Optimal Control, Guidance and Estimation
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Module No. # 11
Lecture No. # 24
MPSP for Optimal Missile Guidance

Hello everybody we will continue our lecture series, last lecture we have seen the generic theory of this Model Predictive Static Programming MPSP. And we also saw this some of this application in the devisable launch vehicle and things like that reentry guidance of the devisable launch vehicle we can think like that. Here, what we are doing here is actually as I told in the last class, will take you through some sort of missile guidance problems.

And I will take you through a number of prime an essentially two class different class of missile guidance problem, I can see the generality of the and applicability of this particular method that I am talking about actually. All these are practical problems and these are not just bench mark examples or think like that actually there. So, we will see that one by one.

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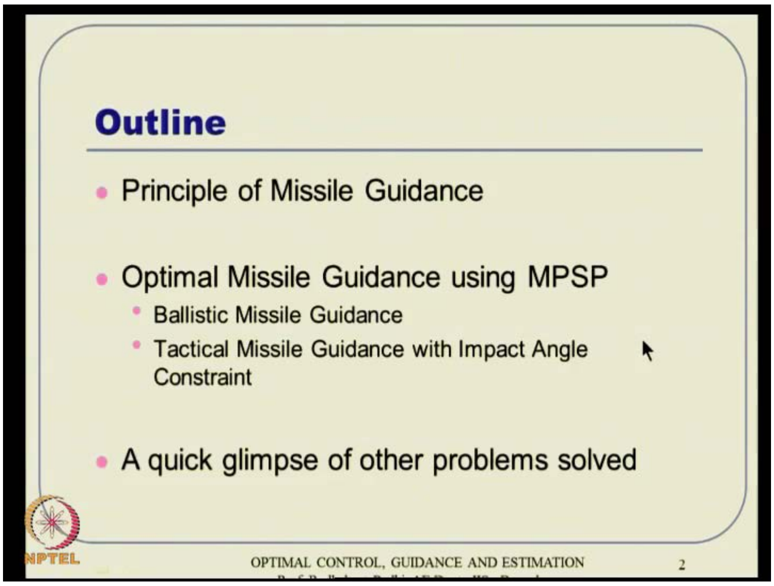

So, outline of this particular lecture some little bit philosophy of a principle of missile guidance. And then I will talk about optimal missile guidance using MPSP, where ballistic missile guidance as well as tactical missile guidance I will talk with impact angle constraint. And also, let me also iterate I mean repeat here, I mean this what I am talking here is more of a theoretical nature, if you want implement any of these in this real vehicle and all you have to do many other things which which is absolutely not part of this this lecture and all that actually. So, do not think that you can just take it and then directly implement in your any vehicle and all that.

There will be several other I mean other thing that you need to verify and verify rigorously rather, before implementing and any think like this actually. But, having set that you can see the the applicability of this particular method and two different class of problems actually under the simplified assumptions that I talking about actually.

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So, first thing is what is this strategic missile guidance this essentially this ballistic missile guidance and all that we talk about. It essentially some vehicle takes off it lifts off from here and then after this some days some height covered really, your dynamic pressure goes down and that is where your you can think of maneuvering your vehicle rather easily actually.

So, then this something called guidance loop closure, so from there onwards the vehicle is actually guided up to that is some sort of preprogrammed maneuver it is not really vertical. But, preprogram maneuver little bit turning sort of thing which will essentially go in the direction of target but, not really very close to that. But, essentially it will also make sure that the vehicle is safe actually.

So, with in that is the first part of phase programming and all that and after there is guidance loop closure and here, onwards the vehicle needs to be turned actually. So, this is what will happen, the vehicle will keep on turning, turning, turning the then, there will be some sort of a either a thrust cut off or there is a burn out thing.

That means if you if you go through the solid motor propelled vehicle, then you have to wait until it burns out or if it is a liquid engine propulsion actually, then you can think of cutting off the thrust anytime you want actually. So, the guidance phase starts somewhere here and end somewhere here and it, at this point what it should happen is your height, mean ultimately the location as well as the the velocity magnitude and it is direction in that is the this flight path angle gamma.

So, satisfy a particular condition, so that further onwards it actually goes through some sort of a free flight and this free flight will be target intersecting actually this, what we talking about is a strategic target sort of thing on the surface of earth. So, essentially it will go to in the free flight mode and what you are looking for is that particular trajectory which will essentially fall on the desired target actually that is what you are is very, very fundamental or basic philosophy think like that actually.

Also you can you can somewhat very cleverly you can see, that essentially this problem is can be extended to launch vehicle guidance as well, that what you are looking for is you are you are looking for some sort of a trajectory, some sort of a condition with a lot more velocity. So, that it will go and actually it will come like that; that means, your earth curvature like that. But, it the trajectory is not intersecting the earth actually; that means, it will go round and round and then it essentially is this this satellite orbit sort of the thing actually.

So, this particular film or method what I am talking here is relevant to both ballistic missile guidance or strategic missile guidance as well as, this launch vehicle guidance ascent phase guidance actually in general alright. So, the also remember the the condition as to be such in I mean as to such that ultimately you will be cover this range angle actually. Essentially this is 5 t the target range angle at from starting from are not actually that way that becomes an additional constraint actually to us that way alright.

So, these are this is what the problem is, philosophy is and before I forget remember all the correction that needs to be done is here. And this happens to be done even though the picture is amplified a little bit for quality, this happens to be very small segment of the trajectory for about something like 5, 10 percent of the trajectory. So, for rest of the trajectory for about 85, 90 percent of trajectory happens to just free flight actually.

And so, in other words we was actually trying to kind of throw a bullet out here and throw a stone or bullet like whatever you think about. So, were the stone after throwing you do not have any control but, you thrown in an such a way. That it will go in this free flight and then, intercept the target actually, that is that is the problem here.

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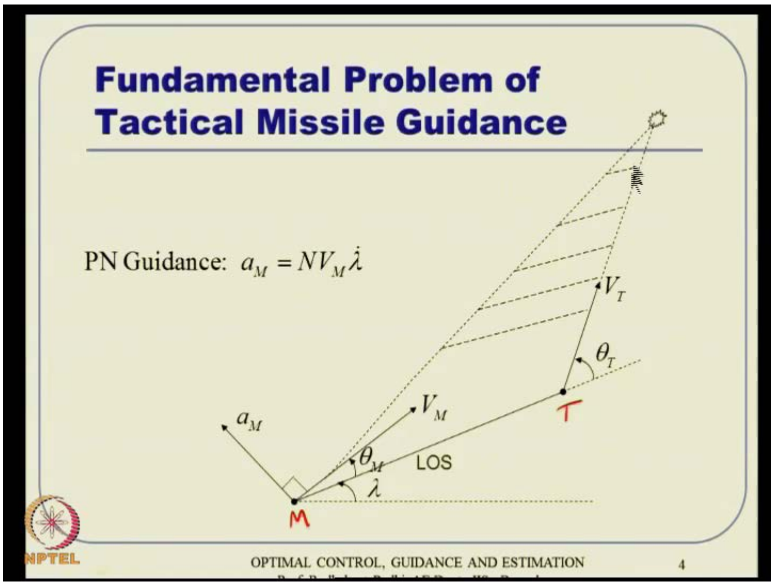

What about the the tactical missile guidance, see you have got a target which is not stationary which is which is kind of intelligent actually. So, target keeps on moving and we have a chasing vehicle. So, we try to kind of chase this vehicle this this let me this this is the missile that we are talking about the target that we are talking about the target has it is velocity V T and missile it is velocity V M actually. And there as couple of terminologies out here this line joining the missile line target is sincerely something called as line of side.

And then from the horizontal reference line if you if you draw some angle I mean this LOS will make some angle that angle is nothing but, LOS angle sort of thing. And then this wait wait LOS angle and with respect to the LOS angle the velocity vector of the missile will make some angle theta a M actually and the target will make some angle theta t. So, remember again that this V T the target is going it is capable of maneuvering and all that actually. So, it can can also change it is velocity vector direction that way. So, what way you can assume that this V M is essentially guided towards that or essentially it should go and hit the target that way.

So, the one idea here is you have this lateral acceleration, generation mechanism through the lift vector. So, well as remember the drag is along velocity vector, opposing that and the lift is perpendicular to that. So, we can think about lift by mass, which is essentially some sort of acceleration quantity and that is called a lateral acceleration actually. So, the idea here is if you generate some sort of a lift traditional lift and all that and what about direction positive or negative we do actually.

Then we are essentially able to change the velocity vector direction that is important actually. So, once you change it essentially it will happen in such a way that, ultimately you you would change a little bit here. So, that if the velocity vector goes, the target goes that way the velocity vector of the missile will be aligned somewhere like that and then it will both will ultimately go towards some sort of a polygon point actually.

And if you little bit closely looking what it turns out that after some time the LOS vector what you are looking looking at this LOS line. The LOS vector remains parallel actually; that means, there is no change in the rotation, I mean there is no rotational LOS vector really actually. So, that is that is what it is fundamental philosophy, see in other words you have to do something, so that this LOS vector should not rotate that is the whole philosophy of a classical missile guidance actually. So, there is very popular missile guidance law which is called proportional navigation guidance, which essentially tells you that you generate this aim that is your essentially the control variable in the guidance flow this way.

It means N times we have into lambda dot, lambda is this and then there is rate of change of lambda ultimately you do not want any rate changes actually. You know that the guidance law refers to be something like this the generator lift vector and this time varying value you know that the generate time in this as for this formula. And V M lambda dot V M is already known to you from your own sensor, lambda dot is against sensed through it is own navigation system and think like that. As well as the the target tracking mechanism; that means, either using radar or signal or rather actually typically these are red games.

So, you see here actually, which will in other way you will directly see the target from using your own instrument actually. So, anyway, so this lambda dot and v m is the only thing that you need to evaluate and then, once you do that n is just a constant number. So, that your typically value is something like three four and maximum five probably like that actually ideally around three. There is a reason for that, we have discussed that in one of the pervious lectures as well actually.

Anyway, so coming back to that this your a M and typically generated that way and it it happens to be kind of an optimal guidance law in a in a restrictive assumption sense and all those things we have discuss before already actually.

Several re several conditions if it meets those conditions this is this is nothing but, an optimal guidance law provided n is lifted as three actually. So, that is the reason why n is hanging around around the three value basically anyway, so that I will not repeat those it is a new one actually. So, this is a very simple guidance law it has been it has been used in many missiles and all that actually it is still continuous to be used and there are many persons around that as well, something like true proportional guide True Proportional Navigation guidance that is called TPN.

Then there is something called PPN Prior Proportional Navigation guidance, something called augmented proportional navigation guidance. And something called modified V N and think like that are various persons of that actually. But, all these can be dumped into clubbed into this classical missile guidance ideas actually this is ultimately, what it happens in the tactical missile guidance sense. And remember this picture is in 2-D and ultimately everything in aerospace happens in 3-D.

So, there are extension of 3-D as well actually you have to apply is lift vector in two different directions actually. And it is possible because, as missile you can you can think of generating lift in two perpendicular directions, which have go through with which are both perpendicular to V M directions. Perpendicular to V M is a plain and in that plain you can. So, construct this two different lines perpendicular to each other and then try to generate in two different directions actually that is how you can take care of this 3-D problems actually.

