Advanced Control System Design
Prof. Radhakant Padhi
Department of Aerospace Engineering
Indian Institute of Science, Bangalore

Lecture No. # 03
Classical Control Overview – II

Before you have studied Laplace transform and its implications and then we have also studied time dominion analysis for first and second order systems the effect of pole zeros and all things. Now in this lecture we will primarily see some stability behavior of the linear systems and can we analyze the stability behavior through transfer functions that is the one of our objectives of this class, followed by we will also study the steady state error of characteristics of linear systems.

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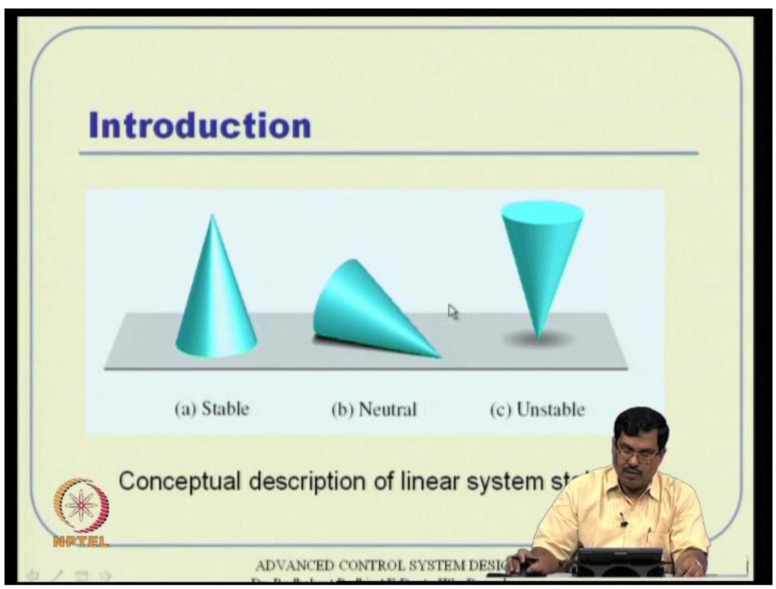

And we are still here in the classical control overview after you are done we will go to the modern control anyway. So first is stability analysis through transfer function. So stability primarily we know that if some objectives standing like this is stable, and if they are inverted sort of thing it is unstable and this is neutral.

So, any small amount of disturbance can make the system go out of this equilibrium point and all these three are sincerely standing on their equilibrium point sincerely. Now this is a stable equilibrium this is a non stable equilibrium somewhat in between is a neutral stability it keep on staying like that exactly. So that is a conceptual description of stability actually.

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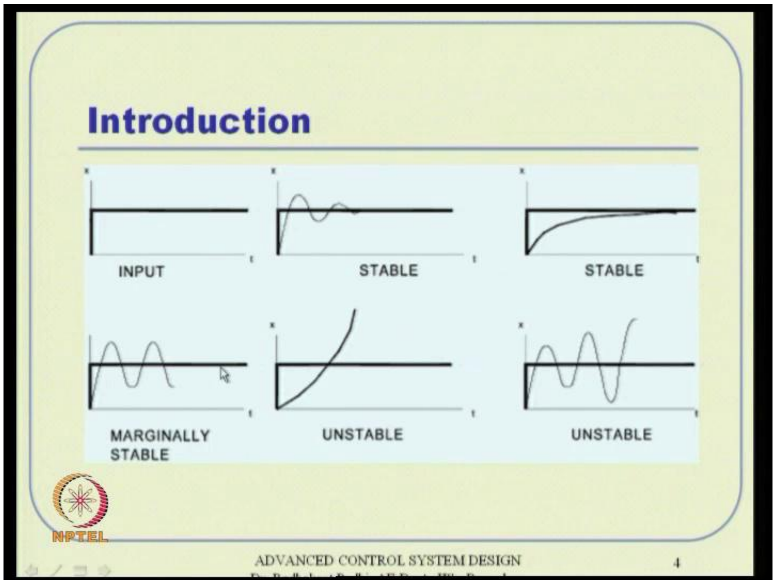

Now as far as the response is concerned the response can be of various nature suppose we will give a constant input, just a stable input starting at time 0, then if the system is stable it can oscillate around that input and stabilize and it can also go one sided and approach to that input. This also stable and this kind of behavior what you see here goes the response goes unbounded that is unstable.

