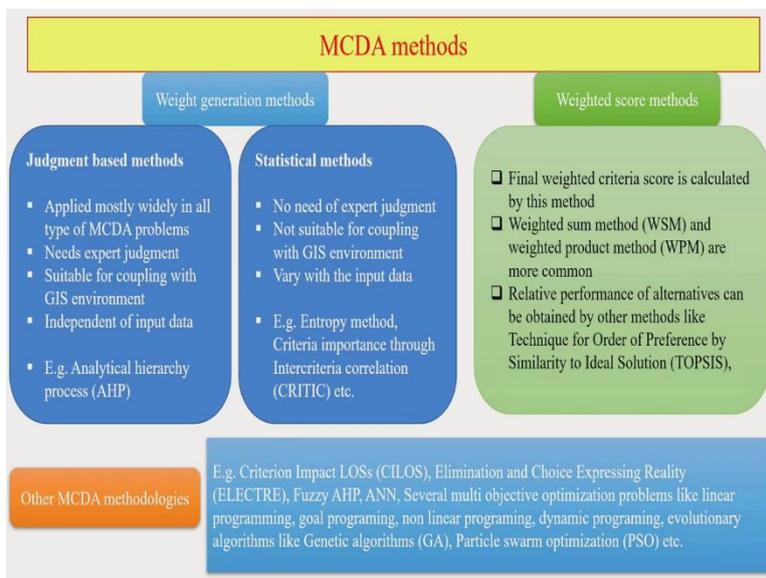
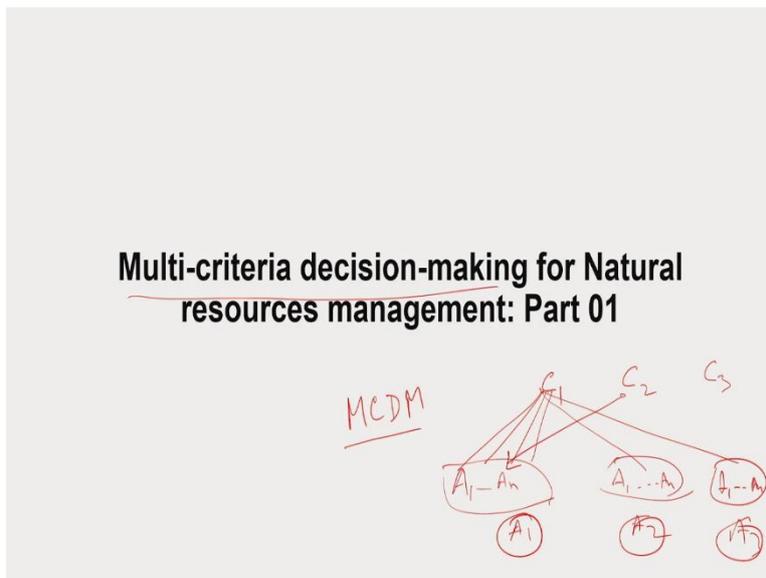


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Week – 09
Lecture - 51

Multi-Criteria Decision-Making for Natural Resources Management: Part-01

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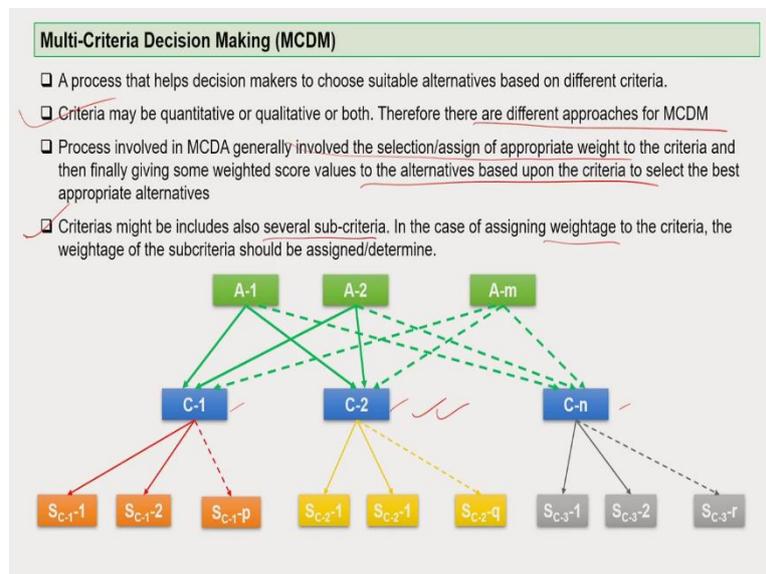
In this lecture today we will discuss about multi-criteria decision making for natural resources management and if you recall that we have already discussed about the introductions of MCDM that we call in brief MCDM so for natural resources management, we also discussed in the introductory lecture or MCDA that how actually it works, what are

the different aspect of that. We talked about different processes methods like AHP, we have talked about analytical, hierarchical process so these are the things we have already discussed.