Anyway these are the fundamental philosophy of missile guidance, both strategic as well as tactical actually. And to summarize many missile guidance laws are actually inspired from observing nature that is as I told proportional navigation guidance law, is based on ensuring this collision triangle. I mean this what you saw here is essentially collision triangle, basically we have to assure that the collision triangle is satisfied actually.

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And there are control theoretic guidance laws also available; that means, you pose some sort of a kinematic relative kinematics. And all moving in a little bit point mass dynamic sense and think like that linearized point mass dynamics think that; and then they there are control theory things available, which essentially tells you that somehow I i know that this line of sight rate should be zero (Refer Slide Time: 12:25). So, what should I do what feedback law should I use, so that the line of sight rate remains zero actually.

So, this kind of ideas have been explored but, the typically use this linearized dynamics actually, because using that you can get a close functional and proceed further for analysis and synthesis basically. However, remember the linearized neither linearization nor nor the this observing natural is actually a very natural way of doing things or very vigorous mathematical way of doing things rather. So, what it happens is the system dynamics turns out to be typically non-linear. And hence non-linear optimal control theory is somewhat in my view a natural tool for obtaining effective missile guidance laws actually.

So, let us see how to do that but, then the as I told if once if you have non-linear optimal control formulation then, it typically gets into this computational issues. However, because of the recent developments like that like MPSP and other things is possible to give some of these actually. By the by, all this this MPSP this, I means head optic critic the SNSC and this theory DN.

And then, all this I will also talk a little bit about model predictive, I mean model predictive static program MPSP is what I am talking about. And then pseudospectral method and also this model predictive control process, these all these things are actually they developing towards this online application sort of things actually.

Anyway so, these are the things, which essentially what I am telling you is this non-linear optimal control theory is a natural tool to obtain effective missile guidance laws actually. But, the problem here is how to address this computational efficiency, I mean issue and as we saw in the last lecture, MPSP happens to be one of the candidates, which essentially guarantees you for computational efficiency. As well as guarantees your terminal conditions being met it is hard constraint, which is more important in the missile guidance actually.

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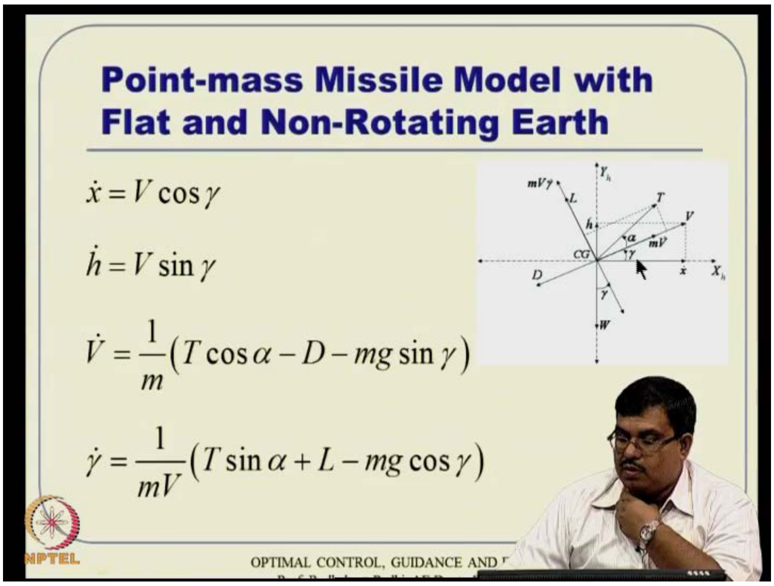

Alright so, this is when you do that attempt to that first you need to have some sort of a relative kinematics or relative dynamics between that. And this what will what will happen point mass missile model with flatten on rotating earth if you take it very easy to analyze that x dot happens to be v sin f this x and this is y lets say. And this x dot is nothing, but v percent gamma and x dot I dot is nothing but, v sin gamma these two are very easy to see. But, these two if you I means v dot and gamma v dot and gamma dot if you see that, essentially this is nothing but, force balance in the direction of V and perpendicular to V.

So, m V dot is essentially the net the total force acting along the along the v direction; that means, m V dot is nothing but, the force component coming out of the thrust, which is T cos alpha minus the drag acting on that. And minus the gravity component m g sin gamma, m g sin gamma will will act on this particular, I mean this m g happens to be like that.

So, m g sin gamma is some some component here in this directions actually which will also oppose to the m V dot component. So, any doubt is t cos alpha minus d minus m g sin gamma and hence is hence v dot is nothing, but one by m of all that actually. Similarly, if you see the force balance net force balance, net force acting on perpendicular to the velocity; that means, the lift direction, then essentially it is T sin alpha T sin alpha plus lift itself minus m g cos and gamma, this direction actually.

So, this thing if you divide it by I mean if you see that there is nothing but, m v gamma dot happens in that direction. You can typically you can think about this this gamma dot acting on that this m v and then think like that actually; that means, it is like a centrifugal force sort of thing actually that way. So, gamma dot is nothing but, one by m d into this quantity.

So, so if you just take a 2-D problem and then you take assume that flat earth and non-rotating earth and all that these are system dynamics, that is necessary, first to is represents the position coordinate. And these two represents the velocity component dynamic variable, again if you go to the 3-D sense, then what you what you need is y dot also here and what you need is I psi dot also here, psi dot are some time called I psi dot actually.

So, v gamma psi together and x y and h together will define the position actually x y and h together will define the position, v gamma and k psi will define velocity vector completely actually. So, the o, what you suggested 2-D problem then these two are kind of not needed four equations are good enough actually and and also remember this t this D and L will contain the alpha terms actually.

So, what you see simply as question alpha and sin alpha here these does not they do not appear just the like that way D is essentially a function of C D and C D is function of alpha actually. Similarly, L is function of C L and C L function is of alpha. So, this happens what happens in MPSP maneuver sort of thing in 2-D but, if you take top 3-D, then this sigma also has to be taken into account velocity vector, I mean the vehicle row and think like that has to be taken on picture actually. And then alpha and sigma will like together some sort of a guidance parameters actually that way.

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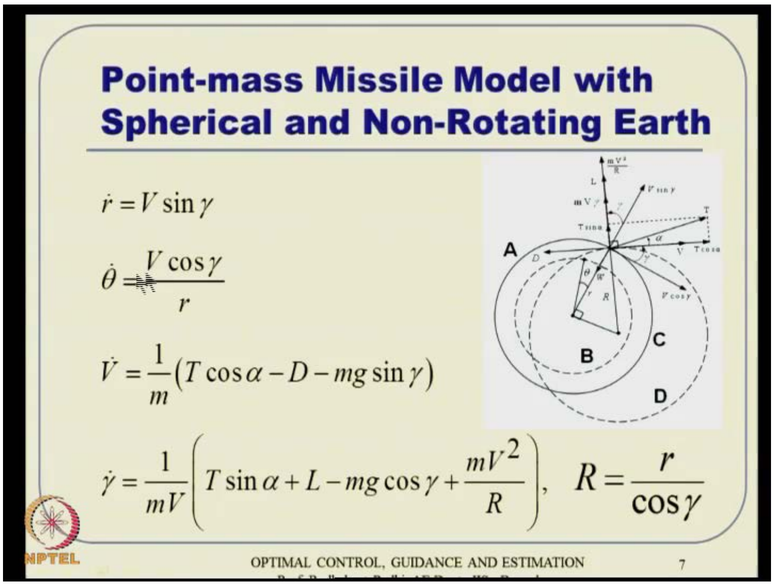

Alright then point mass missile model with spherical and all this is what happen in in flat earth sense flat and non rotating earth. What happens is spherical in non rotating earth that is what we are going more and more towards reality, but the how much reality you want to get for depends on particular machine actually. In other words if your vehicle happens to be in the range of 20, 30 kilometers, you greatly do not bother about this non program in this spherical nature of the earth and all.

But, it it the ray that target range happens to be quite far away they let us say 1000 kilometer, 2000 kilometer like that then you have to account for the curvature of the earth actually. So, essentially it will lead to spherical, now whether you want have rotating or non rotating affect again that depends on your time of flight, and your latitude of launch point and target point different, latitude difference between launch point and target point actually.

So, that is that again the point here is you take that much of complexity, what is required for the problem actually. Now, as less it is possible to handle various degrees of complexity actually. So, the point mass missile model with spherical and non rotating earth typically what we used in in strategic missile guidance and all that is something like this.

So, you no more talk about x dot and s dot and all that, but r dot is is kind of is dot r is nothing, but radius of plus I, so r dot is s dot actually. So, this V sin gamma and this theta dot is now the the reference angle thrust from from, so for a particular reference line actually. So, this theta dot you can think about this may be this point is last point, then joining that to center of earth will define the reference line.

And then with respect to that reference line, you can talk about an angle theta with respect to the current position of the vehicle if you join to the center of the earth. The angle between the two is nothing but, theta that theta dot later change of that angle depends on the the tangential component of the velocity vector, which is nothing but, V cos V cos gamma. V cos gamma happens to like that you can think about, what is happening here in the picture, is this is nothing but, the surface of earth the radius of the I mean circle V.

And circle A is parallel to surface of earth, but happening at the current location of the vehicle actually. That is where the all dynamics happens like that actually, but remember this various velocity vector can be anywhere for this, velocity vector is not tangential to the the circle. So, what we are telling here is let us imagine one more circle here, which will, for which the velocity vector is tangential actually.

And then it turns out that if you join it to the center of this particular circle, can think about something like here. And this this the radius are big capital R and this radius small r and all that actually. And this happens to be this r e radius of earth basically. So, all this little bit force balance analysis, if you do it properly then all that you will end up with this set of equation. Let me not go through the derivation part of it, but I think if you just look a little bit carefully what is m p (Refer Slide Time: 21:03) dot for example, m p dot is all the forces acting along that directions again the component, see here the guidance parameter is the thrust deflection angle.

Let us say that means, how was the thrust is reflected from velocity vector directions, do not get alluded by this this alpha is not really aerodynamic angle of attack for say. But, if you think the vehicle is still under atmosphere and then thrust is a along the body of the I mean body x of x is of the vehicle which is typically true then this alpha happens to be angle of attack as well actually. But, if your x atmosphere machine nothing called angle of attack actually.

So, think about that this alpha being something like a thrust deflection angle with respect to velocity vector, that is how it is the t cos alpha minus d minus m g sin gamma again. So, that that happens to be like that, but gamma dot will have a little more complex see, earlier it was only this much now this this additional term will come because some sort of a centrifugal force. And think about this force what you are looking here is like centrifugal force with respect to this particular circle basically.

And angle gamma dot happens to be in the in that particular direction actually for medical see some some text book and this this dynamic equation derivations all that actually. Now, coming to the first problem, we will talk about velocity missile guidance, which solid motor especially, ballistic missile when I talk is the these are strategic vehicles and all. And again and again I emphasis here these are all theoretical studies, I mean the lot of things are actually. Now, nowhere close to practical things, but essentially we have taken as much the realistic things as possible to experiment in a academic environment. So, that the contractual things can be kind of verified actually, what you are what you are aiming here is verification of the technique.