And the response can also grow unbounded within in a sinusoidal way that is also unstable if it remains the same and marginally stable it will keep on oscillating around that value forever, that is the concept. Now as far as the stability behavior with respect to the pole locations are concerned or we can picture it, it will depict something like this. The poles can come out anywhere I mean in this I mean in this x plane, and if you plot this poles let us say the poles can be on the 0 I mean at 0. It can travel along the real axis both positive side as well as negative side. It can travel along the imaginary axis it can also travel anywhere else actually.

So suppose, it is anywhere in the right hand right half plane it turns out that the system is unstable, we will study little more as we go along, and if it is if it is in the negative half the it turns out to be stable. However, if it is in negative half if it travels along the real line there will not be any oscillatory component. So, it will end as it goes away and away the response becomes I mean starts with a slower (( )) to a faster (( )). If it wants the upper locations go away and away to the left hand side. Similarly, upper locations go away and away in the right hand side it the response grows very fast actually.

So initially, it will I mean the upper location is here the response starts growing some suppose like this then the pole location is here it will travel even faster. Along the imaginary axis the response is marginally stable. However, if the location is here it will continue to oscillate with a lesser frequency if it is goes away and away the frequency of oscillation will be more.

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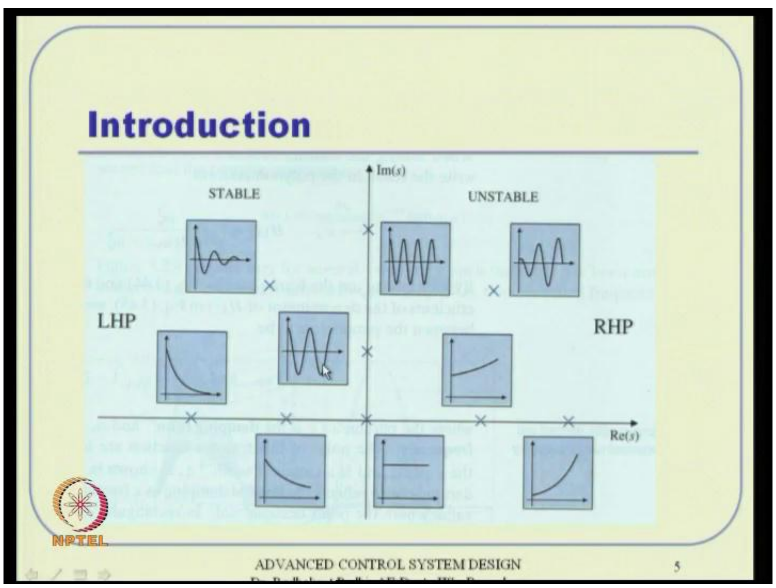

And similarly, if it goes away radially sort of thing the frequency is I mean the oscillation curve is growing basically. It will keep on growing that way and similarly, in the in the other idea of the story I mean the other side of the plane (( )) inside.

It will both be oscillatory, but if it goes in the left hand side sort of thing. So, essentially this is an observation that if the pole locations that are placed in a variety of the waves then, this is what the response turns out to be. So can we study all those, so that so that that gives us an indication that the pole location plays a major role in stability analysis. And coming to a little bit formal study, we already seen that the total response of a system is nothing but sum of the transient and steady state responses. So, this is the total response this is the transient part of it and the steady state part of it.