Today in this part one of MCDM on natural resource management we will be discussing little bit in detail that how particularly MCDM actually work. If you recall that we have seen in the introductory lecture of MCDM that is it uses the mathematical principles for finding out the best possible alternative for a criteria. So, if you recall that we discussed about criteria 1, 2, 3 and then suppose we have alternative A1 to An for each option like this.

Now this one will try from all options A to n and then C1 also will go alternative 1, alternative 2, alternative 3 like that up to n, similar is for C2 so it will try for alternative 1, alternative 2, alternative 3 to alternative n. Now MCDA help us in finding out the best possible combinations for better decision making for natural resources management, there are many other applications are also there in this course I would like to limit two natural resource management.

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Now if you see that we already discussed that MCDM is a process which helps in decision makers to choose suitable alternative based on different criteria, like as I said C1 to Cn any number of criteria, criteria may be quantitative or qualitative or both therefore there are different approaches that we use under MCDM we will discuss about that already earlier lectures talked about all those different methodology. Now this process MCDA generally involved with selection assign of appropriate weight that we give to each of these criteria and

then finally give some weighted scores values to the alternatives based upon the criteria, this we have discussed in the introductory lecture.

Now, criteria might include also several sub-criteria in the case of suppose assigning weightage we when you put to each of the criteria, the weightage of the sub-criteria should also be assigned or determined. if you recall that we discussed about this also in the introductory lecture on MCDM.

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Weighted sum (WSM) and weighted product model (WPM)

- ❑ WSM is a simple and most common process to determine the weighted score of the alternatives.
- ❑ Simple linear combination (sum) of the weight and the values of the criteria
- ❑ Weight may be equal or unequal depending upon the purpose of the MCDM problem and objectives
- ❑ The mathematical formulation of the weighted sum model is as follows

$$S_{A_i} = \sum_{j=1}^n W_j X_{ij}$$

X_{ij} is the normalized criteria values of the alternatives

Criteria weights	W_1	W_2	W_n	S_i
Criteria (j)	C_1	C_2	C_n	
Alternatives(i)					
A_1	X_{11}	X_{12}	X_{1n}	S_{A_1}
A_2	X_{21}	X_{22}		X_{2n}	S_{A_2}
.....					
A_m	X_{m1}	X_{m2}	X_{mn}	S_{A_m}

Example

- ✓ $S_{A_1} = W_1 X_{11} + W_2 X_{12} + \dots + W_n X_{1n}$
- ✓ $S_{A_2} = W_1 X_{21} + W_2 X_{22} + \dots + W_n X_{2n}$
- ✓ $S_{A_m} = W_1 X_{m1} + W_2 X_{m2} + \dots + W_n X_{mn}$

S_{A_i} is the weighted sum score of i^{th} alternatives

- ❑ In many cases the weighted average score is taken instead of the weighted sum

Now, WSM weighted sum model, weighted sum method or weighted sum product model, how actually it works. WSM is a very simple and most common process to determine the weighted score of the alternatives. So, we largely use similar linear combinations of the weight and the values of the various criteria, the weight that we give it may be equal or unequal depending upon the purpose of the MCDM that you are actually trying to use for.

What is the formula that we use for WSM or WPM, very simple S_{A_i} is a function summation of i, j 1 to n , W_j into X_{ij} . Now what are these? X_{ij} is the normalized criteria values of the alternatives. Say, example as I said i is equal to 1 to n , okay, so S_{A_1} you will have $W_1 X_{11}$ suppose 1 and i X_{i1} j also 1 then you have 2 then you have i 1 but for j you have 2 so like that way it will continue up to n .

Similar way you will try for S_{A_2} to S_{A_n} all alternative that you will try to test, now S_i is the weighted sum score of i^{th} alternative, it could be the first or the second or the third, if i value is 1 then it is the first if i is equal to 2 then it is the second if i is equal to 3, then this so and so forth, in many cases the weighted average score is taken instead of the weighted sum, I repeat

once again in many cases the weighted average the mean score is taken instead of the weighted sum, so this is one simple simplest method of MCDM which is known as WSM or WPM.

