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Week - 08
Lecture - 49
Model sensitivity and Uncertainty

In this lecture today, we will discuss about Model Sensitivity and Uncertainty Analysis. If you recall that in one of the previous lectures, we also discussed about the capacity of modeling and how modeling exercise can help us to somehow estimate, evaluate some of the impact of activities even without going into the field.

Because there could be sometimes some situations when you or your team physically may not be able to go to the field and that point of time this kind of modeling exercise will allow you to actually mimic the reality at the field level and also carry out the analysis. Modeling exercise also, if you recall, I mentioned that it can predict also on the basis of the past and present information.

A model can be as robust as is the quality of the data that you provide as input into a modeling system. So, we discussed quite a detail if you can go back and refer to the modeling lectures. Today, what are we going to discuss is that how we can actually analyze the sensitivity of a model, means if you change small parameter here and there, whether the model gets affected or not. And also, we will try to find out that how uncertainty in modeling exercise can be analyzed and also understood.

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Model sensitivity analysis

- ☑ To determine the rate of change in the output of a model with respect to changes in model inputs/ or parameters
- ☑ Ranks the inputs or parameters depend on their contribution to model outputs. That is it helps to determine the most influential inputs on a model
- ☑ Sensitivity coefficients is expressed as

$$S_A = \frac{\partial f}{\partial x}$$
 Numerically

$$S_A = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$
- ☑ Δx vary from some % like 5%, 10%, 20% etc from original value of the inputs or parameters
- ☑ Numerically done in case of a not easily differentiable function
- ☑ If a model has a large set of inputs and parameters it is a time and computation consuming

Now, modeling sensitivity analysis basically we do to determine the rate of change in the output of a model with respect to the changes in model input. Suppose that this is a model kind of a system, functions takes place suppose inside the box. Now, here you have inputs and outputs here. Now, the rate of change here in input, input suppose if we consider, if you recall that we discussed in great detail about crop models.

So, if suppose a crop model is there, then what kind of parameter we need, temperature, then different kinds of wind also, humidity, then soil property like soil nitrogen, carbon, then you need pH, etcetera, etcetera. So, these are basically your input. And in the modeling exercise, you come out with the output where it is for a crop model yield predicting yield.

Now, any change in any of this input parameter, how it could actually impact the output means the yield if you change suppose temperature by one degree centigrade, whether yield goes up or goes down, these are all actually sensitivity analysis. Now, sensitivity analysis also ranks the inputs or the parameters, which depend upon their contribution to the model output. So, the model output here in the crop model is yield.

So, here what sensitivity analysis also allow us to do to rank the input parameter depending upon their role in controlling or regulating the yield say for temperature, wind, humidity, nitrogen, carbon, etcetera, you find that humidity is one which can actually affect the yield parameter quite significantly. So, humidity will go on top, then maybe pH, then maybe temperature, then maybe your carbon, nitrogen like that, so you can rank them.

Why you do this? It helps actually determining the most influential inputs on a model. So, you can identify that which one of these inputs actually are affecting your output that is yield

for a crop model. Now, sensitivity coefficient is one aspect, sensitivity coefficient is the measurement of the sensitivity of a model. Now sensitivity coefficient is expressed as