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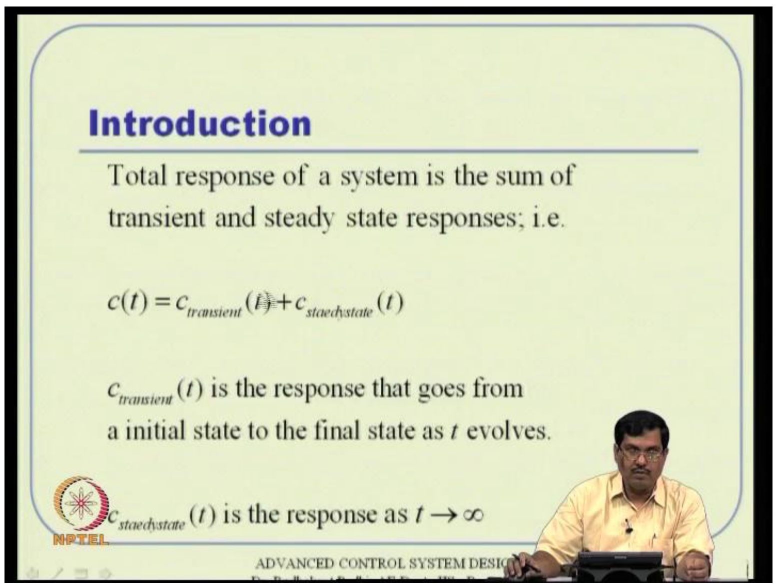

And transient response is the response that goes from an initial state to the final state as t evolves this is the part actually and steady state response is the response as t goes to infinity. Eventually, it will start with something and then it will approach to the steady state response if the system is stable transient part is suppose to decay itself. So that is so that is how the response of the system is all about.

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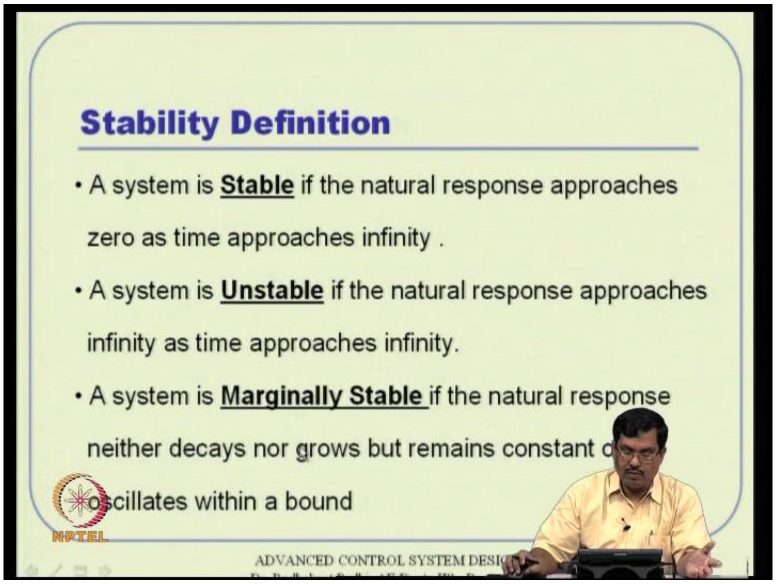

So, stability definition these are formal definitions a system is said to be stable, if the natural response approaches 0 as time approaches infinity and the system is unstable, if the natural response approaches to infinity as time approaches infinity that means both the stable and unstable behavior, as well as marginal stability the system is marginally stable, if the natural response neither decays nor grows because what we just saw in the previous slide, but remains constant or oscillates within a bound which is what we said oscillates within a bound here, because the pole was over here, and it can also remain constant actually.

These are all marginal stability behavior this is also a marginal stability. So these are like formal definition coming from a natural response of the system. Remember the this is the response is transient and steady state part of it and primarily the natural response corresponds to this transient part of it actually. So, anyway so this is the stability definition formally