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Weighted sum (WSM) and weighted product model (WPM)

Weighted product model is another combination of the weight and the values of the criteria for scoring the rank of the alternatives

$$P_{A_i} = \prod_{j=1}^n X_{ij}^{W_j}$$

X_{ij} is the normalized criteria values of the alternatives

Example

$$P_{A_1} = X_{11}^{W_1} \cdot X_{12}^{W_2} \cdot \dots \cdot X_{1n}^{W_n}$$

$$P_{A_2} = X_{21}^{W_1} \cdot X_{22}^{W_2} \cdot \dots \cdot X_{2n}^{W_n}$$

$$P_{A_m} = X_{m1}^{W_1} \cdot X_{m2}^{W_2} \cdot \dots \cdot X_{mn}^{W_n}$$

Criteria weights	W_1	W_2	W_n	P_i
Criteria (j)	C_1	C_2	C_n	
Alternatives(i)					
A_1	X_{11}	X_{12}	X_{1n}	P_{A_1}
A_2	X_{21}	X_{22}	X_{2n}	P_{A_2}
.....
A_m	X_{m1}	X_{m2}	X_{mn}	P_{A_m}

P_{A_i} is the weighted product score of i^{th} alternatives

In case of max-min normalization of the variables (criteria) where $X \in [0,1]$, there in case of the lower limit of X i.e. 0 the entire weighted product will be zero. Which is a major limitation of this model

Weighted sum (WSM) and weighted product model (WPM)

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Simple linear combination (sum) of the weight and the values of the criteria

Weight may be equal or unequal depending upon the purpose of the MCDM problem and objectives

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$$S_{A_i} = \sum_{j=1}^n W_j X_{ij}$$

X_{ij} is the normalized criteria values of the alternatives

Example

$$S_{A_1} = W_1 X_{11} + W_2 X_{12} + \dots + W_n X_{1n}$$

$$S_{A_2} = W_1 X_{21} + W_2 X_{22} + \dots + W_n X_{2n}$$

$$S_{A_m} = W_1 X_{m1} + W_2 X_{m2} + \dots + W_n X_{mn}$$

Criteria weights	W_1	W_2	W_n	S_i
Criteria (j)	C_1	C_2	C_n	
Alternatives(i)					
A_1	X_{11}	X_{12}	X_{1n}	S_{A_1}
A_2	X_{21}	X_{22}	X_{2n}	S_{A_2}
.....
A_m	X_{m1}	X_{m2}	X_{mn}	S_{A_m}

S_{A_i} is the weighted sum score of i^{th} alternatives

In many cases the weighted average score is taken instead of the weighted sum

Now weighted product model, (WPM), weighted product model is another combination of the weight and the values of the criteria for scoring the rank of the alternatives means which of the alternatives highest rank then order of course when you decide take a decision you will go for the highest rank alternative. So, how you do that?

The equation P_{A_i} by j value if is 1 to n and then you have X_{ij} into W_j , X_{ij} to the power W_j , here X_{ij} is the normalized criteria values of the alternatives same as WSM, P_{A_i} , P_{A_i} is the weighted product score of i^{th} alternative, weighted product score here it was weighted sum

score in WSM, in case of weighted product model it is the weighted product score means multiplying.

So see here you are actually summing it up, here you are multiplying this is the difference, now for alternative A1 if i is equal to 1 so alternative A1 you get Xij both 1 W also to the power Wj is equal to 1 then for ith alternative jth option you are getting this and then onward it will be like that. In case of maximum minimum normalization of the variables which are criteria here where X equivalent to 0 it can takes value 0 or 1 there in case of the lower limit of X that is 0 the entire weighted product also will be 0.

If your any X value is 0 then all the product entire product will be 0. This is one of the major limitation of WPM so if in some cases we are not able to use WPM the major reason is this limitation because if your minimum value of X becomes 0 then everything becomes 0.

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Weighted aggregated sum product assessment (WASPAS)

WASPAS is a weighted combination of the WSM and WPM processes to determine the weighted score of the alternatives.

$$R_i = \omega S_{A_i} + (1 - \omega) P_{A_i} \quad 0 \leq \omega \leq 1$$

$$R_i = \omega \sum_{j=1}^n W_j X_{ij} + (1 - \omega) \prod_{j=1}^n X_{ij}^{W_j}$$

Usually in WASPAS ω is taken as 0.5 (but it depends upon the problem and the desired outcome)

Handwritten notes:
 $i=1 = \omega W_j X_{11}$
 $R_i = \text{value}$
 0.5

Next method weighted aggregated sum product assessment which you call WASPAS, WASPAS is a weighted combination of the weighted sum method and weighted product method okay combination of these two which processes to determine the weighted score of the alternatives utilizing both summation and multiplication that means WSM and WPM method how, this is the formula

Ri is equals to W into SAi plus 1 minus W into PAi

Where W score lies between 1 and 0.

