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Week – 09
Lecture - 52
Multi-Criteria Decision-Making for Natural Resources Management: Part-02

So continuing MCDA for natural resource management part 2. So just in part 1 we have discussed about various way that we can actually analyze and find out our best alternative options giving different weightage to different criterias. We tried with two three methods of MCDA like WSM, WPM and WASPASS and we also saw that how in case of beneficial and non-beneficial case, we can identify the best possible alternative for us.

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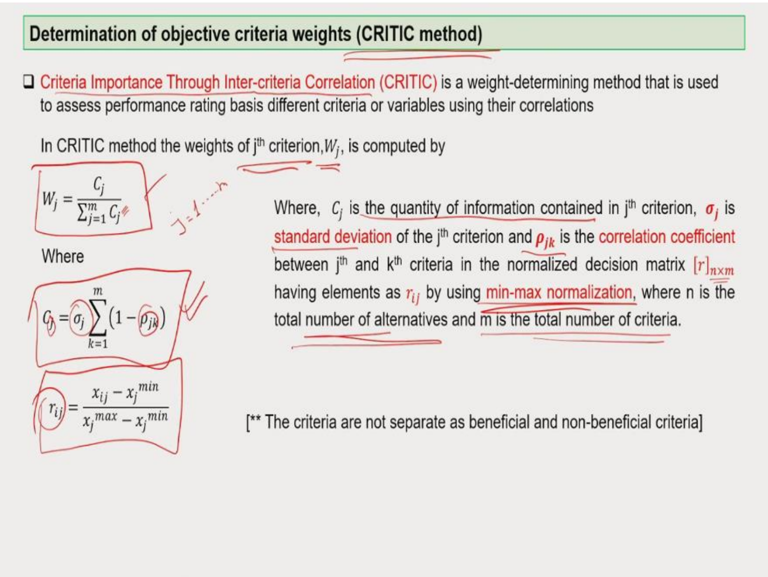

Now here today we will try to see that how to determine the objective criteria weights. We call it as CRITIC method. CRITIC method in full we call it as criteria importance through inter criteria correlation. Its bit long name, this is a weight determining method. See you remember that in previous lecture we discussed that we tried with two different way in case one example we have given with same weight like 0.25 to C1, C2, C3, C4, four different criteria, in the other example we tried that four criterias with different four different weightage.

Here we are talking that that this CRITIC method is a weight determining. How much actually the weight that you are going to give that we determine and that is used to assess the performance rating on the basis of different criteria or variables that we discussed in the previous lecture. Now, in CRITIC method the weights of jth criterion say Wj is computed by this manner

Wj is equal to Cj divided by summation j equal to 1 to m Cj

So Cj this j value could be from 1 to m any value where Cj you can calculate again this way.

Cj is equal to sigma j multiplied by summation k equals 1 to m (1 minus rhojk)

And for another calculation, small calculation we do for rij Now let us see that which one means what, what is Cj? Cj is the quality of information content in the jth criteria sigmaj is the standard deviation of the jth criteria, Cj and rhojk is the correlation coefficient between j and kth criteria, matrix r m cross n having elements as rij as you see here this normalized decision matrix you actually use minimum-maximum normalizations where n is the total number of alternatives and m is the total number of criteria. So this is the way you actually try to find out that how much weightage that you could give to a particular criteria.

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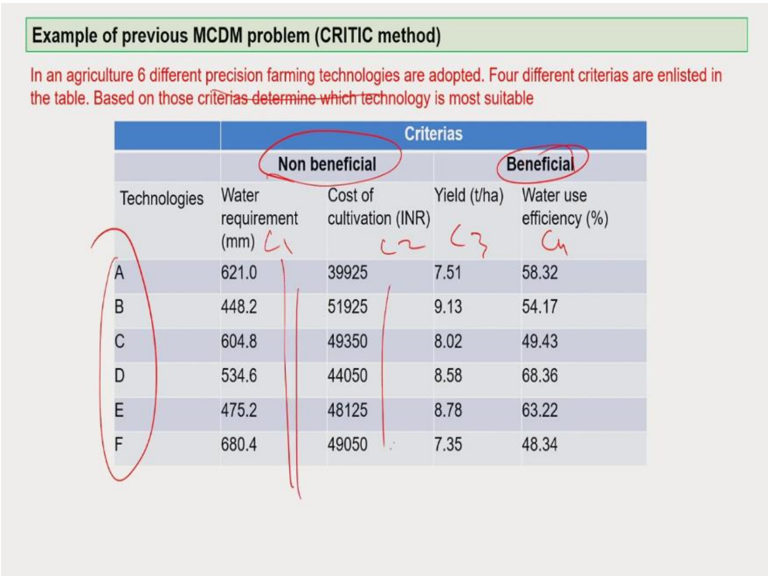
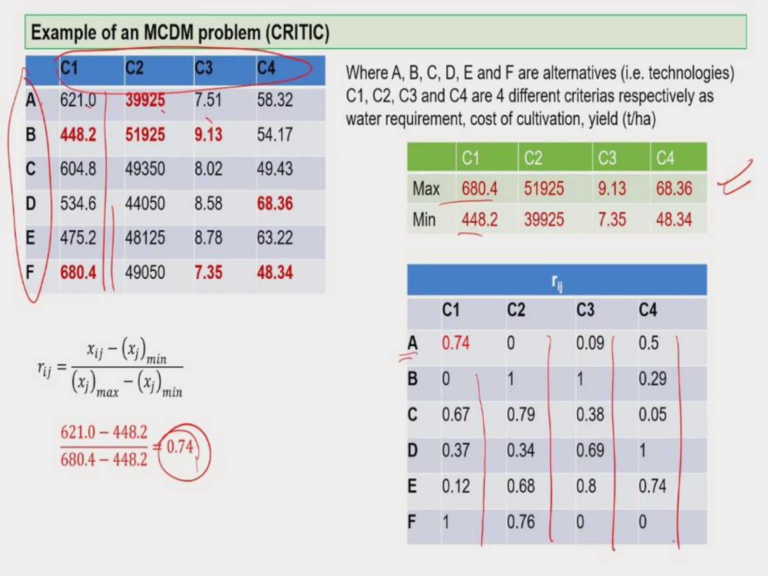