Now, one going for particular machine and trying to kind of mechanism everything around that actually alright. So, just to a little bit of motivation why you want to have a solid motors, first of all this class of vehicles what is what is strategic vehicle or typical meant for deterrence actually; that means, these are not kind of a taking vehicles and all, but if some enemy already attacks us in a way. Then there is no choice you have literately go back and think like that so; that means, the reaction time happens to be small actually; that means, you really want quick firing.

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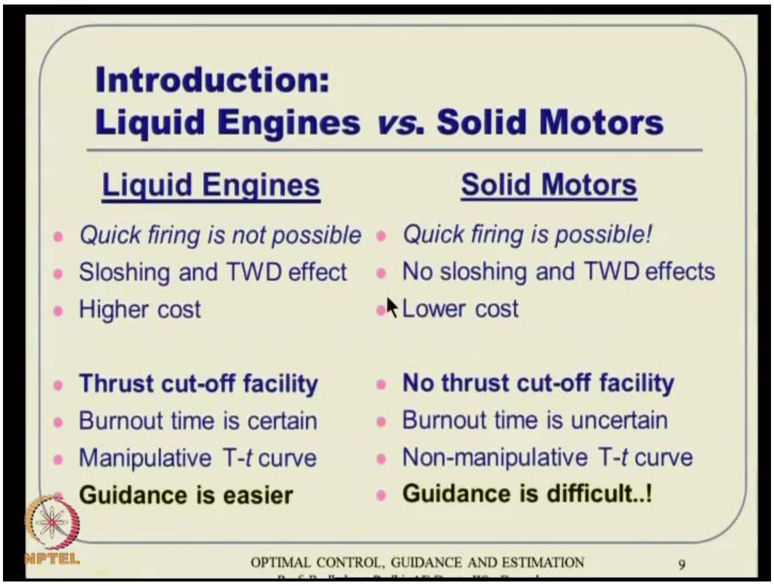

And quick firing is possible only with solid motors not in liquid engines, when you have liquid engine propel vehicle it requires lots of preparation times and all that actually, why for and all the difference reasons for that. The, I mean the fuel tanks has to be pressurized well and it has to be and it cannot be done, it cannot be kept like that for a long time. The liquid has to be pumped in as and when it is required because also corrosive liquid and all that for there are reasons for that actually, why it cannot be done.

But, here it is possible because, once you have a solid motor you just have cursing and then just maintain it keep on doing that actually. Where and when is needed you can simply start firing it actually with very minimal preparation you can go for missile actually. And there are reasons for the other reasons as well essentially if you see solid motors there is nothing called sloshing, because you know no liquid actually.

So, sloshing problem simply not there and there is nothing called tail works dog effect also TWD stands for Tail Works Dog typically dog works tail, what there is concept in aerospace, which talks about tail works dog actually. That means, if you talk about liquid engines there is a huge there is a there is a kind of engine shriveling concept actually; that means, the entire engine is kind of shriveled using this motors and all. And this is dynamic correction is just not one side correction and stop actually it keep on changing left and right like that way.

So, when when the frequency of this isolation happens to be close to the natural frequency of the vehicle the vehicle goes through the violent isolation and all essentially there is a missile failure actually.

So, those things those are not here in solid motor because essentially there is no engine. So, in the engine shivering shivering mechanism is not there actually. So, what is there is essentially this nasal deflection ideas and all, so the for that you do not require that kind of heavy movements and all. So, typically not there actually, cost part I will let it into let it I mean, I will not like to go into that, but typically it turns out that, sincerely solid motors vehicle happens to be slightly cheaper actually, the advantage.

So, what are the penalties actually, so penalties is absolutely no thrust cut off facility and there limited attempts people had done, and all that is the different study with some plugs take out the some apisal drops and then the fire gets extinguished.

But, we do not want to talk about that, in general what happens is there is no thrust cut off facility. Once you fire it has to it has to adjust its energy whatever is there and then it has I mean do the machine actually. So, you do not have this luxury of cutting of the thrust as and when you use actually, then there is this issue of burn out time been uncertain. And there is an issue of non manipulative thrust time curve also, if the this thrust magnitude cannot be increased or decreased on the other hand liquid engine is like some sort of a car engine actually.

If you want more power or more thrust you want more actually the that kind of thing can be done here, but here it cannot be whatever is stipulated or whatever is per selected thrust time curve around that it will operate it. And also remember thrust time curve, whatever is specified then it it is not going to happen exactly like that there will variations around that actually. And part of that will be kind of known variation and part of that is uncertain they are typically not known actually that way.

Nevertheless, it has to happen your guidance must end at the burnout point and that should satisfy the target intersecting trajectory and all that actually. So, essentially the guidance part becomes quite difficult here.

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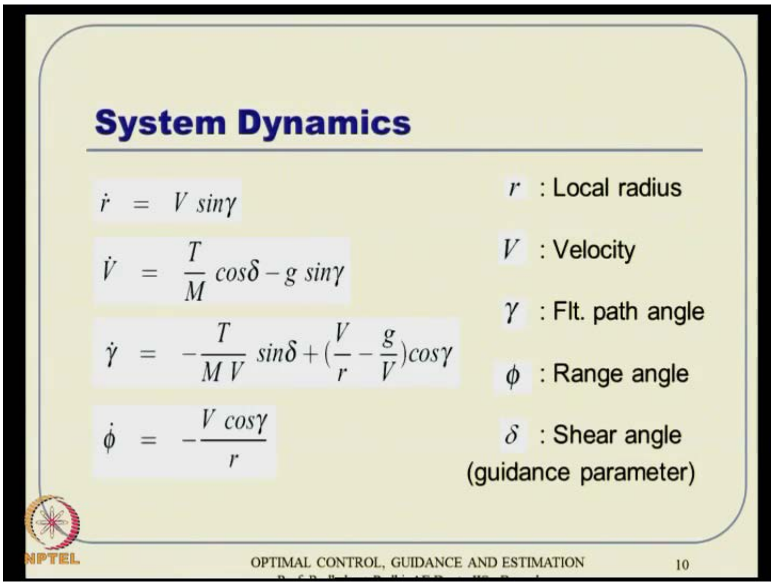

Now, system dynamics these are the variables, what you are looking here and then alpha is redefined as something like shear angle guidance, guidance parameter delta sort of thing. And using that this system dynamics is rewritten something like this and neglecting the earth rotation actually.

Earth curvature is taken into account earth rotation is not actually. So, these are the system dynamics, then there some concept this climate this same picture the beauty part is this free flight equation is kind of known, if you take about a non rotating spherical earth (Refer Slide Time: 27:34). And this part of the trajectory is actually given by this sort of equation is all derived in 1950's and all that actually this is equation which will govern this part of the trajectory.

Now, what happens is this the current this is the any point of time this is r and this is my angle theta basically, what you see here. So, if I really think about my, this target hitting sort of ideas target intersecting sort of ideas, this some this equation from this equation you can drive the. So, called this hit equation and all that what it turns out is just substitute r is r T and then substitute this theta is something like (()) actually.

(()) is nothing but, this 5 t minus 5, 0 sort of things actually; this five burn out this this angle, what you are looking for actually I am sorry. This angle what you are looking for this angle is nothing but, 5, 0 minus 5 t. So, this angle has to be I mean satisfying here basically anyway. So, this is the part, what you are looking for more and that you can see reference book and all that I mean very neat book very popular book actually. All over the world, which is fundamentals of tactical and strategic missile guidance and all actually. So, if you take that you will see some of the details of this free flight equation, hit equation and all that there are papers like ADI VILLAIN and all long time as I told around 1950's.

And all which is, what these equation derived first, anyway we will go back to our system dynamics and then this is our objective. So, this will essentially give us this output that you are that you required at when the vehicle goes here my output, which is my location variable let us say has to satisfy this equation actually, that is all I will say.

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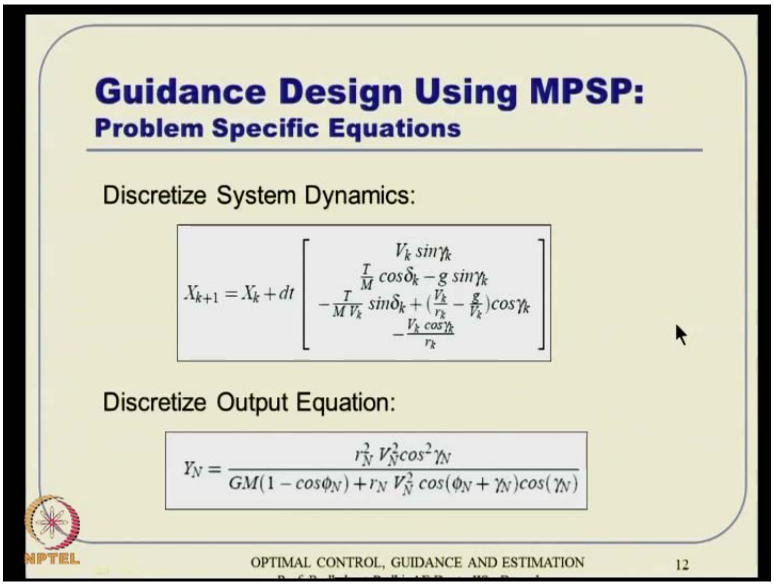

So, we talk about this rest of this system dynamics and then this gives this hit equation gives us this this output equation actually. So, once you have system dynamics discretize system dynamics and there is a discretized out out equation we are ready to apply MPSP actually anyway, we have done that. And then these are the some of the other thing that that is coming into picture the, this discretization of the state equation for control computation have used Euler method.

However for the state prediction part of it we have used r k RK 4 method actually. Remember, you have to predict there is a prediction correction sort of mechanism you have to see the previous control history predict, where it you are going see the error and then correct the control history actually. So, the prediction part you use RK 4 and the correction part you to operate some math algebra and all. So, there you have use this this Euler method and all that actually, so the algebra we can simpler actually.

So, then the step size we taken hundred mille seconds typically this are around that picture actually in in reality. And then guidance loop closer happens after one second of burnout time actually and remember we need a bunch of derivatives they are all computed symbolically. And iteration unfolding and also when implemented but, without that also results are alright actually.

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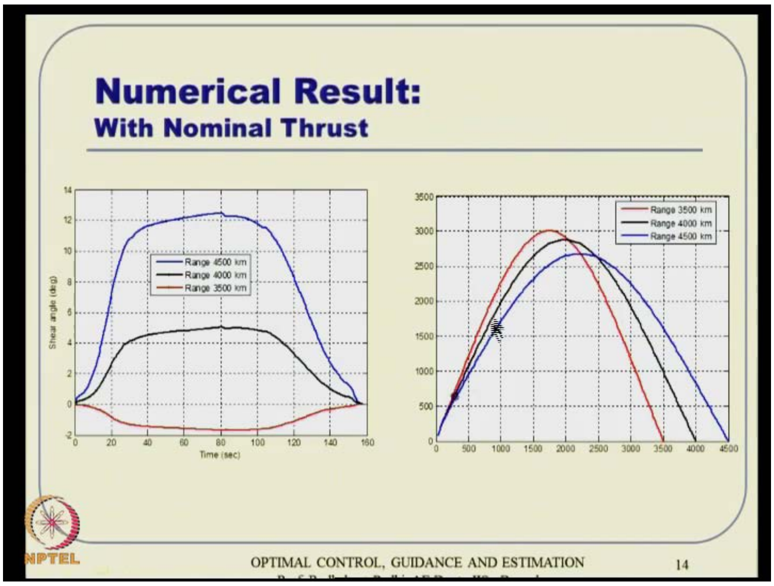

So, these are results you can see that various range actually, for various ranges if you even start with same initial condition the algorithm takes care. And ultimately gives you this this control history it adjust the control history in such way that, no matter where you want the target suppose you want this 3000, 5000, 4000, 500 wherever you want.