Now suppose you put value j equals to 1 to n then how it goes, so in one value if you say for i is equal to 1 so for this value it will be like W and then you will go W here will be 1. So here Wj for j is equal to 1 so Wj will be 1 Xi and j also will be 1 so usually WASPAS W is taken as 0.5 but it depends upon the problem and the desired outcome.

So, the best Ri value under WASPAS method when it is 0.5 that is what actually we consider as one of the best selections for alternatives. So this methodology applies your WSM and WPM as you see that plus as well as also your multiplication both so WSM and WPM both way are utilized.

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Normalization method of criteria values of the alternatives

- It is done to remove the unit mismatch i.e. to take all the variables under a similar scale
- It depends upon the beneficial and non-beneficial (cost) criteria
- Several normalization methods exist

Max-min normalization

- Widely used method
- Beneficial criteria: $X_{ij} = \frac{x_{ij} - (x_j)_{\min}}{(x_j)_{\max} - (x_j)_{\min}}$
- Non beneficial criteria: $X_{ij} = \frac{(x_j)_{\max} - x_{ij}}{(x_j)_{\max} - (x_j)_{\min}}$

Max normalization

- Beneficial criteria: $X_{ij} = \frac{x_{ij}}{(x_j)_{\max}}$
- Non beneficial criteria: $X_{ij} = 1 - \frac{x_{ij}}{(x_j)_{\max}}$

For beneficial criteria higher values are desired
For non-beneficial criteria lower values are desired

Criteria (j)	C ₁	C ₂	C _n
Alternatives(i)				
A ₁	X ₁₁	X ₁₂	X _{1n}
A ₂	X ₂₁	X ₂₂	X _{2n}
.....
A _m	X _{m1}	X _{m2}	X _{mn}

Now normalization method of criteria values of the alternatives, how you normalize the criteria value of each one of your alternatives. It is largely done just to remove the mismatch and so that all the variables come under a similar scale that is the purpose. Normalization also depends upon the beneficial and nonbeneficial criteria, several normalization methods we use probably all of you are using it so few of the widely used methods are like in case of beneficial criteria you use this formula

Xij is equal to xij minus xj min divided by xj max minus xj min

in case of non-beneficial criteria you use this formula.

Xij is equal to xj max minus xij divided by xj max minus xj min

So for beneficial criteria higher values are desired, for nonbeneficial criteria lower values are desired naturally so maximum normalization you see for beneficial criteria this is the formula, for nonbeneficial, this is the formula, so here you will take the lowest value as low as, means lower value means better and here if it is higher it is better because in the positive or beneficial sign.

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Normalization method of criteria values of the alternatives

Sum normalization

- Beneficial criteria

$$X_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}$$
- Non beneficial criteria

$$X_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^m \left(\frac{1}{x_{ij}}\right)}$$

Vector normalization

- Beneficial criteria

$$X_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$
- Non beneficial criteria

$$X_{ij} = 1 - \frac{x_{ij}}{\sum_{i=1}^m x_{ij}^2}$$
- Also used in many MCDM methods (example: TOPSIS)

A1, A2, A3, log, =

Log normalization

- Done either by simply taking log values to the variables

$$X_{ij} = \ln(x_{ij})$$
- Difficult in case if there are zero values or fractions
- Beneficial criteria

$$X_{ij} = \frac{\ln(x_{ij})}{\ln(\prod_{i=1}^m x_{ij})}$$
- Non-beneficial criteria

$$X_{ij} = \frac{1 - \frac{\ln(x_{ij})}{\ln(\prod_{i=1}^m x_{ij})}}{m - 1}$$

i, j.