Example of previous MCDM problem (CRITIC method)

In an agriculture 6 different precision farming technologies are adopted. Four different criterias are enlisted in the table. Based on those criterias determine which technology is most suitable

	Criterias			
	Non beneficial		Beneficial	
Technologies	Water requirement (mm)	Cost of cultivation (INR)	Yield (t/ha)	Water use efficiency (%)
A	621.0	39925	7.51	58.32
B	448.2	51925	9.13	54.17
C	604.8	49350	8.02	49.43
D	534.6	44050	8.58	68.36
E	475.2	48125	8.78	63.22
F	680.4	49050	7.35	48.34



Now let us go to the example. Suppose in an agriculture again six different farming precision farming technologies are adopted, four different criteria C1, C2, C3, C4, similar example like we discussed in the previous lecture based on these criterias. Let us determine which of these technologies is most suitable for my area, this example might help you to get this much more clearly.

So again non beneficial lower the value better for us, beneficial higher the value better for us you have these are the values you calculated, now let us see. Here you see that these are your alternatives, these are your criteria. Now you have certain values that you have calculated for each alternatives against each criteria where A, B, C, D are different technologies and C1 to C4 different criteria.

Now, these values again you try to put down as maximum-minimum which is very easy you find out from the table, now maximum-minimum normalization you will do you got 0.74 for alternative A, for alternative B, C, D you got all for each criteria then it becomes again easy for you to rank them.

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Example of an MCDM problem (CRITIC)

r_{ij}

	C1	C2	C3	C4
A	0.74	0	0.09	0.5
B	0	1	1	0.29
C	0.67	0.79	0.38	0.05
D	0.37	0.34	0.69	1
E	0.12	0.68	0.8	0.74
F	1	0.76	0	0

	C1	C2	C3	C4
σ_j	0.39	0.36	0.40	0.39

Correlation (ρ_{jk})

	C1	C2	C3	C4
C1	1	-0.33	-0.98	-0.51
C2	-0.33	1	0.44	-0.51
C3	-0.98	0.44	1	0.46
C4	-0.51	-0.51	0.46	1

$1 - \rho_{jk}$

	C1	C2	C3	C4
C1	0	1.33	1.98	1.51
C2	1.33	0	0.56	1.51
C3	1.98	0.56	0	0.54
C4	1.51	1.51	0.54	0

Determination of objective criteria weights (CRITIC method)

- Criteria Importance Through Inter-criteria Correlation (CRITIC) is a weight-determining method that is used to assess performance rating basis different criteria or variables using their correlations

In CRITIC method the weights of j[th] criterion, W_j, is computed by

$$W_j = \frac{C_j}{\sum_{j=1}^{m} C_j}$$

Where

$$C_j = \sigma_j \sum_{k=1}^{m} (1 - \rho_{jk})$$

$$r_{ij} = \frac{x_{ij} - x_j^{min}}{x_j^{max} - x_j^{min}}$$

Where, C_j is the quantity of information contained in j[th] criterion, σ_j is standard deviation of the j[th] criterion and ρ_{jk} is the correlation coefficient between j[th] and k[th] criteria in the normalized decision matrix $[r]_{n \times m}$ having elements as r_{ij} by using min-max normalization, where n is the total number of alternatives and m is the total number of criteria.