Starting from the same initial condition, the control history is all happening here at this segment only, after that this part is a free flight trajectory sort of thing. So, the point is here working and working very well rather actually and these are the corresponding thrust anchor that is a that is also there.

But, also remember this is a let us say suppose the middle part is what is what was prescribed to us. However, in reality it can happen both downwards with larger burning time or upwards with smaller one in time actually. For these things you can actually there is a need for online thrust real adjustment, depending on the motor performance actually.

That details I will keep here but, essentially the idea here is as you start flowing you will see were, whether they are actually underperforming this way the thrust is a. The thrust that is expected is actually coming out to the lower or it is over performing that the thrust is coming out to the higher actually. If it comes out to the higher it will burn out faster if it comes out to the lower it will burn slow and hence it will burn out later actually.

So, with that the fundamental idea here is, no matter what it does the idea under the curve remains the same, that mean if I really ignore this trajectory this base line trajectory. And also the area may be curve which is obvious any way and then depending on the performance of the motor online, I can actually redefine this this trajectory thrust in curve.

And this read, I mean this redefined thrust time has to be used for for computing here your guidance problem and and hence you really what you need is in an online algorithm where you can implement this actually. So, this is what it is and also just a small comment the realistic, I means solid motors and all will will be in a very different shape it then they call something called m curve and all that. But, here the experiments are not done with respect with those these are these are the thrust time curve generated artificially from sort of a vehicle sizing study and think like that; from very first fundamental principle using the text book analysis sort of things actually.

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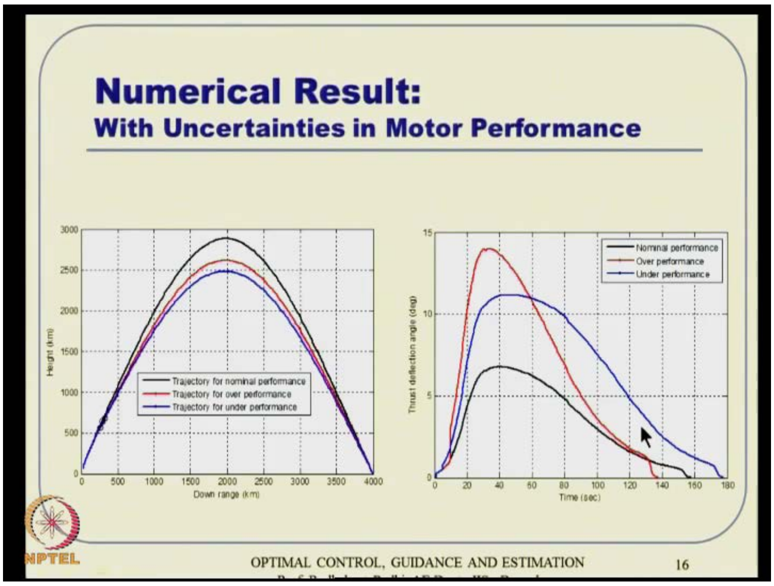

So, like that anyway the point here is not only it meets various target condition but, it also meets the same target with by that the vehicle is under performing, vehicle motor is undergo underperforming or over performing. If if you start with a semi cell condition aim for the same target, what your whether the motor not performing express than predictive normal way, nominal condition.

Where it either it over predicts or not predicts under the thrust time curve is readjusted online. And hence there wherever you start readjusting there will be a different trajectory actually from there. And then the point here is the online algorithm happens to work invariably actually.

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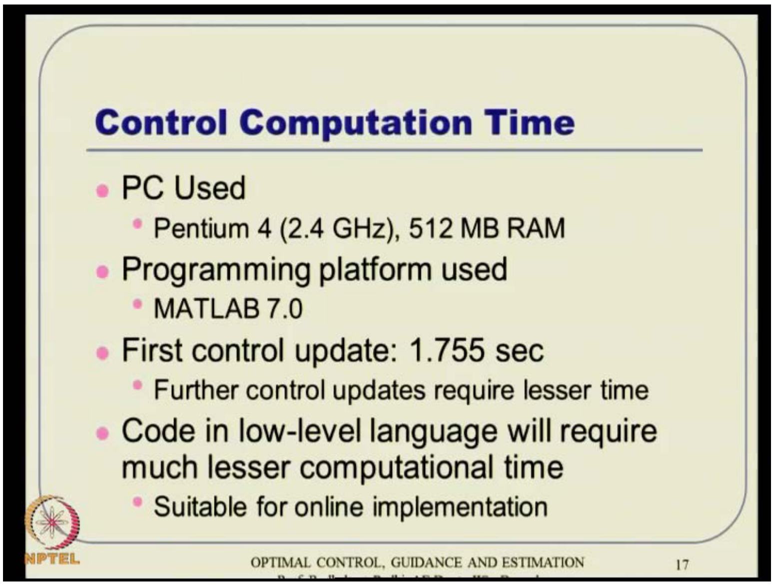

Then how about computational time that is what we have been claimed that this method is computationally quite efficient and all so; obviously, there is a necessity of computational time as well. But, this is typically done in a different platform were are real time computations available and then algorithm needs to be coded and it has to be evaluated properly. And it has to be further evaluated in this online, I mean this onward computer platform and all that we have not done anything of that, I simply taken the same p c that we have use and try to find out evaluate, how much time it takes actually.

And this evaluated in the pentium first, lightly older version of the computer with 512 MB RAM and programming happens to be the mat lab seven platform the first control update, which is the longest one it talks about, I mean takes about 1.75 seconds actually. So, typically a code written in a low level language will require much lesser computational time and if your code is optimized also an it runs only that thing nothing else then; obviously, the the real time is use can be evaluated properly.

But; however, you can think that it can operate safely about one is to hundred is to sort of things actually or may be sometimes it will tell it is one is to two hundred ration rather actually. Remember mat lab is a line interpreter is is compared to that the code will actually generate a compile code and then try to run fast actually. So, the point here is this 1.7 seconds remember this is not minutes or hours seconds actually. So, within two seconds, we are able to get a control stable update actually in mat lab platform and then, where the waylon process are also running actually. So, in that sense I think people do agree that it I mean online implementation might it is possible to do actually.

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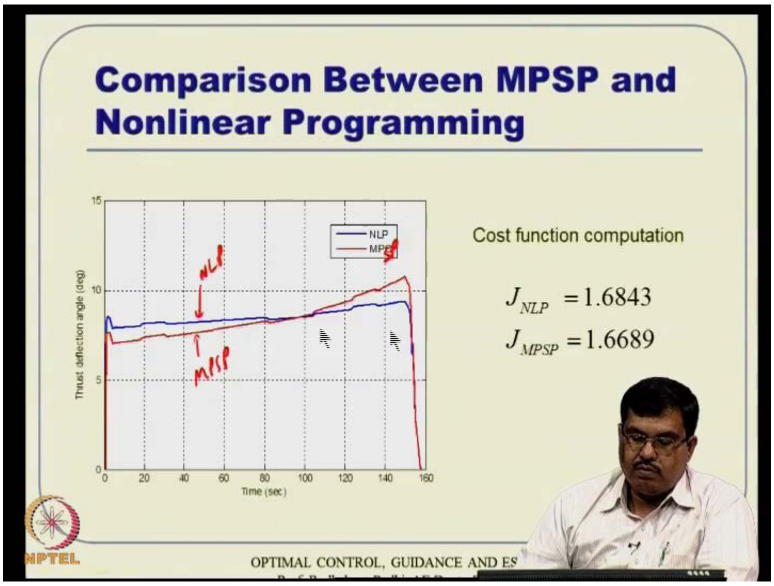

Now what about this performance cost function performance with respect to certain lecture something like true optimal control formulation. Because MPSP is ideally speaking cannot be called optimal method but, rather some people trying to kind of emphasis that it has to be called as sub optimal approach actually. The reasons for that again and one of the happens to be because we are stopping.

We because, we are actually bothered about output convergence, we are not waiting until cost function convergence and all that actually. But, how about looking at this actually suppose the output is converse, then where those the cost function lead to, it may be a slightly different control history for say. However, it the cost function seems is essentially leads to almost the same value actually, because there is some energy loss here, I mean area loss here or again here and think like that way or probably you can think about this is the N L P solution. The blue one is the small mark one, let us say I solve from direct transcriptions which I this using N L P approach and all as I i will discuss in some of the lectures later basically. Essentially it actually solves tries to solve the full optimal control problem as it is actually.

So, compared to that this particular methods that, we are talking about well this has to be is not MPSP it has to be S P basically. So, this this part this this part is a M P S P and this part is NLP as it is transcription method sort of thing actually. Alright, so, this this is the closeness, that we are looking at, so what message is the MPSP actually led leads to very close to optimal control solution actually.

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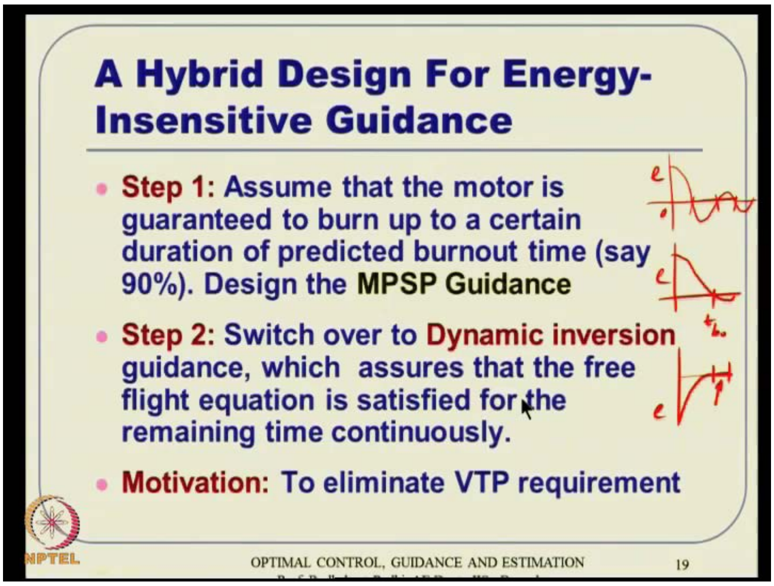

Then there are issues the problem does not stop there, there are issues like the we have assumed that the thrust anchor the area of the thrust anchor remains constant. But, it so, happens that it it need not be accurately known basically; that means, there may be uncertainty on that value itself basically. So, whatever scaling we are doing or even the nominal profile, that we are using is actually an incorrect, because the energy contained may be different actually.

