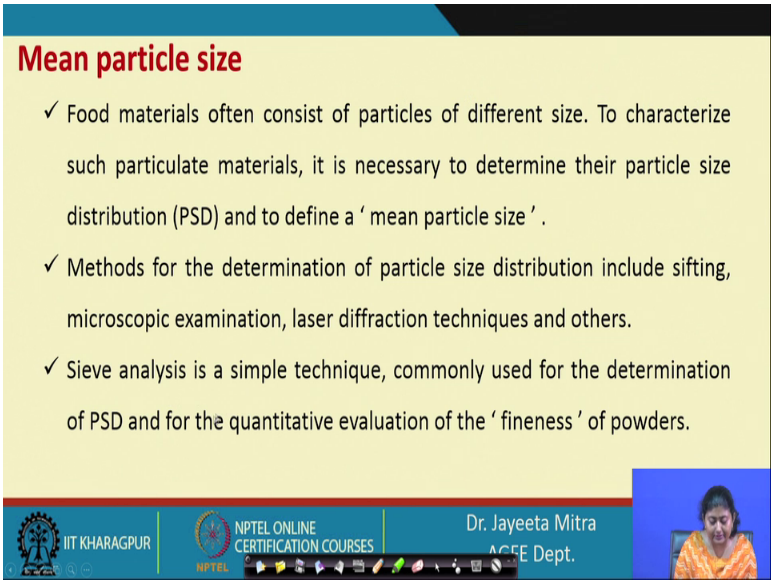

Now, food material of an consists of different size, to characterize such particulate material it is necessary to determine their particle size distribution as we are doing now. So, method for determination of particle size distribution include many that we have discussed already. Out of that sieve analysis is the simple technique commonly used for the determination of particle size distribution of the quantitative evaluation of the fineness of the powder.

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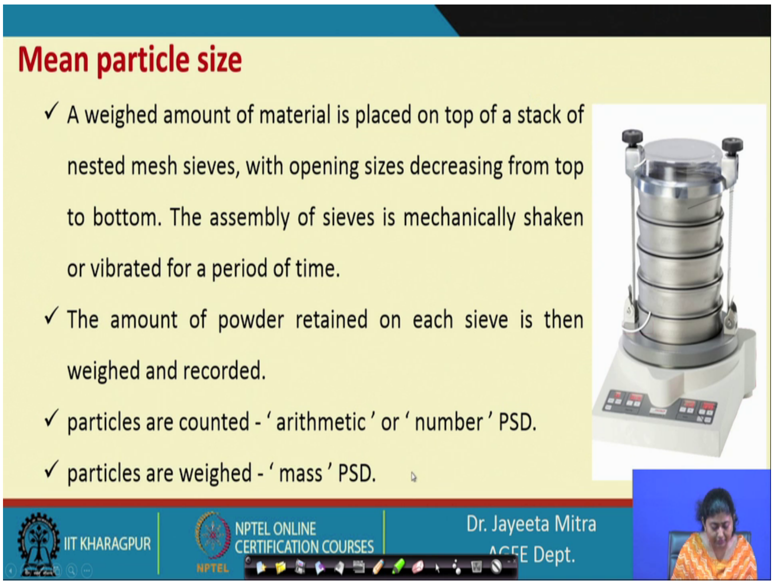

So, this is the sieve analysis set look like, a weighed amount of the material is placed on top of a stack of the nested mesh sieve; with opening size decreasing from top to bottom. And generally this sizes changes with a power of 2 sometime root 2 or 2 to the power 1 by 4 like that. And the assembly of sieve is mechanically shaken or vibrated for a period of time.

The amount of powder retain on each sieve is then weighed and recorded. So, particles are counted based on arithmetic or number mean distribution or also the mass mean distribution we can calculate.

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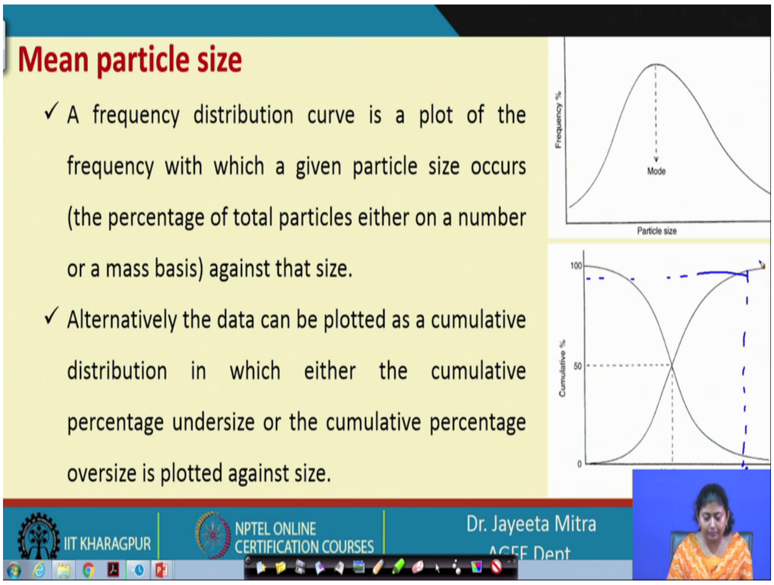

Then a frequency distribution curve as I mentioned that the dx by dd with particular size is plotted. And which give the particular particle size the percentage of the total particle either on number or mass basis against that size so, that we can identify that which is the mode of the distribution. Alternatively, that data can be plotted as a cumulative distribution, in which either the community percentage undersize or the cumulative percentage oversize is plotted against the size.

So, if we see that this particle sorry, so here we can see that; when we see this curve so we are getting 50 percent of the size lower than this particular. Or if we this one is the undersize and if we go by this curve so we will get the percentage oversize of that particular material. So, like that for each size if we suppose get this one; that means, suppose we get this size so; that means, this much fraction has already passed through this particular diameter opening so that way we can analyse.

So, both the way cumulative mass fraction and frequency distribution is done. And from this we can get very good insight of you know distribution of the particle, in terms of how many fractions are dominating, which is the maximum size, which is the mean surface you know diameter? So many conclusion we can take out of this particle size analysis, particle size distribution analysis right. So, we will stop here and we will continue in the next class.

Thank you.