[** The criteria are not separate as beneficial and non-beneficial criteria]

Example of an MCDM problem (CRITIC)

	C1	C2	C3	C4
A	621.0	39925	7.51	58.32
B	448.2	51925	9.13	54.17
C	604.8	49350	8.02	49.43
D	534.6	44050	8.58	68.36
E	475.2	48125	8.78	63.22
F	680.4	49050	7.35	48.34

Where A, B, C, D, E and F are alternatives (i.e. technologies) C1, C2, C3 and C4 are 4 different criterias respectively as water requirement, cost of cultivation, yield (t/ha)

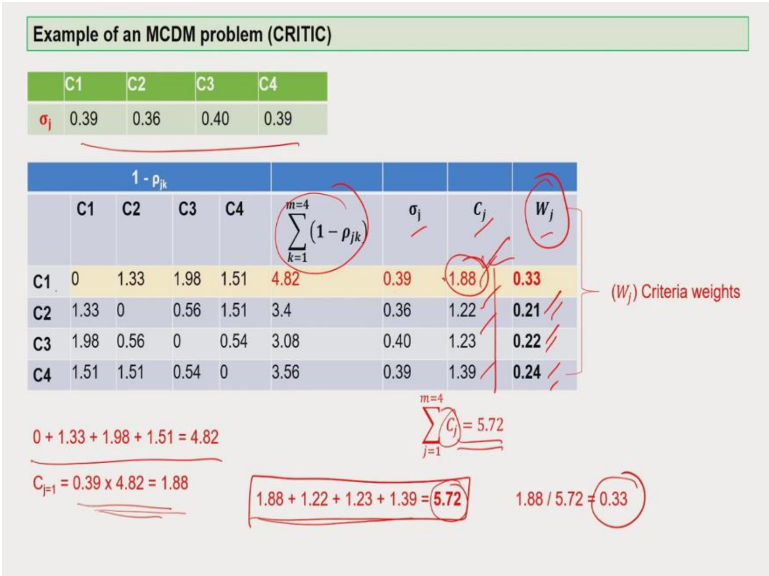
	C1	C2	C3	C4
Max	680.4	51925	9.13	68.36
Min	448.2	39925	7.35	48.34

$$r_{ij} = \frac{x_{ij} - (x_j)_{min}}{(x_j)_{max} - (x_j)_{min}}$$

$$\frac{621.0 - 448.2}{680.4 - 448.2} = 0.74$$

r_{ij}

	C1	C2	C3	C4
A	0.74	0	0.09	0.5
B	0	1	1	0.29
C	0.67	0.79	0.38	0.05
D	0.37	0.34	0.69	1
E	0.12	0.68	0.8	0.74
F	1	0.76	0	0



So this is your rij value, weightage value, then you go for correlation that is rhojk so C1, C1 of course you have 100 percent correlation like that you will get the correlation matrix then you calculate 1 minus rhojk value which is again you get this table, so basically what you try to do, you are trying to find out all these factors to calculate the final value for your identifying the best technology.

Now here rij you have xij value which is 621 for alternative A for criteria 1 then xj value you have 442 jth alternative means second alternative then you have 680.4 for the last alternative that you have minus again the value of xj so maximum minus minimum, maximum minus minimum, so here you are now calculating this value of rij which you got in this table.

Now, this will allow you to go towards your sigma j or your standard deviation value which is 0.39, 0.36, 0.4 and 0.39 for four criterias, now basically you get all the values from this calculation from the table and then comes your SD value and then 1 minus rhojk that below that this table that you have got.

So, finally you get for criteria, one table for criteria weightage that how much actually weightage that you are going to give for which criteria so for C1 then we find that the weightage that we need to give is, you calculate on the basis of these values that you have already calculated.

Now let us calculate the weightage that we are going to give Cj your value finally comes 5.72 how you get that, you calculate each one of on the basis of your criteria so C1, C2, C3, C4 you get each one of them a value, now for all these criterias, four criterias you got four different value 1.88, 1.22, 1.23, 1.39 the Cj values then you add them up you get 5.72 so now you are trying to find out the weightage so for C1 you actually divide C1 the Cj value divided by the summation of Cj s.

So that actually we saw in case of the formula that we have shown discussed earlier here in this particular slide. So basically you are trying to find out that for each criteria how much weightage that you are going to give weightage is Wj so for C1 your weightage comes 0.33, for C2, 0.21, 0.22, 0.24 like that. Now you got the weightage for each criteria once weightage is there then you go back in the previous lecture that I mentioned unequal weightage, so for C1, C2, C3, C4 you get all this unequal weightage and then of course it is easy to calculate for WSM or WPM whichever method you actually goes to use.