SA is equals to $\frac{\Delta f}{\Delta x}$

Now, numerically sensitivity coefficient

SA is equal to $\frac{f(x + \Delta x) - f(x - \Delta x)}{2 \Delta x}$

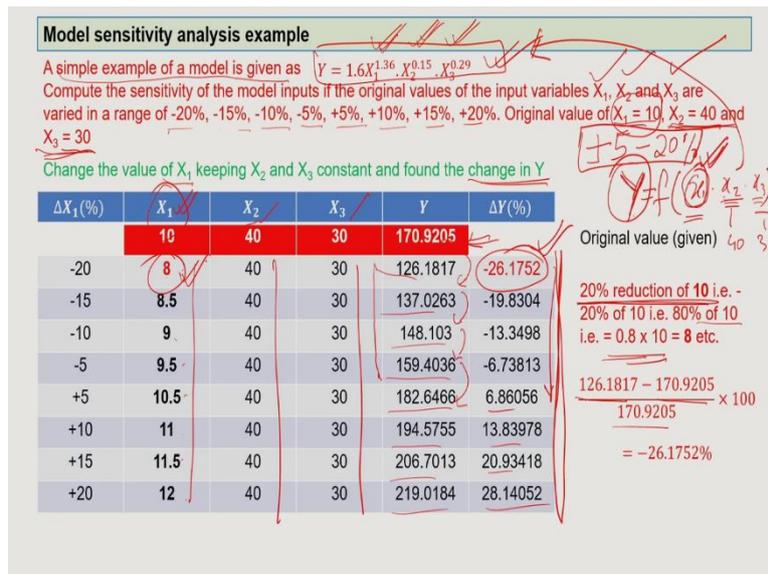
Now, I will explain what are these, Δx varies from some percentage like say 5 percent, 10 percent, 20 percent, etcetera, from its original value of the inputs. Here, we are talking about the inputs. So, the variation in any of the inputs in the value by 5 percent, 10 percent, 20 percent, that is depicted by Δx .

Numerically in case of not easily differentiable function if you find, then what happened is that you try to do it completely in a mathematical manner, in case the non-differentiable function is not easily available or easily doable. So, then you can do it numerically. If a model suppose has a large set of inputs 15, 20, 30 and parameters, then you need more time for modeling, even sometime we find that with 4 or 5 parameters, when we try to run model and with iterations, it takes a lot of time sometimes even hours together.

So, to reduce this kind of consumption of time sensitivity analysis also helps. So, sensitivity allows you to rank your parameter. So, accordingly you can take. Suppose, you are finding that your modeling exercise is taking huge amount of time, you can actually then identify the top 3, top 4 parameter and then you go ahead because they may be capturing almost 70 to 80 percent of the ill prediction or sensitivity.

So, in that case the bottom 3 or 4 you can neglect. Because see, you have to see that it cannot take such a long time, because time is also an important parameter, also resource for you. So, that is need to be seen.

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Now, with an example, I will try to explain the sensitivity analysis. Suppose a simple example of a model, simplest that you can think of,

Y is equals to 1.6 multiplied by X_1 to the power 1.36 multiplied by X_2 to the power 0.15 multiplied by X_3 to the power 0.29

So, these are 3 different parameters X_1 , X_2 , X_3 . And you have values like 1.3, 6.1, 5.29. Now, let us see how we can analyze the sensitivity of the simplest one model.

So, to compute the sensitivity of the model inputs, if you find the original values of the input here it is X_1 , X_2 , and X_3 are varying in the range of 20, 15, 10, 5 percent plus 5, 10, 15 and plus 20 percent, that means, you can say plus minus 5 to 20 percent, so that is the range variation that you have. So, the original value of X_1 suppose is 10, X_2 is 40, and X_3 is 30. How can we calculate the sensitivity of this very, very simple model?

Now, if you change the value of X_1 , change the value of X_1 keeping X_2 , and X_3 constant, you will find a change in Y . So, because your Y is a function of X_1 , X_2 , and X_3 if you see that this model. So, here first case we are changing the value of X_1 only keeping X_2 and X_3 constant which is $X_2 = 40$, $X_3 = 30$. See here, so, 40 and 30, X_2 , X_3 is constant. What we are going to do? We are going to play with X_1 .

So, we would see like to see that how X_1 if we change affects the output which is yield. So, that will allow me to understand that if I keep my other 2 parameter X_2 , X_3 fix and change X_1 , if I get different kinds of Y data like here you see, we are changing X_1 original value is 10, we are changing 8, 8.5, 9, 9.5, 10.5, 11, 11.5, 12 different value rise and these all

should range as I said between plus minus 5 to 20 within that range we are changing the value of X_1 keeping X_2 and X_3 same.

You see we are going to get different kinds of values. Now, we calculate the delta Y that means, the differences between the original value, this is the original value and then this calculated value as per the change in the value of X_1 . If I make 8.5, then how much. If I make 9, then how much. So, this is the difference, means the original value 170, the other values is go up to 159 which is less than 170 this much less minus 26, minus 19, minus 13, minus 6 and then it goes at the plus range higher than original value 6, 13, 20, 28. So, you got a set of values here by changing only one parameter that is X .