Weighted sum (WSM) and weighted product model (WPM)

Weighted product model is another combination of the weight and the values of the criteria for scoring the rank of the alternatives

$$P_{A_i} = \prod_{j=1}^n X_{ij}^{W_j}$$

X_{ij} is the normalized criteria values of the alternatives

Example

$$P_{A_1} = x_{11}^{W_1} \cdot x_{12}^{W_2} \cdot \dots \cdot x_{1n}^{W_n}$$

$$P_{A_2} = x_{21}^{W_1} \cdot x_{22}^{W_2} \cdot \dots \cdot x_{2n}^{W_n}$$

$$P_{A_m} = x_{m1}^{W_1} \cdot x_{m2}^{W_2} \cdot \dots \cdot x_{mn}^{W_n}$$

Criteria weights	W_1	W_2	W_n	P_i
Criteria (j)	C_1	C_2	C_n	
Alternatives(i)					
A_1	X_{11}	X_{12}	X_{1n}	P_{A_1}
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.....
A_m	X_{m1}	X_{m2}	X_{mn}	P_{A_m}

P_{A_i} is the weighted product score of i^{th} alternatives

In case of max-min normalization of the variables (criteria) where $X \in [0,1]$, there in case of the lower limit of X i.e. 0 the entire weighted product will be zero. Which is a major limitation of this model

Sum normalization is another method where you add up your value to calculate beneficial criteria under sum normalization this is the formula

X_{ij} is equal to x_{ij} by summation i equal to 1 to m x_{ij}

and for nonbeneficial criteria this is the formula,

X_{ij} is equal to $1 - \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}$

so if you wish you can note down this formula otherwise anyway this transcript of these lectures also will be provided to you.

Vector normalization, for beneficial this formula we use,

X_{ij} is equals to x_{ij} by square root sumetion I equals to 1 to m x_{ij} square

for nonbeneficial we use this formula.

X_{ij} is equals to $1 - x_{ij}$ by square root sumetion I equals to 1 to m x_{ij} square

Now vector normalization method are often used also for another MCDM technique. TOPSIS, you recall that we discussed in the first lecture of MCDA about various methods. So TOPSIS is another method of MCDA where vector normalization can be used.

Log normalization this done by simply taking the log values of the variables, you have all the alternatives, so these values whatever values you have those values actually will take log of those values, log of those values, so this particular normalization methodology log normalization is difficult in case if there is zero values or fractions like you remember that in case of weighted product model there is chance that you can get the X lower limit as low as 0 so in that case you might find difficulty.

So how you normalize using log normalization process for beneficial criteria this is the formula,

X_{ij} is equals to natural log of x_{ij} by natural log of multiplication I equals 1 to m x_{ij}

for non beneficial this is the formula.

X_{ij} is equals to $1 - \text{natural log of } x_{ij}$ by natural log of multiplication I equals 1 to m x_{ij} whole divided by m minus 1

So, it looks complicated but when you get the value of each one of them suppose i j when you get the value then actually it becomes much easier to calculate.

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Example of an MCDM problem

In an agriculture 6 different precision farming technologies are adopted. Four different criterias are enlisted in the table. Based on those criterias determine which technology is most suitable

Technologies	Criterias			
	Water requirement (mm)	Cost of cultivation (INR)	Yield (t/ha)	Water use efficiency (%)
A	621.0	39925	7.51	58.32
B	448.2	51925	9.13	54.17
C	604.8	49350	8.02	49.43
D	534.6	44050	8.58	68.36
E	475.2	48125	8.78	63.22
F	680.4	49050	7.35	48.34

C₁
C₂
C₃
C₄

Just let me try to explain these things with an example, suppose in an agriculture experiment 6 different precision farming technologies are adopted, 4 different criterias are C1, C2, C3, C4 different four criteria listed, now based on those criteria, let us determine which technology is most suitable for any given area. This is a very common question, most of the time any of us face, which one, three-four technologies are there say I had given if you recall the example of irrigation, you have suppose drip irrigation, surface, groundwater, which one in which area is suitable this is the way you can actually find out the answer.

Now, you have suppose six technology okay now there are two type of criteria non beneficial and beneficial. Now, water requirement in case of non beneficial we discussed as lower the value better for us so you see that water requirements 621, 448, 604, 534, 475, 680, which one is the lowest, this one, which is the technology, B, cost of cultivation this one if you see which one, which one, this one is the lowest, yield, beneficial, higher is better so here we have the highest here, water use efficiency, higher is the better, here you have the highest where, here.

Now you see that for any single technologies which one having better benefit for us, here you have one water requirement less yield high looks like a good option right, so what a requirement is less yield high but of course here cultivation cost is relatively higher but at the same time you have to see that water is a very important natural resource again it comes the giving some value to our natural resources, if you feel that water is just free then it is different. So this is one combination that we have let us see.