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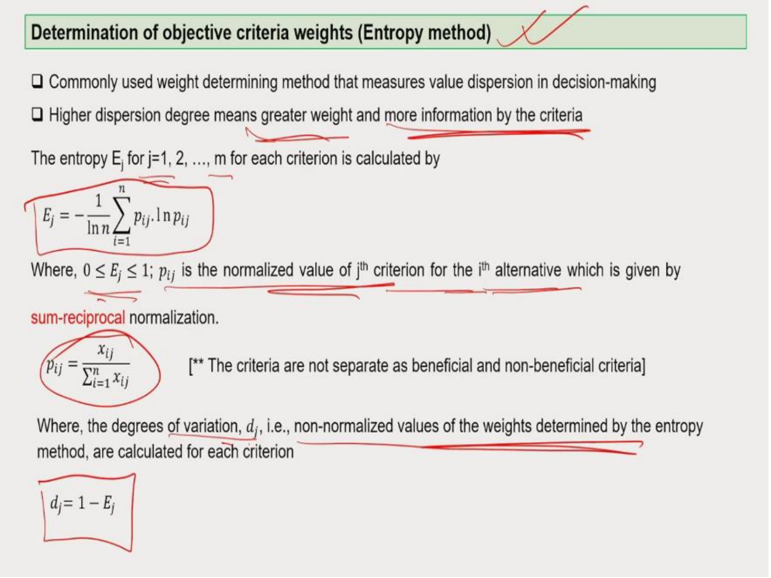

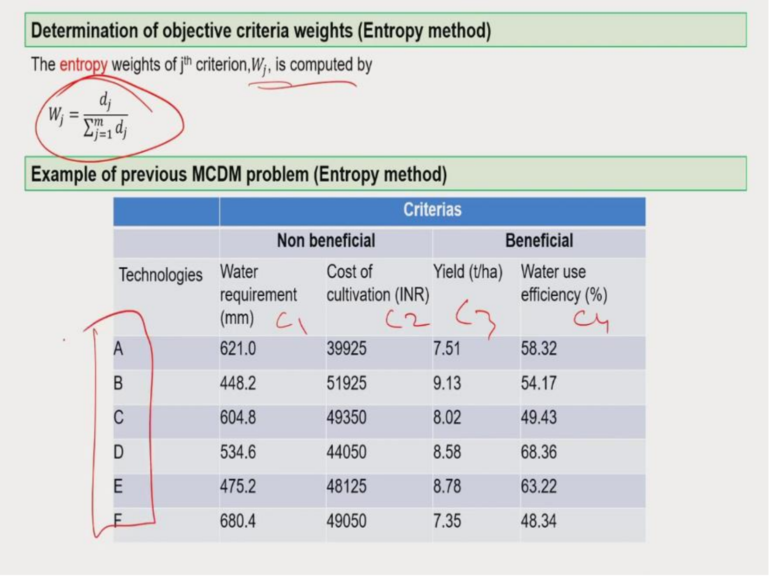

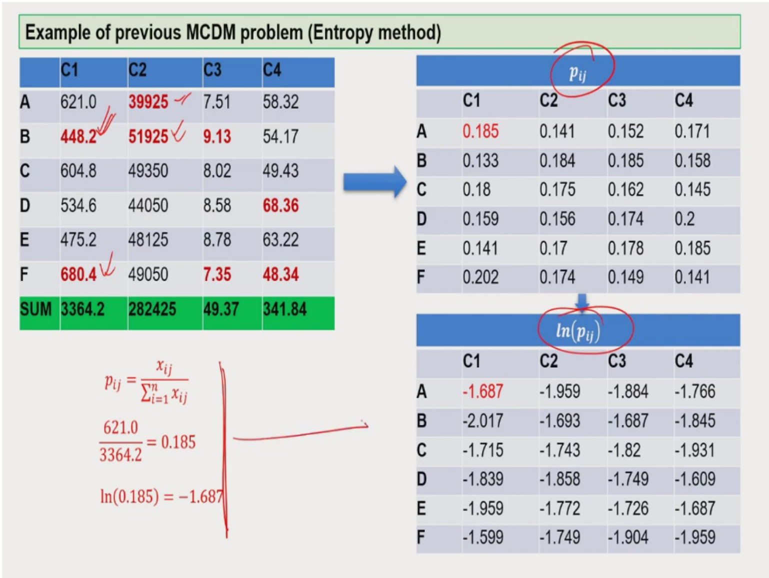

Now for coming to find out the weightage criteria weightage an entropy method is also used for determining the weightage of your criteria. So higher dispersion degrees means greater weight and more information by the criteria. The entropy Ej for j is equal to 1 to m for each criteria is calculated by this formula.

Ej is equals to minus 1 by ln of n multiplied by summation 1 equal to 1 to n pij into ln of pij

So you calculate the entropy Ej where the Ej value lies between 1 to 0 pij is the normalized value of jth criteria for the ith alternative which is given by some reciprocal normalization, you add up and normalize it.

So pij again can be calculated in this way now where dj stands for degrees of variation that is nonnormalized value of the weight which are determined by the entropy methods and this dj you can calculate by dj is equal to 1 minus entropy value of that jth criteria. So this is the way that we can actually also calculate the entropy value and this entropy weights of jth criteria which is Wj is can be computed by this formula.

So once you know dj then you can actually can calculate also the Wj, so example for this particular entropy method again you have C1, C2 same set of criteria and technologies then you calculate all the values then you go for the maximum and minimum identification, as you got it here maximum minimum like that for all the criteria you can easily calculate and then you go for pij calculation and then you go for log normalizations. After that basically you plot this table, this is your log normalization methodology, from here you get these values.