So, in that sense, we have actually the extended the work little bit further I am not going to talk you too much on that, because that is out of this optimal control domain sort of things actually. But, the whole idea is we first assume that up to 90 percent it is actually easier ninety percent of the predicted energy is actually available without fail.

So, up to that, we will design an MPSP guidance and then further remanding part. We will switch over to this dynamic inversion based guidance details of which you can find it in our references all that, which actually ensures it this MPSP guidance will ensure, that you you the error goes to zero, but what is more important is error should stay at zero also basically.

So, essentially the whole problem here based on because the the error function can cut the point several things actually. So, this this is error and this is zero value. So, there are several places it can cut, now if it is liquid engine you can stop anywhere and the job is done. But, if it is solid engine it the I mean it has to it has to cut where the burn out point actually it has to cut at t v o only, that is the major restriction.

What happening in addition to that, what happens here is what we are talking here is error value should whatever this positive or negative either way. Wherever it it starts it should go to zero, but going to zero is not sufficient it has to stay at zero at a final segment actually. Now, if it stays at zero then any time any time the thrust burn out happens in the segment.

What you are looking for in this segment, happens to be in energy insensitive; that means, the error error is still zero anyway basically. So, so going to zero part this is the first part that is that this part is ensured by MPSP but, staying at zero is is ensured by its dynamic conversion. And also remember there was a necessity that, if you aim something like that, then you also aim that while going to zero the derivative should be also close to zero.

The derivative is the other words the approach should be like the way I have shown you actually. The approach is somewhat like this let us say then, you are talking about an infinite slope here and then suddenly correcting to zero, there will control discontinuity and and any discontinuity at the end is extremely difficult to track, we just cannot do it actually. So, this is not not an option basically.

So, what you really need sorry what you really need is something like this kind of a thing ah sorry, where they are also not only go to zero but, it should stay at zero; while going to zero the derivative should also approach to our zero actually. So, then maintaining becomes easier actually. So, that thing is we done a little way, I am not going to talk too much on that, but essentially it it has been done it is assured and the whole idea here is it actually kind of eliminates this this.

So, called VTP requirement is a Velocity Twining Packers they call, so what it does is even though talk about a solid motor, then along with your warhead you take essentially take a small liquid engine. And then derive the benefit that, the liquid engine can be manipulated like one it can be used to cut off the thrust any time, you want actually and because it is it is sitting on the along with the warhead the correction can can can be done any time along the path actually.

It remember this you are going back to this this thing imagine a situation where the there is a war, I mean this there is a war keeps on going here. So, along with the warhead there is a small liquid engine. So, any amount of inaccuracy that you see somewhere, it can be corrected any point during the free flat actually; the entire free flat is available to you. So, again this is done in a initial phases as much possibly, because that is where the sensitivity is high.

And hence the control effectiveness is high. So, very close I between this point one or point to that I mean this, we assume height point with in that the the solution is I mean the correction has typically applied actually in general anyway. So, that is the velocity trimming package they call, they just small liquid engine, which goes along with the war but, the problem with that is when anything, that you have along with the warhead. The there are two things one is it hits out the space that and it also hits two pollard weight and here, what we are doing is actually you are talking about something like a compromise in a in warhead size or your come talking about compromising something like the range actually.

So, the whole idea is can you really eliminate VTP (Refer Slid Time: 42:13) or at least minimize that requirement to a very great value basically. So, that is what we are claim that it probably can be done looking at the results it it turns out to be slightly more promising actually.

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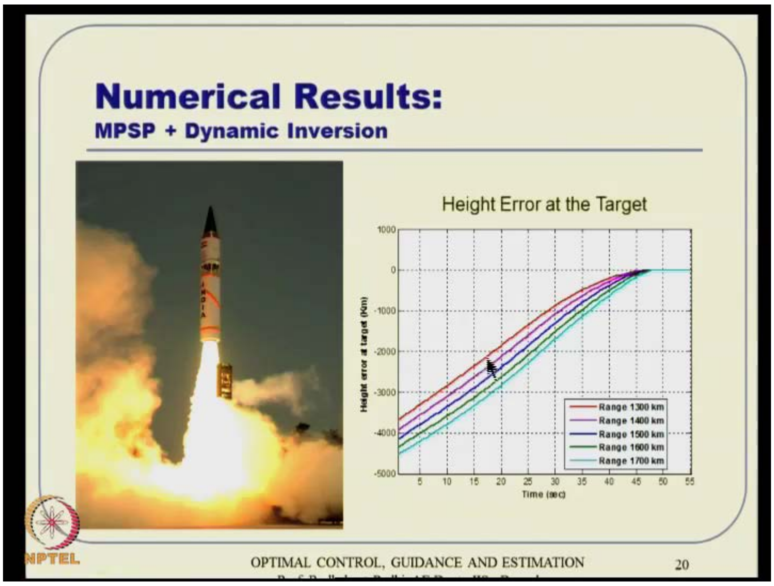

The result part you can see the all the results not only they are going to zero, but they are staying at zero towards the end. That means, any time that is the burnout can happen but, still here error will continue to be zero and hence all of that will target intercepting actually alright.

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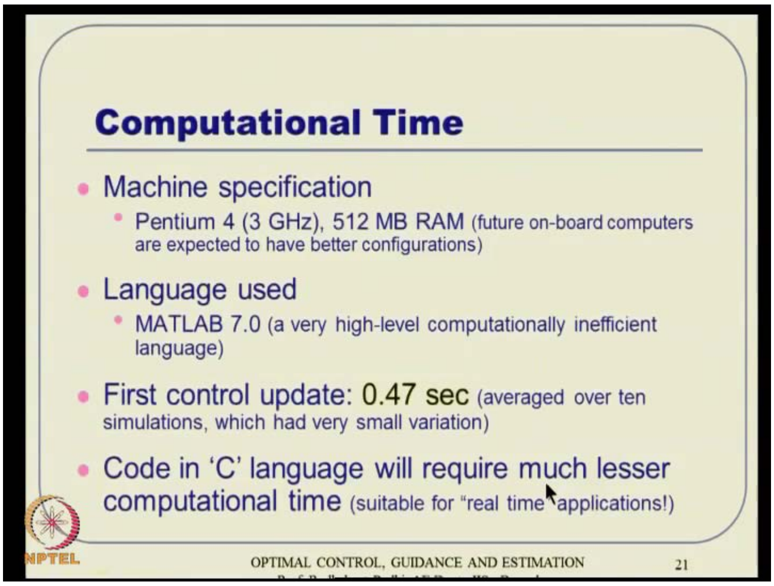

So, this is what it is actually and then again computational time point of view, this time it was some part of a some sort of a similar platform and all we also use first control update is actually 0.47 seconds. And as happens to be quite smaller than the other one and so, basically.

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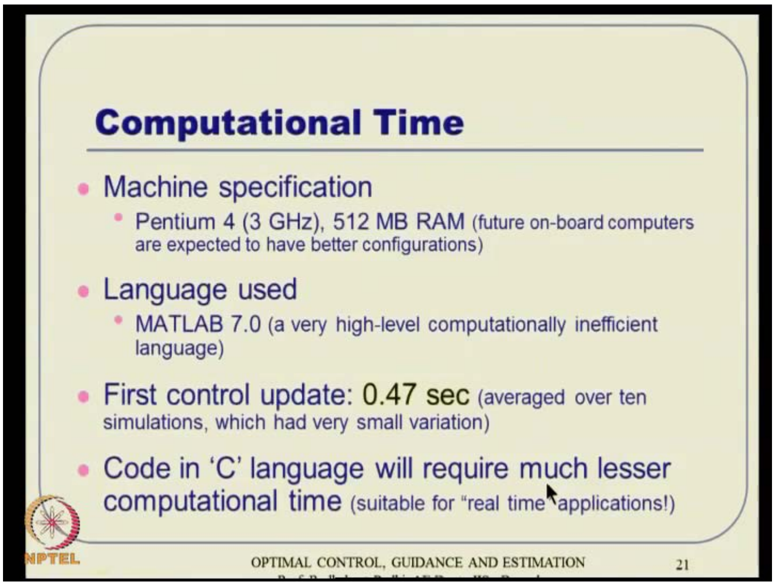

(Refer Slide Time: 43:02) So, the conclusion of this particular problem it I mean it talks about the algorithm is is successfully works and tested for various ranges it leads to high accuracy. And remember accuracy in the sensor plus or minus actually, is nothing I mean what you are looking for is even, if the accuracy happens to be of the order of 2, 3 kilometers it is very much. And what we are talking here is is strategic warhead sort of things, it that wild area kind of destruction basically. So, having plus or minus 1 meter is extremely good actually is is the hard constraint.

This happens because the final constraint is put as a hard constraint actually; also observe there is computationally quite efficient. And hence it is possibility of implementing it online in future actually; also we have done comparisons study with NLP solution, which slow which actually slows some sort of very close and nice actually. This is about the velocity missile problem.

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Then there are energy insensitive of guidance design extension of that using this dynamic inversion along with MPSP. Essentially meets the specification at takes about some sort of robust guidance algorithm. So, it essentially it makes it kind of energy insensitive a limited sense actually at least.

Alright, what next as I told it does not necessarily turn only strategic thing it the conventional guidance also it works. Let us go quickly go through that that kind of idea, so here I am talking about some missile guidance problem with impact angle constraint as well. So, what you mean is not only you to hit the target a conventional target of course, with very, very good accuracy but, at the time of hitting the target also there should be some angle constraint for the velocity vector; and there are reasons for that also which are summarize here.

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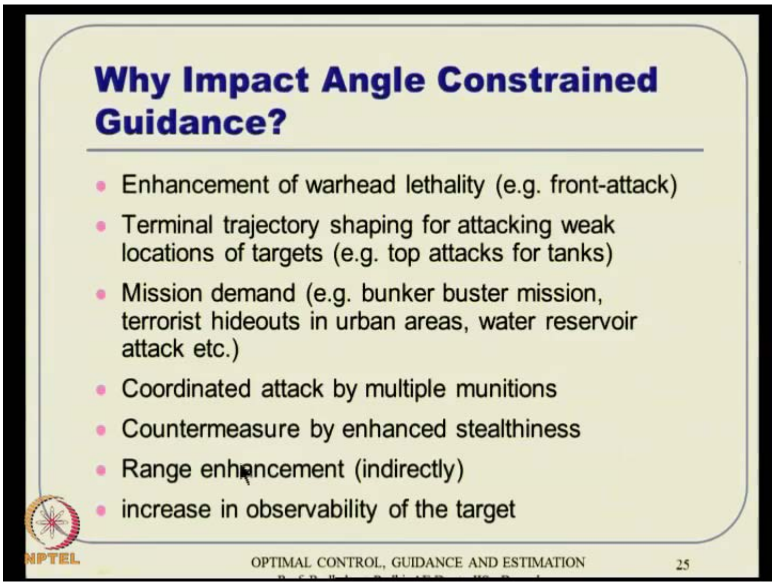

Why angle constrained the very first thing that you can see is enhancement of warhead lethality, that are certain targets for why there are vulnerable parts either front or top and think like that. So, if you talk that kind of target then your enhancement of the warhead lethality happens only when you attack from the front actually or attack from the top like that actually.