Now, here if you see that 20 percent reduction of 10 that is 20 percent of 10 and 80 percent of the 10 that is 0.8 into 10 that means 8. So, here you got 8. So, how you calculate that, then you get the value 126 and your original 170. So, the difference is divided by the original value multiplied by 100 you get the percentage term, percentage term of differences between the original and the estimated one after changing the value of parameter X_1 .

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Model sensitivity analysis example

Change the value of X_3 keeping X_1 and X_2 constant and found the change in Y

$\Delta X_3(\%)$	X_1	X_2	X_3	Y	$\Delta Y(\%)$
	10	40	30	170.9205	
-20	10	40	24	160.2102	-6.26623
-15	10	40	25.5	163.0518	-4.60371
-10	10	40	27	165.7771	-3.00925
-5	10	40	28.5	168.3969	-1.4765
+5	10	40	31.5	173.3561	1.424972
+10	10	40	33	175.7106	2.802548
+15	10	40	34.5	177.9904	4.136355
+20	10	40	36	180.2008	5.429601

Original value (given)

20% reduction of 30 i.e. -20% of 30 i.e. 80% of 30 i.e. = $0.8 \times 30 = 24$ etc.

$$\frac{160.2102 - 170.9205}{170.9205} \times 100 = -6.26623\%$$

$\pm 14 - 6.2$

Model sensitivity analysis example

Change the value of X_2 keeping X_1 and X_3 constant and found the change in Y

$\Delta X_2(\%)$	X_1	X_2	X_3	Y	$\Delta Y(\%)$
	10	40	30	170.9205	
-20	10	32	30	165.2942	-3.29176
-15	10	34	30	166.8042	-2.40831
-10	10	36	30	168.2405	-1.56798
-5	10	38	30	169.6105	-0.76645
+5	10	42	30	172.176	0.734537
+10	10	44	30	173.3816	1.439921
+15	10	46	30	174.5415	2.118559
+20	10	48	30	175.6594	2.772563

Original value (given)

20% reduction of 40 i.e. -20% of 40 i.e. 80% of 40 i.e. = $0.8 \times 40 = 32$ etc.

$$\frac{165.1817 - 170.9205}{170.9205} \times 100 = -3.29176\%$$

$+0.7 - 3.2$

Model sensitivity analysis example

A simple example of a model is given as $Y = 1.6X_1^{1.36} \cdot X_2^{0.15} \cdot X_3^{0.29}$

Compute the sensitivity of the model inputs if the original values of the input variables X_1 , X_2 and X_3 are varied in a range of -20%, -15%, -10%, -5%, +5%, +10%, +15%, +20%. Original value of $X_1 = 10$, $X_2 = 40$ and $X_3 = 30$

Change the value of X_1 keeping X_2 and X_3 constant and found the change in Y

$\Delta X_1(\%)$	X_1	X_2	X_3	Y	$\Delta Y(\%)$
	10	40	30	170.9205	
-20	8	40	30	126.1817	-26.1752
-15	8.5	40	30	137.0263	-19.8304
-10	9	40	30	148.103	-13.3498
-5	9.5	40	30	159.4036	-6.73813
+5	10.5	40	30	182.6466	6.86056
+10	11	40	30	194.5755	13.83978
+15	11.5	40	30	206.7013	20.93418
+20	12	40	30	219.0184	28.14052

Original value (given)

20% reduction of 10 i.e. -20% of 10 i.e. 80% of 10 i.e. = $0.8 \times 10 = 8$ etc.