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Example of previous MCDM problem (Entropy method)

p_{ij}	C1	C2	C3	C4
A	0.185	0.141	0.152	0.171
B	0.133	0.184	0.185	0.158
C	0.18	0.175	0.162	0.145
D	0.159	0.156	0.174	0.2
E	0.141	0.17	0.178	0.185
F	0.202	0.174	0.149	0.141

$ln(p_{ij})$	C1	C2	C3	C4
A	-1.687	-1.959	-1.884	-1.766
B	-2.017	-1.693	-1.687	-1.845
C	-1.715	-1.743	-1.82	-1.931
D	-1.839	-1.858	-1.749	-1.609
E	-1.959	-1.772	-1.726	-1.687
F	-1.599	-1.749	-1.904	-1.959

$p_{ij}.ln(p_{ij})$	C1	C2	C3	C4
A	-0.312	-0.276	-0.286	-0.302
B	-0.268	-0.312	-0.312	-0.292
C	-0.309	-0.305	-0.295	-0.28
D	-0.292	-0.29	-0.304	-0.322
E	-0.276	-0.301	-0.307	-0.312
F	-0.323	-0.304	-0.284	-0.276

0.185 x (-1.687) = -0.312

Example of previous MCDM problem (Entropy method)

	C1	C2	C3	C4
A	621.0	**39925**	7.51	58.32
B	**448.2**	**51925**	**9.13**	54.17
C	604.8	49350	8.02	49.43
D	534.6	44050	8.58	**68.36**
E	475.2	48125	8.78	63.22
F	**680.4**	49050	**7.35**	**48.34**
SUM	3364.2	282425	49.37	341.84

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}}$$

$$\frac{621.0}{3364.2} = 0.185$$

$$\ln(0.185) = -1.687$$

p_{ij}	C1	C2	C3	C4
A	0.185	0.141	0.152	0.171
B	0.133	0.184	0.185	0.158
C	0.18	0.175	0.162	0.145
D	0.159	0.156	0.174	0.2
E	0.141	0.17	0.178	0.185
F	0.202	0.174	0.149	0.141

$ln(p_{ij})$	C1	C2	C3	C4
A	-1.687	-1.959	-1.884	-1.766
B	-2.017	-1.693	-1.687	-1.845
C	-1.715	-1.743	-1.82	-1.931
D	-1.839	-1.858	-1.749	-1.609
E	-1.959	-1.772	-1.726	-1.687
F	-1.599	-1.749	-1.904	-1.959

Example of previous MCDM problem (Entropy method)

$p_{ij}.ln(p_{ij})$	C1	C2	C3	C4
A	-0.312	-0.276	-0.286	-0.302
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C	-0.309	-0.305	-0.295	-0.28
D	-0.292	-0.29	-0.304	-0.322
E	-0.276	-0.301	-0.307	-0.312
F	-0.323	-0.304	-0.284	-0.276
SUM	-1.78	-1.788	-1.788	-1.784
E_j	0.993	0.998	0.998	0.995
d_j	0.007	0.002	0.002	0.005
W_j	0.438	0.125	0.125	0.313

$$E_j = -\frac{1}{\ln n}\sum_{i=1}^{n=6} p_{ij}.\ln p_{ij} \qquad n = 6$$

$$d_j = 1 - E_j$$

Example, 1 – 0.993 = 0.007

$$W_j = \frac{d_j}{\sum_{j=1}^{m=4} d_j} \qquad m = 4$$

$$\sum_{j=1}^{m=4} d_j = 0.007 + 0.002 + 0.002 + 0.005 = 0.016$$

$$\frac{0.007}{0.016} = 0.438$$

	C1	C2	C3	C4
W	0.438	0.125	0.125	0.313

(Nearly)

Now once you got this pij table and log normalization then you come here in this table from this table we go for the next is pij into log of pij which is very easy for each criteria actually you will multiply these two values so for each criteria you multiply these two value you get another value which actually is your entropy value.

So now for each criteria you will get ultimately a set of value and then you calculate the sum. Sum normalizations you have Ej with you, dj and Wj you can calculate here in this methodology which we also earlier in other example we have showed how you can calculate Ej entropy dj and also weightage Wj value.

So once these are done then actually you try to calculate the summation value of dj s which is again divided by the individual values of dj of each criteria then you get a value of 0.438 which is the weightage of that particular criteria C1. So for C1 you have dj value this and the summation of all the dj values under criteria C1 is 0.016. So that means 0.007 that is the dj value of the entire set of alternatives for C1 divided by the summation value you get the weightage for C1 and this is actually again you get it by another method which we call as entropy method.

Basically what we are trying that just to confirm that that whatever we are choosing is actually almost near to the reality, we are not making any mistake. So you can actually try choosing the alternatives by more than one method also to cross check whether you reach to the same conclusions.