So, that is one reason, why you want to put angle constraint guidance and there are some certain things like well, I will retranslate little bit here, there is something called en I mean enhancement of warhead lethality. In other words if you approach the target in a particular direction at least in the velocity vector sense and all, the essentially the if once you explored your own warhead. Then the damage on the target will be more actually.

So, for example, front attack and all that the target is target is coming there, target is coming something like that if you go in a front attack model then explored it there is a very large chance that you will capture the target actually. But, if you if you go some other direction then then suppose if have attack it I mean some from this direction let us say the we never miss an objectives, but it it will explore this way, but it. But, by the time if there the fragments cross these the target should be somewhere here if it is here or here it has already crossed or it is about to cross before closing, then it was virtually useless actually.

So, that is what the warhead lethality is all about actually. Now, coming to that there are certain vulnerable parts or I want to attack the weak location of the target for example, top attacks for tanks or the tank where is a main hole for that. So, that the driver can can met can enter the time and hence there is there are some weak parts in the tank is not actually hammered heavily from the top.

Whereas, it is hammered heavily with a with a thick metal seats and all that from the side and all that actually. So, if you really want to attack a tank then better do that a top attack of a tanks actually. And there are certain the missile demands for example, bunker busters and and terrorist hideouts in urban areas and like that, there where you again want to go in a particular direction only, either to minimize collateral damage or you to minimize the mission demands that actually. The bunker buster means there are certain laws of concrete that you have to explore go deeper and deeper and think like that.

So, unless the velocity vector is perpendicular suppress the vehicle may escape and go inside the earth and it is does not serve any purpose actually. So, then similarly if you have this terrorist hideouts in urban areas, there will be multiple buildings and all unless you come from the top or in a particular direction sense you will lecture your rather structures and all may come on the way actually.

So, it will not do the job basically, so the mission itself demand missions demands that you in a specific direction sort of things, then there are ideas like coordinated attack by multiple munitions; that means, you should three four things let us say small a, b, c. And all and they have to go and attack from a particular direction only then only combined effects will maximum actually, if a individual if they talk in a different sense, different location, different direction and all that the combined effect may not be much.

But, if I had to go all of them go to a some location in the same direction sort of thing then the effectiveness is much more actually. Now, we will talk about not only the same location and same angle but, they also talk about same time actually; that means, the 4, 5 years it should be there, they will all go at the same time same location same angle of the target actually. The effectiveness would be much, much, much higher basically that way, then there are also reasons for something like countermeasures for enhanced sustain less.

For example, if you are radar is looking in some particular direction you want to I mean then we radar is looking certain particular direction, you do not want to go in that direction and then make yourself busy to any.

So, if you go in the backward direction sense and all that, so for them in those situations it is you can think of it as something like a counter measure actually. So, it has done for for for enhanced stealthiness is you can introduce this this countermeasures actually also think about range enhancement it is it is typically not a range enhancement formulation, but it happens indirectly. But, if you if you happen to something like let us say top at then you have to actually top it off if, we if we target which is at the same altitude or above view basically.

Let us say then what happens the vehicle the mission leads to claim even for the the that atmosphere density comes down by flying at the high altitude. And once the atmosphere density is is down the drag is less if the drag is less the range can be more actually. So, this is what is done at the range enhancement indirectly sort of thing. Then sometimes it is done to increase the observe ability of the target also; that means, you purposely a maneuvers side wise and all that. So, that you can see the target better actually.

So, that is the is another reason, why you want to have this this angle constraint guidance and all. So, there are several reasons I have listed out only about six seven but, there are some of the reasons also for which we really need this this angle constraint guidance actually.

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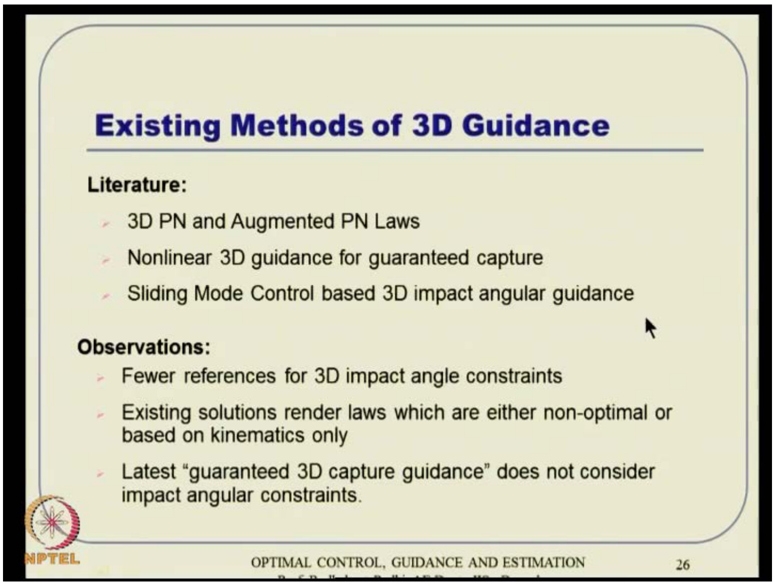

Now, existing methods of 3-D guidance this particular thing we are interested in talking about guidance in 3-D sense and then angle constraint will happen in two angle sense actually. So, what is the motivation for that, the 3-D PN and augmented PN laws available but, I mean there are couple not too any there are couple of them are available.

However the very less of them, we will actually talk about 3-D impact angle constraints, angle constraint will be talk discussed if you see the literature most of them will talk about orientally actually. But, here the we are aim has to kind of a experiment in 3-D domain and then existing solution which we have which we have already there there either non optimal or based on kinematics only. Now, we want to extend it further we want to talk about the full non-linear dynamic point mass equation actually; and then you using the equations you want to design guidance laws actually.

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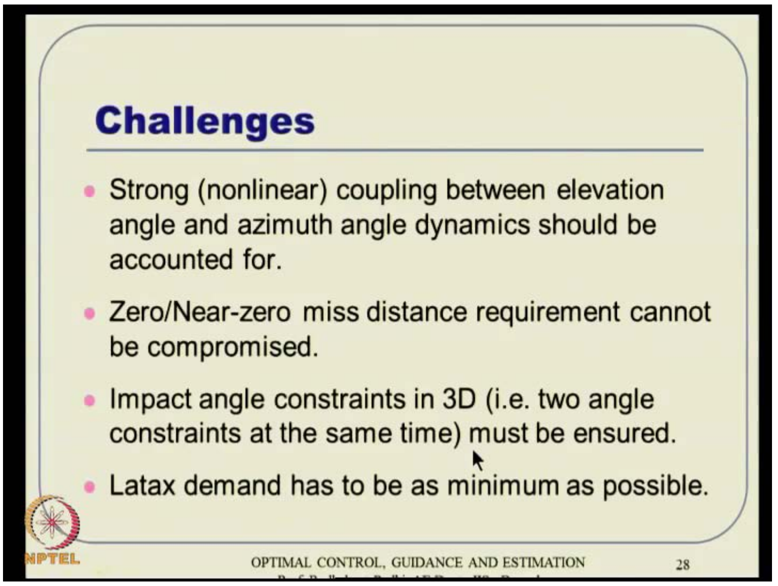

And so, the the challenge is first thing is strong non-linear coupling between the elevation angle and azimuth angle. Basically, that should be accounted for what you really need is zero or near zero miss distances remember, unless there is zero miss distance sort of thing most of the angle constraints are remaining less actually. In other words if you only when you hit target, then only angle becomes much important actually. Otherwise it is there but, it is not that much important actually in a way alright.

So, as I told two angle constraints at the same time has to be ensured that is called 3-D angle constraint. And then latex demand has to be minimum on the way also later latex demand has to be as minimum as possible on the way actually.

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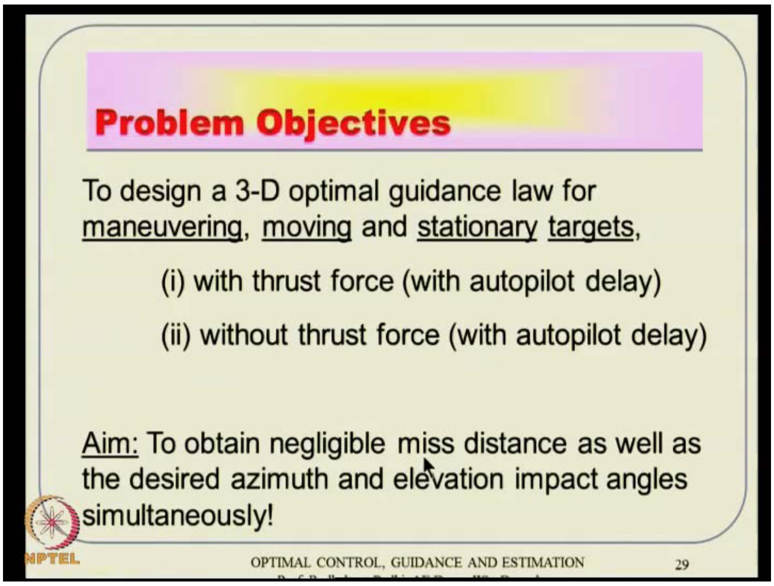

What is the problem objective, now the problem objective is design a three d optimal guidance law for maneuvering moving or stationary targets on the ground with thrust force with autopilot delay or without thrust force thrust force. Either you can simply release a vehicle from the aircraft and it will go there, it can also on it is own thrust force, I mean depending on the mission of course, actually.

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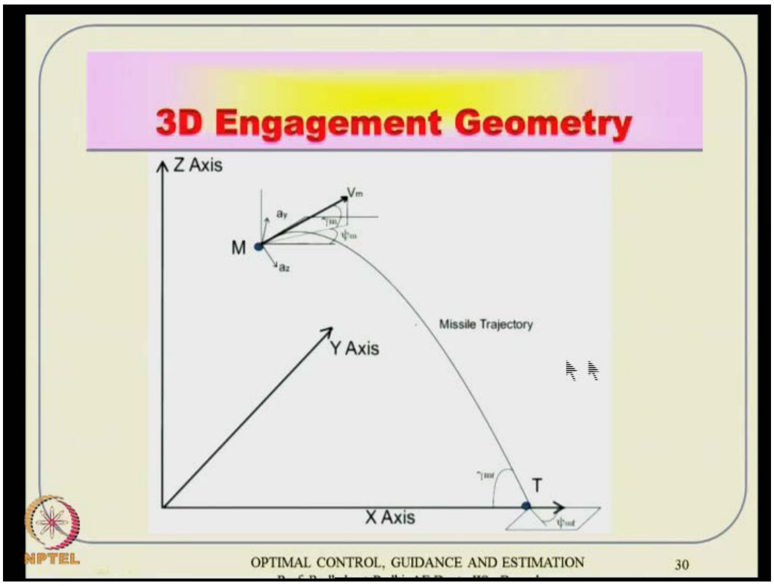

So, this is the type of thing what you are looking at, you want to release a vehicle from here. Let us say in the target ground moving target actually it can be stationary, it can go in constant direction, it can take some maneuvering path. Whatever it may be the road condition is like that or it takes intensive maneuvers as intensive maneuvers just we got the target or what and just we got the attacking missiles something like that actually. So, it can be either stationary or moving in a constant direction or moving in some sort of a time varying resolution actually.