$$\frac{126.1817 - 170.9205}{170.9205} \times 100 = -26.1752\%$$

$+5 = 20\%$
 $Y = f(X_1, X_2, X_3)$

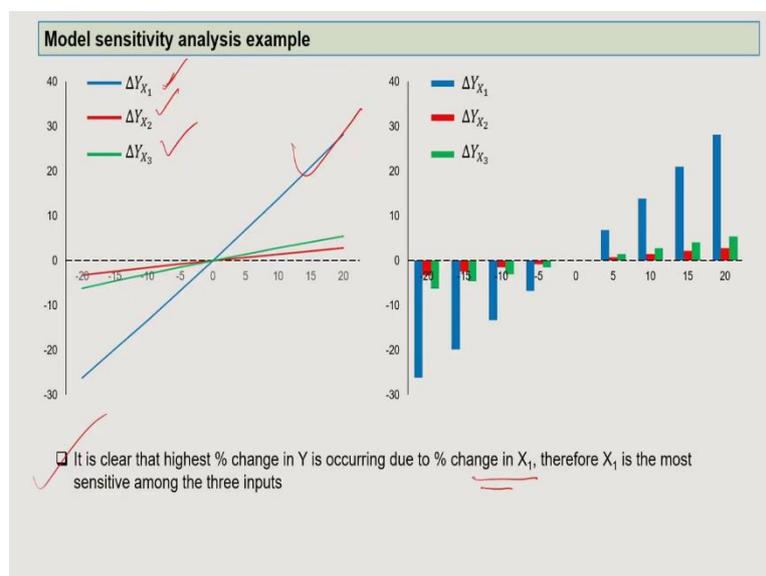
Now, change the value of X2 keeping X1 and X3 constant and then let us see that how much differences we are going to get in Y. You remember here we are getting the differences you see that 26, 19. So, between 6 percent plus minus to almost 28 percent. Now, here X1 we are keeping, original value fix, we are changing X2, X1 and X3 are fixed. Now, you will see that whether it is affecting the yield more than X1, changes in X1 or not.

Now, we have changed here the value from original and we are getting corresponding value of Y. Now, you see the differences. So, its range more or less plus minus 0.7 to 3.2 maybe. So, this is the range whereas, here is the earlier one you saw the range much higher. Same way here 20 percent reduction, we are doing 20 percent reductions of the value of 40 which is 32. So, 32 we got here.

Now, the value is 165, Y value, how much is the difference, divided by the original and then you get minus 3.29 percentage of differences. Now, see last one, X3 we will change X1 and X2 will keep constant. Here, you see that how much percentage of yield is actually change, because of change in X3. Till now our changes in Y yield is in case of X1 higher than X2, sensitivity. So, same, this is same.

Now, here we are changing again here 20 percent reductions. So, you get the value 24, you fix it at 24 here, then use get the value here and the original value here, subtract it, multiplied by 100, you get the value 6.26. So, you see the range here, more or less the range is 1.4 Plus minus 6.2, that is the range. So, the lowest we find that in case of X2, when we change the value of X2.

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Model sensitivity analysis example

A simple example of a model is given as $Y = 1.6X_1^{1.36} \cdot X_2^{0.15} \cdot X_3^{0.29}$
 Compute the sensitivity of the model inputs if the original values of the input variables X_1 , X_2 and X_3 are varied in a range of -20%, -15%, -10%, -5%, +5%, +10%, +15%, +20%. Original value of $X_1 = 10$, $X_2 = 40$ and $X_3 = 30$

Change the value of X_1 keeping X_2 and X_3 constant and found the change in Y

$\Delta X_1(\%)$	X_1	X_2	X_3	Y	$\Delta Y(\%)$
	10	40	30	170.9205	
-20	8	40	30	126.1817	-26.1752
-15	8.5	40	30	137.0263	-19.8304
-10	9	40	30	148.103	-13.3498
-5	9.5	40	30	159.4036	-6.73813
+5	10.5	40	30	182.6466	6.86056
+10	11	40	30	194.5755	13.83978
+15	11.5	40	30	206.7013	20.93418
+20	12	40	30	219.0184	28.14052

Original value (given) 40 30

20% reduction of 10 i.e. -
 20% of 10 i.e. 80% of 10
 i.e. = $0.8 \times 10 = 8$ etc.