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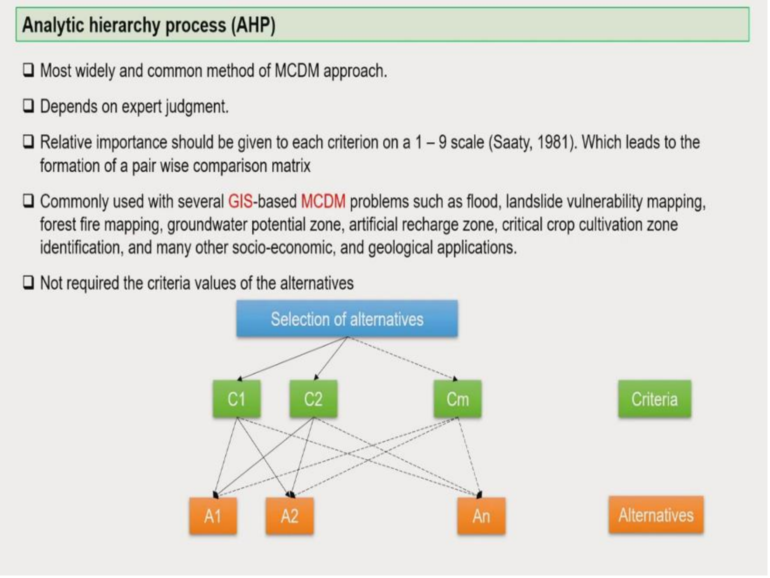

Analytic hierarchy process (AHP): previous example problem

❑ Scale of relative importance is 1, 3, 5, 7, 9 and intermediate values are 2, 4, 6, 8

Index	1	3	5	7	9
Importance	Equal	Weak	Strong	Demonstrated	Absolute

C1	C2	C3	C4
Water requirement (mm)	Cost of cultivation (INR)	Yield (t/ha)	Water use efficiency (%)

Pairwise comparison matrix				
	C1	C2	C3	C4
C1	1	3	7/9	5/3
C2	1/3	1	4/7	2
C3	9/7	7/4	1	5
C4	3/5	1/2	1/5	1

→

Pairwise comparison matrix				
	C1	C2	C3	C4
C1	1	3	0.778	1.667
C2	0.333	1	0.571	2
C3	1.285	1.75	1	5
C4	0.6	0.5	0.2	1

Analytic hierarchy process (AHP): previous example problem

Pairwise comparison matrix				
	C1	C2	C3	C4
C1	1	3	0.778	1.667
C2	0.333	1	0.571	4
C3	1.285	1.75	1	5
C4	0.6	0.25	0.2	1
SUM	3.218	6.25	2.549	9.667

→

	C1	C2	C3	C4
C1	0.311	0.48	0.305	0.172
C2	0.103	0.16	0.224	0.207
C3	0.399	0.28	0.392	0.517
C4	0.186	0.08	0.078	0.103

1 / 3.218 = 0.311

	C1	C2	C3	C4	W
C1	0.311	0.48	0.305	0.172	**0.317**
C2	0.103	0.16	0.224	0.207	**0.174**
C3	0.399	0.28	0.392	0.517	**0.397**
C4	0.186	0.08	0.078	0.103	**0.112**

(0.311 + 0.48 + 0.305 + 0.172) / 4 = 0.317

←

$W_j \times C_{ij}$				
W	0.317	0.174	0.397	0.112
	C1	C2	C3	C4
C1	0.317	0.522	0.309	0.187
C2	0.106	0.174	0.227	0.224
C3	0.407	0.305	0.397	0.56
C4	0.19	0.087	0.079	0.112

Ex. 0.317 x 1 = 0.317, 0.317 x 0.333 = 0.106 etc.

Analytic hierarchy process (AHP): previous example problem

$W_j \times C_{ij}$							
W	0.315	0.209	0.378	0.098			
	C1	C2	C3	C4	SUM	W	SUM / W
C1	0.317	0.522	0.309	0.187	**1.335**	**0.317**	4.211
C2	0.106	0.174	0.227	0.224	**0.731**	**0.174**	4.201
C3	0.407	0.305	0.397	0.56	**1.669**	**0.397**	4.204
C4	0.19	0.087	0.079	0.112	**0.468**	**0.112**	4.179

0.317 + 0.522 + 0.309 + 0.187 = 1.335

1.335 / 0.317 = 4.211

Consistency index (CI)

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

n = Number of criteria
n = 4 (Here)

$$CI = \frac{4.199 - 4}{4 - 1} = 0.0.066$$

$$\lambda_{max} = \frac{4.211 + 4.201 + 4.204 + 4.179}{4}$$

$$\lambda_{max} = 4.199$$

Random index (RI)

n	1	2	3	4	5	6	7	8	9	10
RI	0.0	0.0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Consistency Ratio (CR) < 0.1

$$CR = \frac{CI}{RI} = \frac{0.066}{0.9} = 0.073 < 0.1$$

Hence

C	C1	C2	C3	C4
W	0.317	0.174	0.397	0.112

Now, AHP we have discussed in the introductory lecture of MCDM, so I am not going to repeat much here. So AHP method is one of the most commonly used method and here in analytical hierarchy process we give relative importance of values to each of the criteria and now suppose your criteria again you have four criteria here how you are going to actually calculate AHP.