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Example of an MCDM problem

	C1	C2	C3	C4
A	621.0	39925	7.51	58.32
B	448.2	51925	9.13	54.17
C	604.8	49350	8.02	49.43
D	534.6	44050	8.58	68.36
E	475.2	48125	8.78	63.22
F	680.4	49050	7.35	48.34

Where A, B, C, D, E and F are alternatives (i.e. technologies) C1, C2, C3 and C4 are 4 different criterias respectively as water requirement, cost of cultivation, yield (t/ha) and water use efficiency. C1, C2 are non beneficial and C3, C4 are beneficial criteria

	C1	C2	C3	C4
Max	680.4	51925	9.13	68.36
Min	448.2	39925	7.35	48.34

Normalized decision variable matrix				
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

Max-min normalization

- Beneficial criteria
- Non beneficial criteria

$$X_{ij} = \frac{x_{ij} - (x_j)_{min}}{(x_j)_{max} - (x_j)_{min}} \quad X_{ij} = \frac{(x_j)_{max} - x_{ij}}{(x_j)_{max} - (x_j)_{min}}$$

$$\frac{8.02 - 7.35}{9.13 - 7.35} = 0.38$$

$$\frac{680.4 - 621.0}{680.4 - 448.2} = 0.26$$

Example of an MCDM problem

In an agriculture 6 different precision farming technologies are adopted. Four different criterias are enlisted in the table. Based on those criterias determine which technology is most suitable

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Normalization method of criteria values of the alternatives

It is done to remove the unit mismatch i.e. to take all the variables under a similar scale

It depends upon the beneficial and non-beneficial (cost) criteria

Several normalization methods exist

Max-min normalization

Widely used method

- Beneficial criteria
- Non beneficial criteria

$$X_{ij} = \frac{x_{ij} - (x_j)_{min}}{(x_j)_{max} - (x_j)_{min}}$$

$$X_{ij} = \frac{(x_j)_{max} - x_{ij}}{(x_j)_{max} - (x_j)_{min}}$$

For beneficial criteria higher values are desired

For non-beneficial criteria lower values are desired

Max normalization

- Beneficial criteria
- Non beneficial criteria

$$X_{ij} = \frac{x_{ij}}{(x_j)_{max}}$$

$$X_{ij} = 1 - \frac{x_{ij}}{(x_j)_{max}}$$

Criteria (j)	C ₁	C ₂	C _n
Alternatives(i)				
A ₁	X ₁₁	X ₁₂	X _{1n}
A ₂	X ₂₁	X ₂₂	X _{2n}
.....
A _m	X _{m1}	X _{m2}	X _{mn}

Criteria (j)	C ₁	C ₂	C _n
Alternatives(i)				
A ₁	X ₁₁	X ₁₂	X _{1n}
A ₂	X ₂₁	X ₂₂	X _{2n}
.....
A _m	X _{m1}	X _{m2}	X _{mn}

Now here A, B, C, D, E, F are the alternatives as you saw and C1, C2, C3, and C4 are the criteria, four different criterias like water requirement, cost of cultivation, yield and water use efficiency. So, C1, C2, C3, C4 criteria. Now, C1, C2 are nonbeneficial and C3 and C4 are beneficial. Now, you see in case of C1 maximum is this, minimum is this value C2 maximum is this minimum is this, for C3 maximum minimum, maximum minimum, now we know that we have to do a normalization.

So, maximum-minimum normalization process can be applied here, now then you calculate use this formula

X_{ij} is equals to x_{ij} minus x_j min divided by x_j max – x_j min

X_{ij} is equals to x_j max x_{ij} divided by x_j max – x_j min

remember the normalization process we talked about, so here maximum minimum normalization this is the one we can use because we have very clear cut maximum and minimum values of the different criteria.

Now, here we calculate we find that for beneficial criteria our maximum minimum normalization value is 0.38, whereas for nonbeneficial it is 0.26 is this is the value so this is the normalized decision variable matrix you can say for C1 C2 C3 C4 for all. Now for C1 this is the value lowest, for C3 this is the value or you can say normalize value.