$$\frac{126.1817 - 170.9205}{170.9205} \times 100$$

$$= -26.1752\%$$

$$Y = f(X_1, X_2, X_3)$$

$\pm 6 - 281$

Model sensitivity analysis example

Change the value of X_2 keeping X_1 and X_3 constant and found the change in Y

$\Delta X_2(\%)$	X_1	X_2	X_3	Y	$\Delta Y(\%)$
	10	40	30	170.9205	
-20	10	32	30	165.2942	-3.29176
-15	10	34	30	166.8042	-2.40831
-10	10	36	30	168.2405	-1.56798
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+20	10	48	30	175.6594	2.772563

Original value (given)

20% reduction of 40 i.e. -
 20% of 40 i.e. 80% of 40
 i.e. = $0.8 \times 40 = 32$ etc.

$$\frac{165.1817 - 170.9205}{170.9205} \times 100$$

$$= -3.29176\%$$

$\pm 07 - 3.2$

Model sensitivity analysis example

Change the value of X_3 keeping X_1 and X_2 constant and found the change in Y

$\Delta X_3(\%)$	X_1	X_2	X_3	Y	$\Delta Y(\%)$
	10	40	30	170.9205	
-20	10	40	24	160.2102	-6.26623
-15	10	40	25.5	163.0518	-4.60371
-10	10	40	27	165.7771	-3.00925
-5	10	40	28.5	168.3969	-1.4765
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+15	10	40	34.5	177.9904	4.136355
+20	10	40	36	180.2008	5.429601

Original value (given)

20% reduction of 30 i.e. -
 20% of 30 i.e. 80% of 30
 i.e. = $0.8 \times 30 = 24$ etc.

$$\frac{160.2102 - 170.9205}{170.9205} \times 100$$

$$= -6.26623\%$$

$\pm 14 - 6.2$

Now, let us plot it, all the value that you have got. Model sensitivity analysis now will be much more clear to this. See, we plot it now, ΔY_{X1} , ΔY_{X2} , ΔY_{X3} means when we have changed the value of X1 then the yield X2 then the yield in red X3 then the yield in green. Now, where you are finding larger differences? Certainly, here in the case of X1. So, it is clear that highest percentage change in Y is occurring due to percentage change in X1.

So, therefore, X1 is the most sensitive among the 3 inputs. I hope that I could clear your understanding, this is the simplest example that I can think of to help you to understand the sensitivity. I shall repeat it again for the benefit of all others participants who are not from mathematical background. So, imagine that it is a just a simple function Y, is a function Y is yield as a function of X1, X2 and X3, 3 parameters, these 3 parameters all of them may affect your yield that is output, that is the simplest model.

Now, we want to analyze the sensitivity of this model. Means, which one of these 3 parameters affect my output or yield maximum. So, what I have to do, in first case, I have to keep 2 constants and one I will change, second time I will fix these 2 and change suppose X3, third time I will fix X1 and X3 I will change X2 and then I will calculate the Y values and then also the differences as I explained here in a very simple way and then you find out the ΔY the differences and range of that like here you found that when you change X you find the differences almost plus minus 6 to 28 percent, 26 percent, 27 percent that range of changes if you change X1.

If you change X2, then you find roughly around 0.7 to 3 percent changes which is much less. That means X1 is affecting more than X2. Then we change X3, here also we find that the changes in yield is 1.4 to 6.2 that is the range which is much less than X1. So, that means in my out of 3 parameter X1, X2, X3 it is the X1 which is impacting my yield the most.

That means, X1 is the most sensitive parameter in my exercise, I hope that this is clear to all of you about modal sensitivity. Now, the simple the way I have explained the same thing now can be much more complicated when you bring in several input parameters. But the basic philosophy basic understanding of sensitivity analysis is exactly that this which I have just now explained and I hope that it is almost clear to your understanding.

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Model uncertainty analysis

- ☐ Uncertainty is the deficit of sureness in the phenomenon.
- ☐ A set of possible states or outcomes where probabilities are assigned to each possible state or outcome.
- ☑ In a model uncertainty occurs in output due to various reasons those are as follows
 - ❖ Uncertainty in model input variables
 - ❖ Uncertainty in model parameters
 - ❖ Uncertainty in model initial conditions, boundary conditions
 - ❖ Uncertainty in model assumptions, algorithms, objectives, stopping criteria
 - ❖ Spatiotemporal variation of parameters

Temp
Rain
 $Y = f(x_1, x_2, x_3)$

Now, sensitivity is one thing, the next thing comes model uncertainty analysis. If you remember the modeling lecture that I have discussed in one of the previous lectures, we discussed about the uncertainty. Model can mimic the real situation, but not exactly the same way, it can never be. We can try our best to go as close as to the original in the field or in the nature. So, there will be certain amount of uncertainty in modeling exercise.