So, not only the vehicle has to be the vehicle has to be guided or the missile has to be guided in such a way that the ultimately it is not only fall on the target. But, the velocity vector should also give raise to this desired I mean flight path angle gamma I may and desired psi and may both together actually. So, this is problem that you are looking at actually here.

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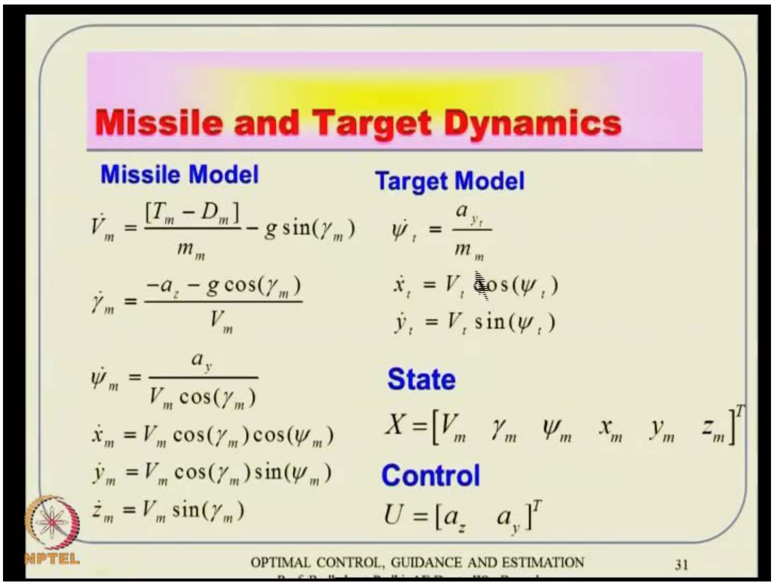

The dynamics missile model and target model are typically done that way, remember for even though instruments are available for a state and target information it cannot be full information for say. So, typically target models are done in kinematic sense, whatever the missile model here the it is a sensor is system and not only that the sensors information can be expected for the further to pilot actually.

So; that means, the this the sensor information is more detail in as far as your own vehicle is concern. And hence detail model can be used for the missile target is simplified kind of a kinematic kinematic model basically; state equations are like that control equations like that again there was a necessity if this this number normalization and all.

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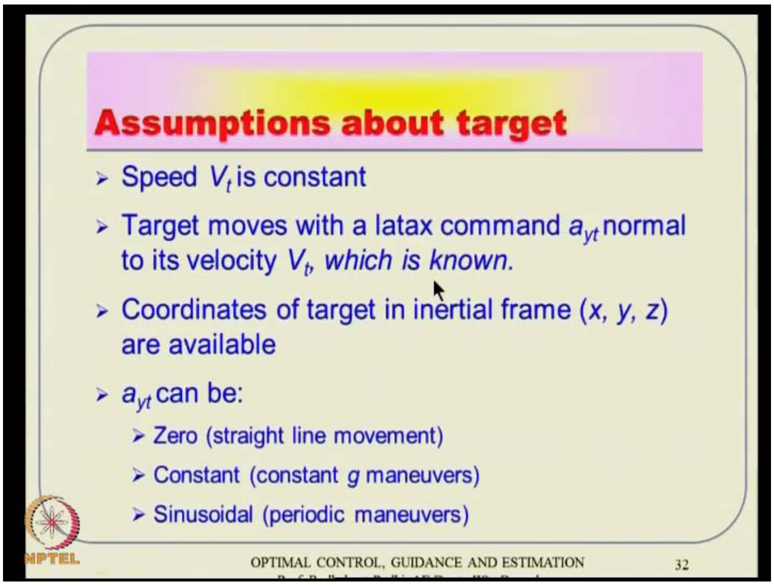

So, we have done this normalization and as I told the target explanation can have can be either zero; that means, just a straight line movement it can be constant a y t; that means, some sort of a constant turning. And it can have also this viewing sort of sinusoidal sort of things periodic maneuvers actually. So, in let it be anything still the algorithm has to work actually and also remember this can this can they change actually; that means, target can start with the straight line movement on the way it can change it is strategy to constant or may be centrifugal or combination of that.

Now, what your assuming here is the information is known to us actually the what the what the target is doing is actually extracted in from an estimation algorithm onward. But, that part we are neglecting here we are assuming that that part is done separately and it is the in necessary information is available here, actually alright. So, these are the normalized dynamic and all. So, substitute that these are the discrete static equation and all using same all are integration and all that and then you these are some of the reasons actually.

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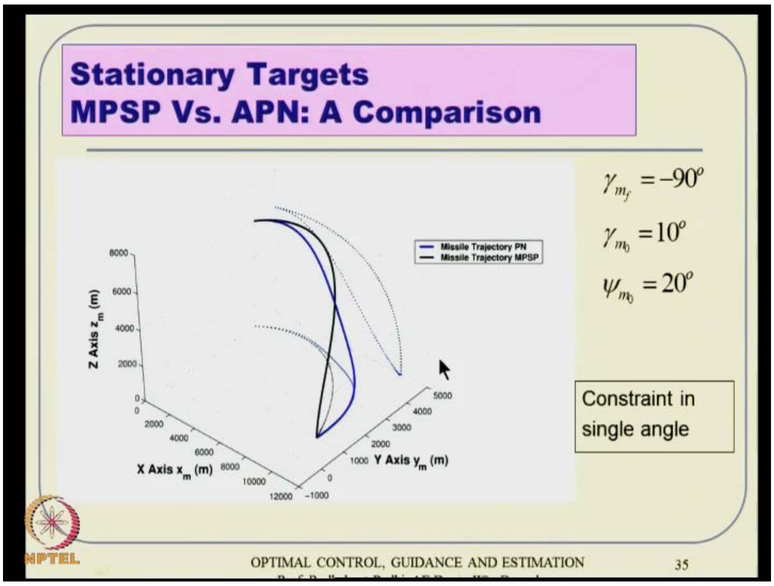

You can see that we put only a single angle constraint here, what you see here, this is this what the PN gives, but this is what we want and if there is a angle constraint here, it has to be be some sort of a vertical here. So, it happens to be like that.

And these dotted lines are are the projection curves actually, on the this 2-D plains one happens to be the X Y plain that is the one. And the the happens to be in the X Z plain actually and from X Z plain, you can see the angle is 90 the here and there is no angle constraint in the X Y plain. So, it is free anyway actually.

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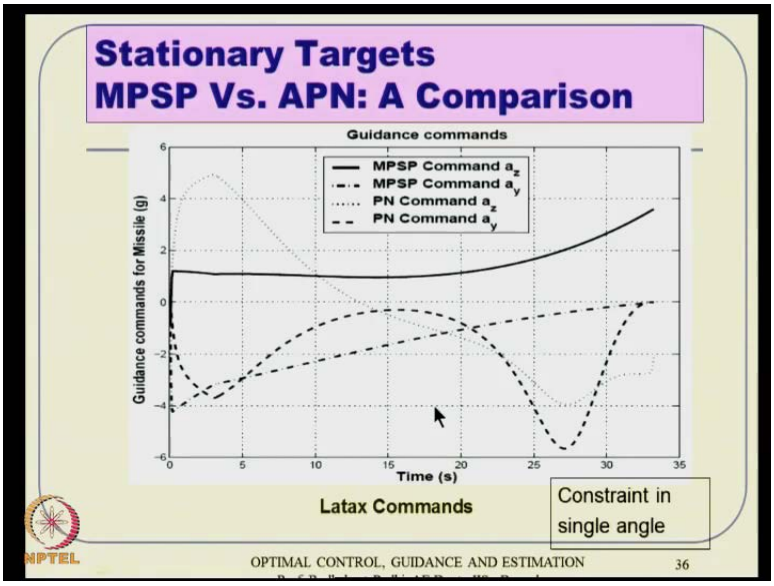

And also see, there are the comparison with respect with this lateral s phase and studies and all that see this this is one and third happens to be the same a, one coming from MPSP, other coming from PN. And then two and four same they are also similar. So, if you see one and third you can also see that the one much more smoother, then what is what is happening is third.

And the the maximum value also happens to be low this happens to be maximum value is something like 3, 3.5 around, but here the maximum value is around 5 actually. So, it is smoother it is I mean magnitude wise also there is lot of advantages actually other the similar advantage happens to be other other phase as well actually. If you compare these two this line happens to be much more smoother actually and the maximum value also happens to be lower any way.

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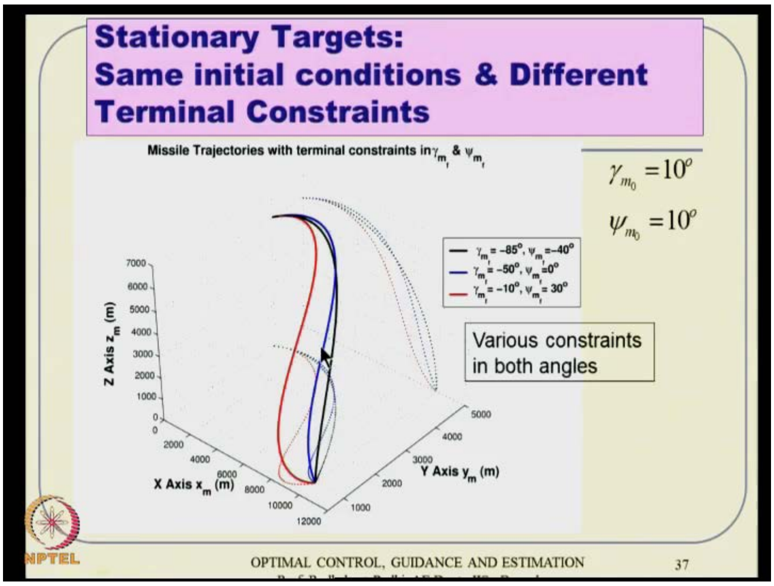

So, this is what it is the, what about two angles now giving I mean two desired angles and all that, you can still do that the formulation allows that. So, the various combinations of three different cases, we have saw, now we to I mean different, different desired angles and all that actually.

We start with the same initial condition but, suppose your desired angles are then what happens actually. You can very clearly see that no matter what you desire the algorithm is able to find out and I emphasize here it is not extensive twinning which is typically in industry and all that. case by case depending on situation and think like that, you there are various sense of PN guidance law which will actually try to at kind of view some angles.

And all that we can attempt doing that but, it requires augment of twinning and scheduling and all sort of thing this is absolutely not need here. We just you just code these numbers, whatever numbers you feed it to the algorithm of before launch and then it will take you there actually.