(Refer Slide Time: 18:01)

Example of an MCDM problem (cont.): Weighted Sum Model (WSM)

Normalized decision variable matrix				
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

W	0.25	0.25	0.25	0.25
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

	S	Rank
A	0.46	4
B	0.57	3
C	0.24	5
D	0.75	1
E	0.69	2
F	0.06	6

Where W is the weight associated with each criterion (let equal weight is assumed for each criterion i.e. 0.25 each)

Apply the weighted sum model

Example for row-1 i.e. for alternative A

$$(0.26 \times 0.25) + (1 \times 0.25) + (0.09 \times 0.25) + (0.5 \times 0.25) = 0.46$$

(Handwritten notes: C1 ✓, C2 ✓, C3 =, C4 x 1 = 0.46)

Now on this basis you will go for now a weighted sum model, now normalize decision variable matrix you have this with you we calculate 0.26, 0.38 as the normalized value. So, here now we will go for, where W is the weight associated with each criterion say equal weight is assumed for each criteria that is 0.25 each.

Now, you try to apply the model so row 1 that is for alternative A1 all the criteria, so you have 0.26, you are giving weightage 0.25, then you have C2 you are giving weightage again 0.25, then you have C3, you are giving weightage 0.25, then you have C4, you are again giving 0.25, how much is the value is coming WSM 0.46.

So your alternative A for all criteria value is 0.46, similar way you will try for alternative B, C, D, E, F, then you get actually all the value. Now, you see the ranking, so your alternative D becomes first ranker and the one that you have calculated for A it becomes rank 4. So this way you can clearly see that out of your all alternatives which one is the best one that you should go for is not it is a good idea or easy way to solve a complex situation of choosing an alternative especially when you have alternative options more than one.

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Example of an MCDM problem (cont.): WSM

If W is the weight associated with each criterion is **unequal** then
 Let weights are 0.05, 0.8, 0.05 and 1 respectively here

W	0.05	0.8	0.05	0.1
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

→

	S	Rank
A	0.87	1
B	0.13	6
C	0.21	4
D	0.69	2
E	0.41	3
F	0.19	5

Example for row-1 i.e. for alternative A
 $(0.26 \times 0.05) + (1 \times 0.8) + (0.09 \times 0.05) + (0.5 \times 0.1) = 0.87$

Example of an MCDM problem (cont.): Weighted Sum Model (WSM)

Normalized decision variable matrix				
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

→

W	0.25	0.25	0.25	0.25
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

→

	S	Rank
A	0.46	4
B	0.57	3
C	0.24	5
D	0.75	1
E	0.69	2
F	0.06	6

Where W is the weight associated with each criterion (let **equal** weight is assumed for each criterion i.e. 0.25 each)
 Apply the weighted sum model

Example for row-1 i.e. for alternative A
 $(0.26 \times 0.25) + (1 \times 0.25) + (0.09 \times 0.25) + (0.5 \times 0.25) = 0.46$

So we continue with the now WSM. So W is the weight associated with the each criteria, suppose if we give the weight which are different weights or unequal weights; previous one we have given 0.25 to all as you saw that we multiply with 0.25. but here if we give four different types of weight to four different criteria then how it happens, so we have this value for alternative A, now let us see how it goes.

So C1, C1 we have given 0.5, C2 0.8, C3 0.05, C4 we have given 0.1, now we get the WSM value 0.87 so we put it here for against alternative A. Now, that is the way we calculate all others and we find 0.87 is the highest value so it goes as the rank 1. So this is when our weightage to each criteria C1, C2, C3, C4 given four different weightage, previous one we tried with single weightage of 0.25, but in real situation it always may not happen you may

have four different or three different like that kind of weightage value so like that you can also calculate.

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Example of an MCDM problem (cont.): Weighted Sum Model (WPM)

W is the weight associated with each criterion (let **equal** weight is assumed for each criterion i.e. 0.25 each). The same example of the weighted sum model. Apply WPM in the example.

W	0.25	0.25	0.25	0.25
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

→

	P	Rank
A	0.33	3
B	0	5
C	0.19	4
D	0.73	1
E	0.64	2
F	0	5

Example for row-1 i.e. for alternative A
 $(0.26)^{0.25} \times (1)^{0.25} \times (0.09)^{0.25} \times (0.5)^{0.25} = 0.33$

- ❖ In the places where any value of the normalized decision matrix is 0 there the entire product score of the row (i.e. the alternative) is 0.
- ❖ If such cases for both alternatives then in all those cases weighted product will be zero
- ❖ In this example as B and F both weighted product as 0 hence both ranked 5

Now you can also see that how WPM the product model can work. Suppose if we try same way giving weightage same weightage 0.25 to all then how it happens? Here is your alternative then you actually try like 0.26 that is C1, C1 to the power 0.25, C2, C3, C4, so then what you get you ultimately get a value of 0.33. So that is your value for A, now calculate all others. You see here if your criteria is 0 remember I said that in product model we have limitation you will get 0 values and that is why sometime we do not use it. So here you find that your rank number 1 is alternative D so this is the way you can calculate.