Now, let us see that how that uncertainty we can also analyze and understand about a model. Uncertainty is the deficit of sureness in any kind of phenomenon within the model that you are using. Say suppose you are using a model to predict rainfall. Now, your that model is based on past data.

Now, if the past data quality is not very good, which suppose you have gathered from various weather monitoring stations or maybe third-party data source, if they are not very good, then to your prediction of rainfall will have certain, any model exercise will have certain uncertainty. So, uncertainty of model is that the deficit of sureness in any phenomena. Next, a set of possible states or outcomes where the probabilities are assigned to each possible state or outcome.

I repeat, a set of possible outcome where probabilities, so suppose rainfall, rainfall your modeling exercise can give with some probability that okay there is 60 percent probability that there will be rain tomorrow, you cannot say with 100 percent certainty, tomorrow it is going to rain. So, each of the outcome, suppose rainfall is one outcome, wind, humidity, etcetera, temperature, different outcome.

So, you can associate certain probabilities with those parameters or phenomena. In a model uncertainty generally occurs in output due to various regions while you actually run your model. Now, what are those regions, already some of them we have discussed, still uncertainty in your input variables, past data in the model that you are actually putting.

So, previous example, I saw that Y is a function of X1, X2, and X3. So, these are your data input data. So, the uncertainty in these input data is also one region of the uncertainty in the outcome. Uncertainty in model parameters, that is also another region it could be. Uncertainty in model initial condition and boundary condition. Again, I would request you to refer to the modeling lecture that we discussed previously.

So, if your initial and boundary conditions also has certain uncertainty, you are bound to get certain amount of uncertainty in your outcome. Uncertainty in model assumptions, the algorithms that you choose the objectives that you actually work for and the stopping criteria, means at what point of time you will actually stop your run, model running. Then come spatiotemporal variation of parameter. These also most of the time you will get there is a certain amount of uncertainty and it is very difficult to get rid of these uncertainties completely we have to live with that.

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Model sensitivity analysis additional example

In a simple model for drop inlet spillway which is constructed for soil and water conservation purpose, the output of the is related to the input variables such as length of pipe (L), velocity of flow (V), entrance head loss coefficient (K_e) and acceleration due to gravity (g) as follows.

$$H_f = K_e L \frac{V^2}{2g}$$

Determine the sensitivity (%) of the variables when L=20 m, K_e = 0.4, g = 10 m/s² (Constant) and V = 5m/s when the value changes 10% from its original values

X	X + ΔX	X - ΔX	H _f (X + ΔX)	H _f (X - ΔX)	$S_A = \frac{H_f(X + \Delta X) - H_f(X - \Delta X)}{(X + \Delta X) - (X - \Delta X)}$	$S_X(\%) = \frac{S_A}{\sum S_A} \times 100$
L	22	18	11	9	0.5	1.69
K _e	0.44	0.36	0.4	0.4	25	84.75
V	5.5	4.5	4.5	5	4	13.56

$S_A \times 100 = \sum S_A$

Model sensitivity analysis example

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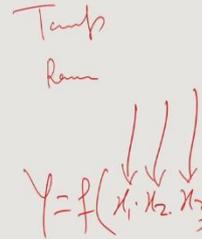
$$= -26.1752\%$$

$$Y = f(X_1, X_2, X_3)$$

$\pm 6 - 281$

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Model sensitivity analysis

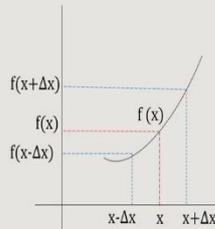
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- Ranks the inputs or parameters depend on their contribution to model outputs. That is it helps to determine the most influential inputs on a model