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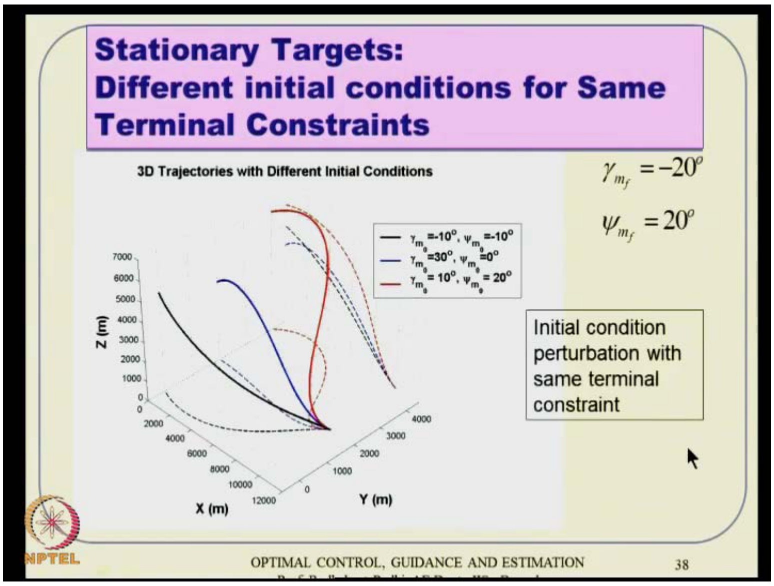

So, then this different one, so we start with three different initial conditions but, as ultimately aim to the same conditions actually same position and same angles also basically. So, this what is also again happening here the later let listen advantages as well actually.

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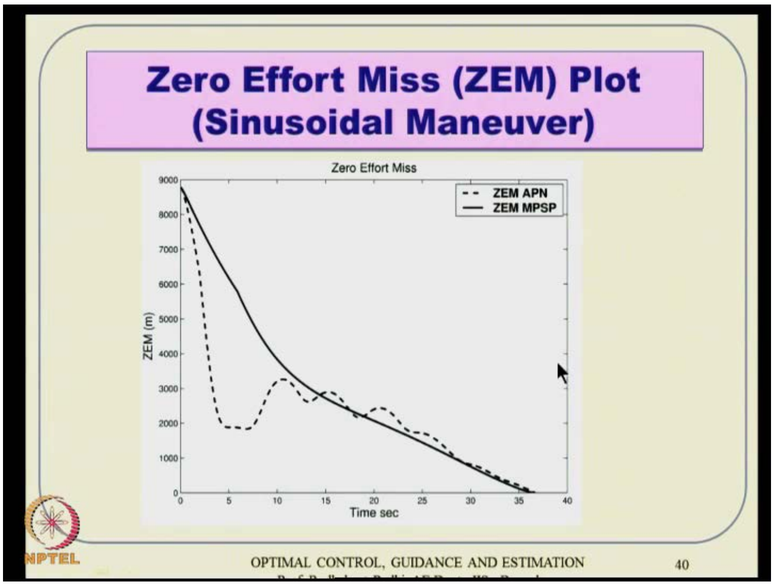

Then other plot there is something called as zero effort miss, I mean the details you can see in mission guidance proof and zero effort is if from wherever I am from there onwards if I go with zero effort. That means, lift is zero actually from there onwards, then what happens actually and you can see that the solid line and happens to be MPSP; which is much more less oscillatory much, much smoother and less oscillatory and hence there are lot of additional advantages as well actually.

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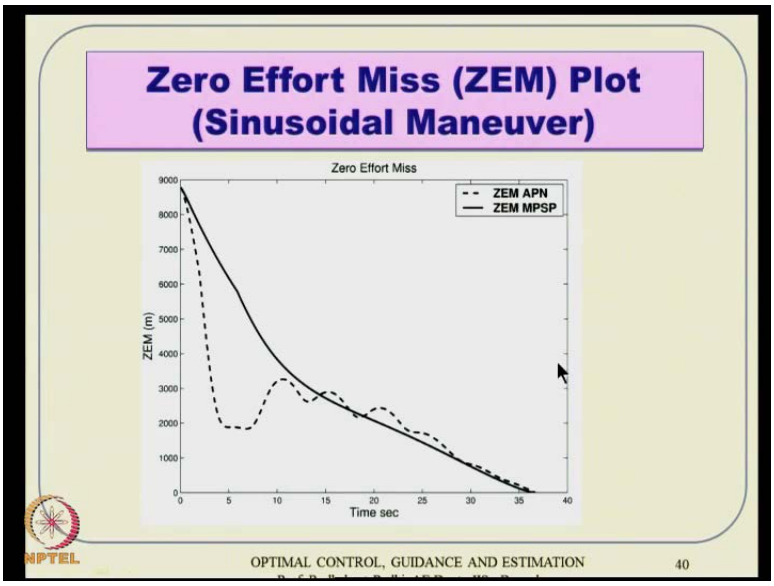

Coming to another class of problem this different class of problem, this incoming things is actually ballistic missile, we want to define it before it destroys you and all that actually; in those settings also you have experimented all this logic.

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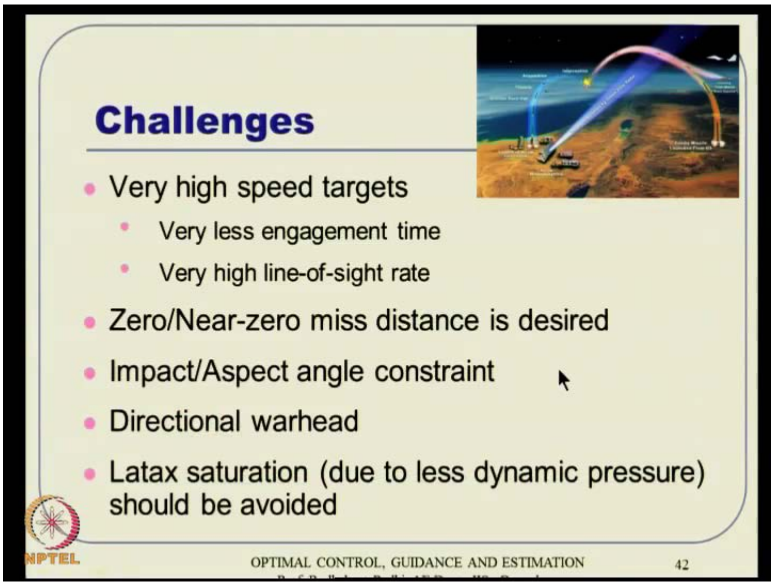

But different class of challenges again here there is a very high speed target and hence very less engagement time very high line of sight rate. And ultimately what you really require is zero miss distance rather and there are has to be impact angle constraint direction warhead is also necessity, but but latex saturation has to be accounted for the due to less dynamic pressure you you want to engage the target in high altitude actually.

So, high altitude means less dynamic pressure and latex saturation is constraint there actually.

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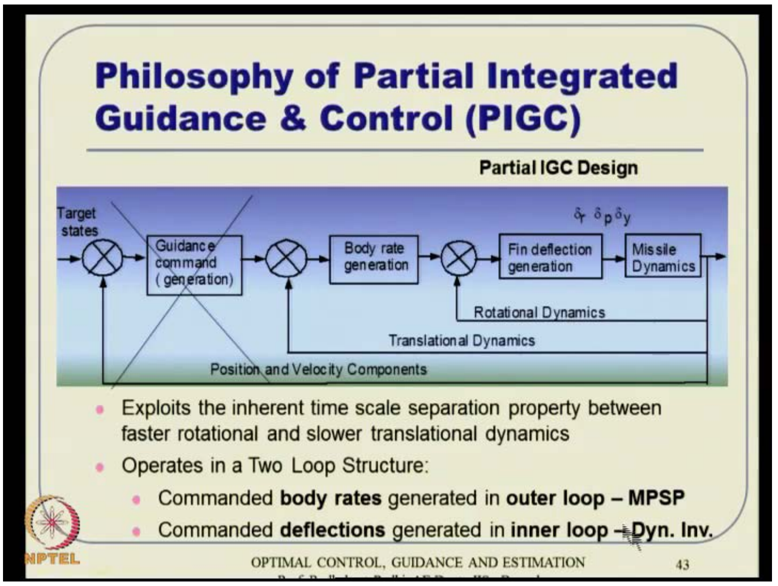

So, here what we have done is the you extended the logic of this loop design and all, you can think of taking out the loop this is what the conventional three loop thing. And the what you have done is we have taken out this the the outer loop sort of thing and this essentially leads to this. So, called this partial integrated guidance and control loop integrated guidance and control designs sort of things.

So, this is this is what it is and this this there are advantages and all I will encourage that you you read this I mean references and all that actually. I will not take you through that, but essentially the this outer loop and there is inner loop outer loop operates in MPSP inner loop operates on dynamic inversion actually. So, that is also application there there are papers available also read.

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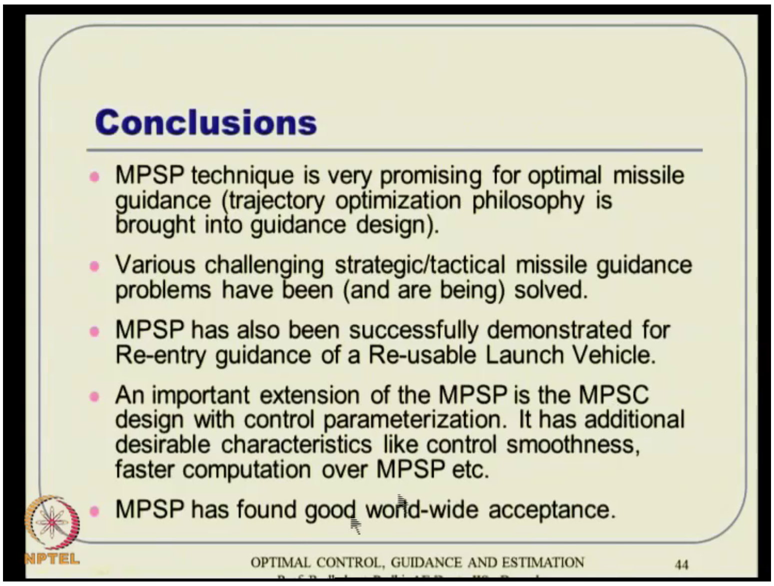

So, in summary and conclusion MPSP technique is a very promising technique for fro optimal guidance in general. And essentially the it various challenging problems we have solved and I also encouraged all of you to kind of put your hands on various different problems, else I will be I am very sure you will be surprised to see the result in a very good way actually. Alright, so this is there are extensions available for MPSP as I told in the previous lecture we have propose this MPSP technique, MPSC techniques. And there are other techniques as well we have generalize MPSP and all I will talk those in next class. And as I told this technique is found worldwide acceptance slowly actually, some of the references about missile guidance the the classical this a air to ground missiles.

And all that the impact angle constraint problem and see that in this particular reference is already published. And there is another application which talks about this mid course guidance sort of things with alignment angle constraint, very close to the impact angle constraint for for sort of problems. But, a different class of requirement sort of things actually and see that also in this this angle paper actually.

And as I this another journal paper, which talks about energy insensitive guidance the velocity missile sort of thing also available you can see details out there. As if the first paper which from from which I the I discussed the previous lecture as well, which talks about this the very basic paper it talks about, the very first idea available here alright. So, with that ill stop here this lecture thanks a lot.