Here we go for pairwise comparison matrix which we discussed earlier. So for pair wise comparison matrix you will get a matrix with value like this and from there you will go for calculation of your different weightage and once you get the weightage then you go for finally you plot them and you get your ranking. Ranking for each one of the criteria so these are the basic weightage that you have got for different criteria through different processes that I have mentioned earlier.

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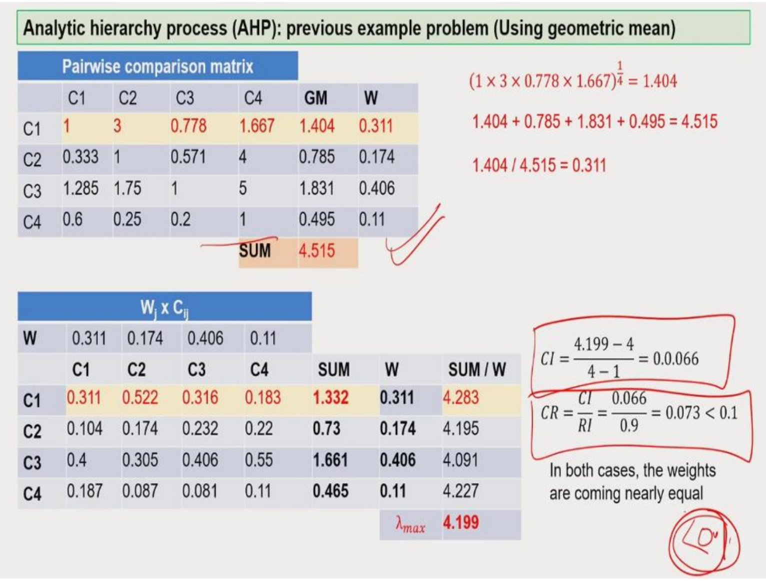

And then once you get the value of weightage of each criteria you put them in a matrix and from this matrix you get now the value of individual criteria as well as total sum of their criteria values, so once up it is done then you calculate the CI and CR value if you again recall the introductory lecture we mentioned about this, this you can calculate easily and CR values it should be less than 0.1 because this is the value that we considered as acceptable in case of AHP.

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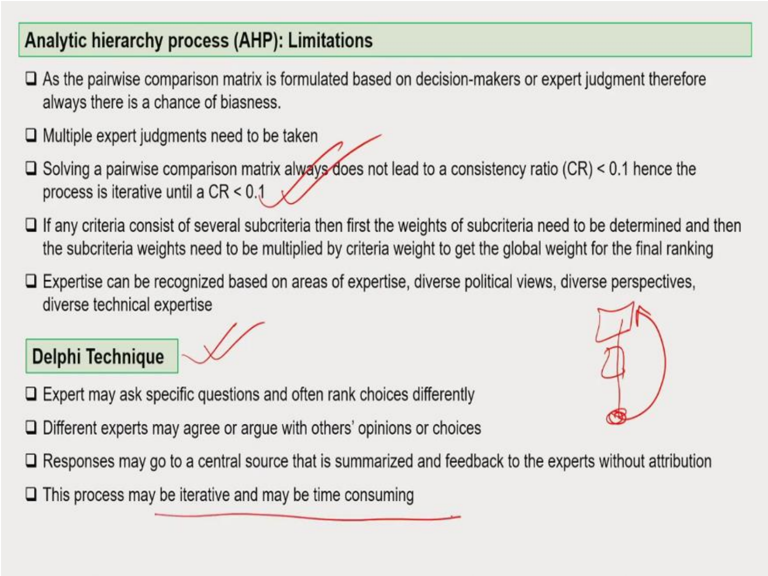

So AHP though I said that it is a very commonly used methodology for multiple criteria decision analysis but it has certain limitation. You see that when you do the pair wise comparison then you will find the decision-making process or expert judgment there are always a chance of certain amount of biasness in the pairwise comparison matrix. Multiple expert will give for one particular criteria different values or weightage.

So, solving a pair wise comparison matrix always does not may lead to a consistency ratio less than 0.1. Hence the process is iterative means repeating you have to continue repeating giving different values and unless until you get rich CR value less than 0.1, that is one of the major aspect of AHP. If you find if any criteria consists of suppose several sub-criteria then we first give the weightage to sub criteria and then the sub-criteria weightage need to be multiplied by the weight of the criteria to get the global weight for the final ranking.

So, sub-criteria weightage you first give and then you multiply it with the main criteria, this is how we go for final criteria weightage ranking. So expertise sometime as I said that can be diverse in different political views, it can have diverse perspectives and that is why sometime in AHP utilizing experts suggestion comments could be a time little bit difficult.