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Numerically

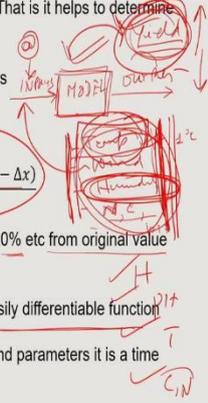
$$S_A = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$



Δx vary from some % like 5%, 10%, 20% etc from original value of the inputs or parameters

Numerically done in case of a not easily differentiable function

If a model has a large set of inputs and parameters it is a time and computation consuming



Now, model sensitivity analysis. Another example, I will try to give where you can understand a little bit more. Say again we take a simple model for suppose drop inlet spillway, drop inlet spillway normally it is used for water management activity which is constructed for soil and water conservation purposes.

The output of this particular phenomena which is related to the input variable such as the length of the pipe, how much water actually going per unit time. So, the length of the pipe is one, velocity V will be another one, entrance head loss coefficient K_e is one, the point this is the pipe the entrance where actually this pipe you connect with the water source, entrance head loss coefficient and then acceleration due to gravity which is g .

So, here you can calculate H_f which is a head force which is a function

H_f is equals to K_e multiplied by L multiplied by V square divided by $2g$

V is the velocity. Now, while determining the sensitivity percentage of the variables like length of pipe, velocity, entrance head, how you can do that. Suppose, the pipe length is 20 meter, your entrance head coefficient is 0.4, gravity acceleration is 10 meter per second square and then which is suppose constant that you are taking because that will not change.

And then velocity 5 meter per second and when the value changes 10 percent from its original value. Now, here you will see length 20, head 0.4, velocity 5. Now, we are changing slowly, X plus delta X , that means, which is 2, 20, 22, 0.4, 0.04, 5.5 means 0.5. So, these are the variations that you are making. Similar way it can be having again X minus delta X another value.

Here, you add plus 2, here minus 2, here plus 0.4 and here minus 0.4, here plus 0.5, here minus 0.5. Now, H_f into X plus delta X and then you get H_f into X minus delta X . So, 2 different value we will get. So, here when you are using X plus delta X , one set of value. And where here you are using X minus delta X , another set of value. Now, you come to your sensitivity coefficient analysis, which we discussed earlier, SA, remember?

Now, this we are trying to calculate numerically to find out the sensitivity coefficient. Is a H_f into this minus this divided by this by this, all will come from these values. So, you get basically 0.5, 25 and 4, 3 different value for 3 different parameters. Now, then you calculate the sensitivity percentage.

Sensitivity percentage is sensitivity coefficient divided by summation of sensitivity coefficients into 100, summation of all these 3 coefficients that is your SA. So, each of the SA of the parameter divided by summation of SA into 100 will give you the percentage sensitivity. So, numerically you can actually also analyze the sensitivity of your model.

So, again I just you know for your benefit once again have a look at that that how we are actually doing the sensitivity analysis, sensitivity coefficient and sensitivity coefficient can be calculated numerically. So, here you can actually calculate it numerically in this process, which we have followed in at the last example, and then here I showed you through a very simple example, why output is a function of 3 parameters and then how 3 parameters changing one at a time to keep in constant and then you find that which one is the more sensitive one.

And then we have another example, where we try to show you know that how you can calculate numerically also the sensitivity coefficient. And then finally, the significant percentage. So, this particular lecture is dedicated to your sensitivity analysis and also to understand the uncertainty in the model analysis and why these uncertainty take place.

So, once again that any model for any purpose whether it is for water or soil or crop the most important part is that you understand the sensitivity of your modeling exercise. Once you understand that among your input parameters, which are the most sensitive parameters, which actually regulating your outcome or output of modeling exercise very strongly, you can rank them.

If you see that your modeling exercise is taking too much of time you can make a call, you can actually choose the first few parameters, because they are the parameters which are strongly affecting your outcome. Most of the cases that top few ones will actually capture 60 to 70 percent or sometimes even more than that depending upon the case, the outcome of your modeling exercise. So, that is the one decision that as a user, you or me we have to take it.