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Example of an MCDM problem (cont.): Weighted Sum Model (WPM)

W is the weight associated with each criterion (let **unequal** weight). The same example of the weighted sum model. Apply WPM in the example.

W	0.05	0.8	0.05	0.1
	C1	C2	C3	C4
A	0.26	1	0.09	0.5
B	1	0	1	0.29
C	0.33	0.21	0.38	0.05
D	0.63	0.66	0.69	1
E	0.88	0.32	0.8	0.74
F	0	0.24	0	0

→

	P	Rank
A	0.77	1
B	0	5
C	0.19	4
D	0.69	2
E	0.38	3
F	0	5

Example for row-1 i.e. for alternative A
 $(0.26)^{0.05} \times (1)^{0.8} \times (0.09)^{0.05} \times (0.5)^{0.1} = 0.77$

If you have unequal values weightage for your all different criteria then you get like this C1 0.5 suppose C2 have given 0.8, C3 have given 0.05 and then again C4 you have given 0.1, four different weightage you get 0.77 as your WPM value. You write it here calculate all others you find that A is the highest so rank number 1 so you go for alternative 1 when you use the methodology WPM but remember this is your one limitation.

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Example of an MCDM problem (cont.): WASPAS $R_i = \omega S_{A_i} + (1 - \omega) P_{A_i}$

W	0.25	0.25	0.25	0.25					
	C1	C2	C3	C4	WSM	WPM	WASPAS Rank		
A	0.26	1	0.09	0.5	0.46	0.33	0.4	5	Equal weight $(0.5 \times 0.46) + ((1-0.5) \times 0.33) = 0.4$ Weight aggregated sum product assessment with $\omega = 0.5$
B	1	0	1	0.29	0.57	0	0.29	3	
C	0.33	0.21	0.38	0.05	0.24	0.19	0.22	4	
D	0.63	0.66	0.69	1	0.75	0.73	0.74	1	
E	0.88	0.32	0.8	0.74	0.69	0.64	0.67	2	
F	0	0.24	0	0	0.06	0	0.03	6	

W	0.05	0.8	0.05	0.1					
	C1	C2	C3	C4	WSM	WPM	WASPAS Rank		
A	0.26	1	0.09	0.5	0.87	0.77	0.82	1	Unequal weight $(0.5 \times 0.87) + ((1-0.5) \times 0.77) = 0.82$
B	1	0	1	0.29	0.13	0	0.07	6	
C	0.33	0.21	0.38	0.05	0.21	0.19	0.2	4	
D	0.63	0.66	0.69	1	0.69	0.69	0.69	2	
E	0.88	0.32	0.8	0.74	0.41	0.38	0.4	3	
F	0	0.24	0	0	0.19	0	0.1	5	

Example of an MCDM problem (cont.): Weighted Sum Model (WPM)

W is the weight associated with each criterion (let **unequal** weight). The same example of the weighted sum model. Apply WPM in the example.

W	0.05	0.8	0.05	0.1				
	C1	C2	C3	C4				
A	0.26	1	0.09	0.5				
B	1	0	1	0.29				
C	0.33	0.21	0.38	0.05				
D	0.63	0.66	0.69	1				
E	0.88	0.32	0.8	0.74				
F	0	0.24	0	0				

	P	Rank
A	0.77	1
B	0	5
C	0.19	4
D	0.69	2
E	0.38	3
F	0	5

Example for row-1 i.e. for alternative A
 $(0.26)^{0.05} \times (1)^{0.8} \times (0.09)^{0.05} \times (0.5)^{0.1} = 0.77$
 $C_1 \quad C_2 \quad C_3 \quad C_4$

WASPAS is another methodology where you can try as similar way giving same weightage to all criteria and different weightage to different criteria and then you try to calculate values for equal weight and unequal weight. Here you get 0.4, here you get 0.82. So that is the way that we could basically calculate, you can analyze the alternatives and you can choose which alternative actually is the best one out of the four or five options that you have.

In the field it helps really a great way to decide which methodology which alternative people will go for it could be irrigation choices, it could be fertilization choices, it could be also for any other any other kind of activity related to agriculture or non-agriculture also basically these methodologies actually simplifies your decision-making process that is why you call multiple criteria decision making or methodology.