So Delphi technique is one which actually is a kind of a feedback taking process and then again you send it back to the people so two to three times of this iteration takes place. Expert may ask some specific questions different experts may agree or argue with others opinion, responses sometime may go to the central source that is summarized and feedback of the experts without any attribution.

So that means from this process you come down and here after synthesis it can go back again to the responder so that means you check it one or two times to avoid any kind of biasness. So this process basically it iterative and often it is time consuming.

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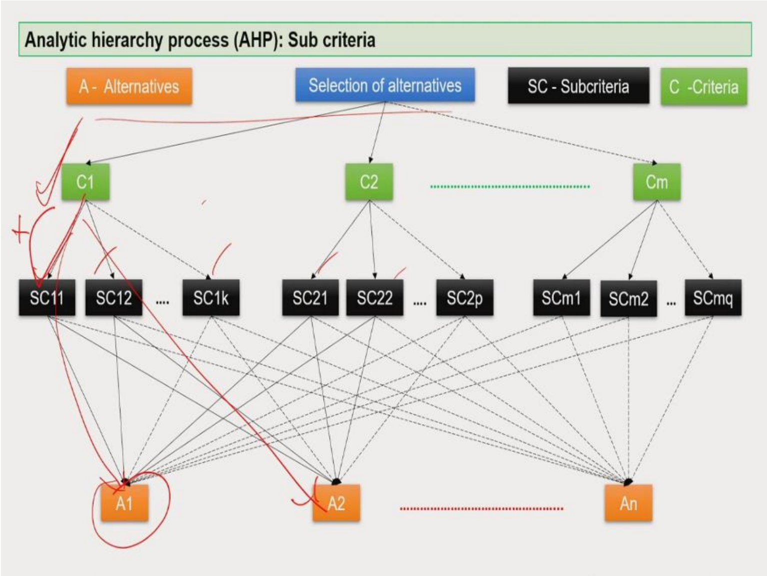

Yes, when we discussed about sub-criteria it looks like this so in AHP you have this is your criterias and within criterias you have sub criteria. But when you have sub criteria then you have to choose your alternative in this way so your criteria and sub-criteria these two will be multiplied then you get a global value and that value will be tried with all alternatives. So this becomes little bit time consuming to identify finally the scores and the ranking of the alternatives.

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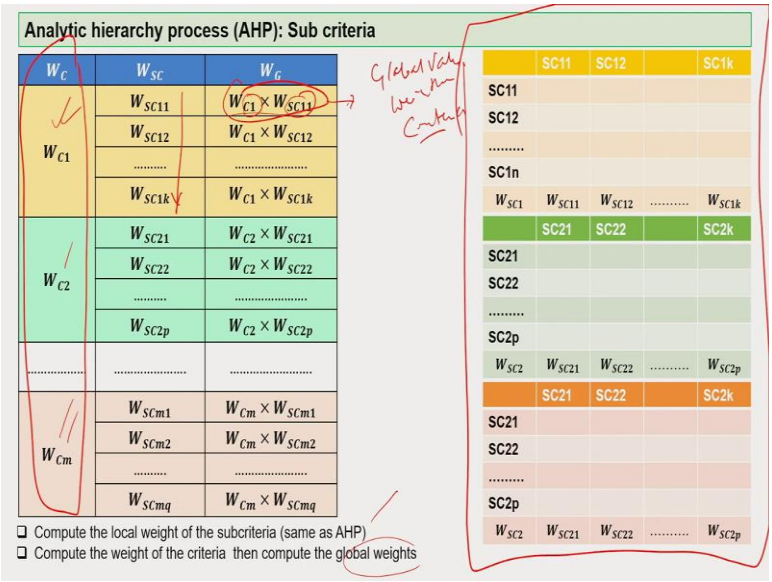

So this is what I discussed and finally when you go for sub-criteria level of analysis criteria and sub-criteria then of course your task become little time consuming as you see that you have a criteria then you have sets of sub-criteria and then for each criterias, sub-criteria you have different weightage so you multiply and then you get a global value with weightage of criteria.

So this is the after multiplication that value you get finally you take it further for ranking process. Same way you tried for all the criteria so you should compute the local weight of as I said the sub criteria and then the criteria then compute the global weights. These are the global weights basically you get by multiplying sub criteria weightage with criteria weightage and once you get that then you plot them and then you find out that which one of the criterias basically are coming on the top or at the bottom and then you choose it for your purpose.

So AHP again analytical hierarchy process is one of the MCDM technique which you will find very commonly used especially for in natural resource management businesses. So I hope that these processes you would also try to apply whenever you actually work in the field of natural resource management and you get perplexed with what you call legacy of having plenty full of alternatives in your hand.

But in many cases if you do not find more than one, suppose you have only one choice, suppose irrigation, you have only one choice surface irrigation then there is no need of doing all these analysis but if you have more than one then the question comes should I go for this or that, if you are confused then these exercises of MCDA it helps you to identify the best alternative.