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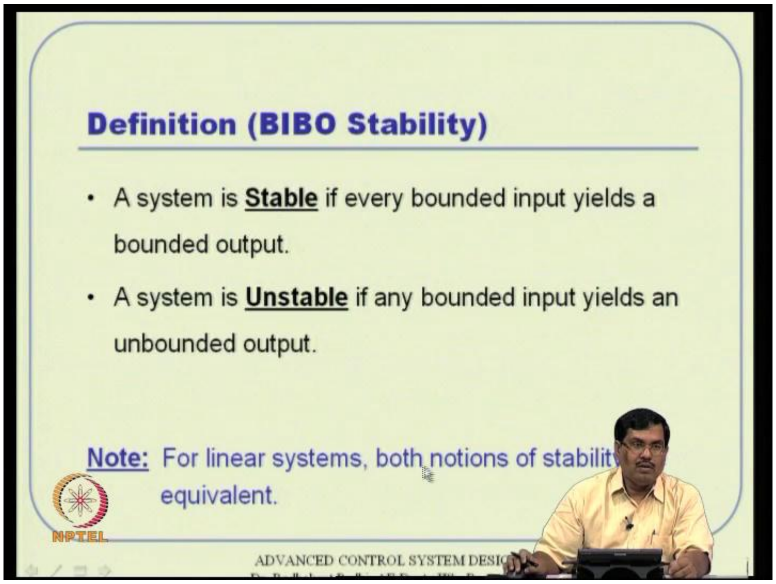

And another definition which comes from something called bounded input bounded output that means if the system, the system is said to be stable, if every bounded input yields a bounded output that means the if you exert the system in a bounded input sense the input itself should not grow to infinity. So, its it can oscillate it can stay constant whatever, but it should be bounded then the response also remains bounded may not be within the same bound the response bound can be can be larger than the input bound that is still as long as it does not go to infinity sort of thing.

If system is unstable if any bounded input results in a non bounded output. If you have just one bounded input which will result in an unbounded output then the system is a unstable. Other wise, for every bounded input the system response is to be bounded. So that these are like actually this behavior stability plays a major rather important role in non-linear systems.

Because notion stability will differ where however for linear systems both notions of stability are equivalent actually. If one is stable in one sense it is also stable in the other sense so that is where it is.

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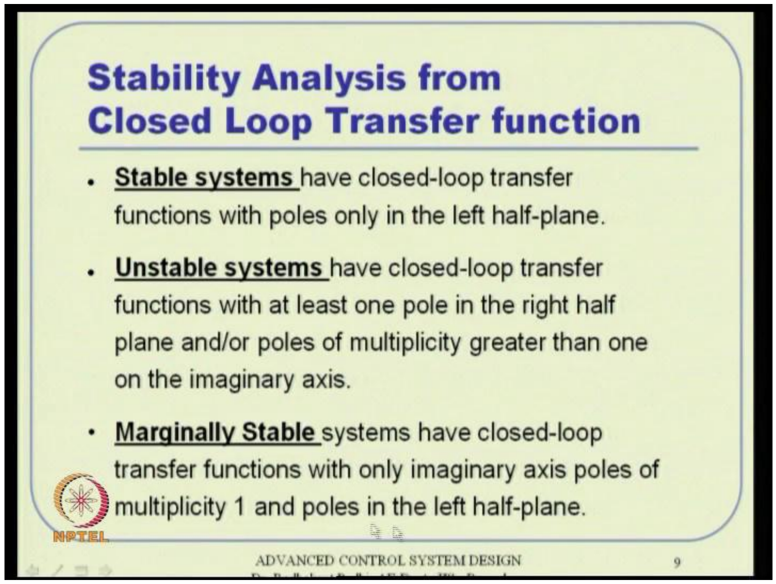

So, stability analysis from closed loop transfer r function suppose somebody gives us a transfer function of a closed loop system, then how do you analyze whether the system remains stable or not actually.

And we have already seen some of the characteristic plots in the previous slide. So stable systems have closed loop transfer functions with poles only in the left half plane and unstable systems have closed loop transfer functions with at least one pole, if at least one pole is in the half plane or and or poles of multiplicity greater than one that that means it either one pole in the your half plane or number of poles exactly at the same location on the imaginary axis. So that can also leads to instability, and marginally stable systems have closed loop transfer functions with only 1 imaginary axis poles of multiplicity and the poles and the remaining poles should be in the left half planes.

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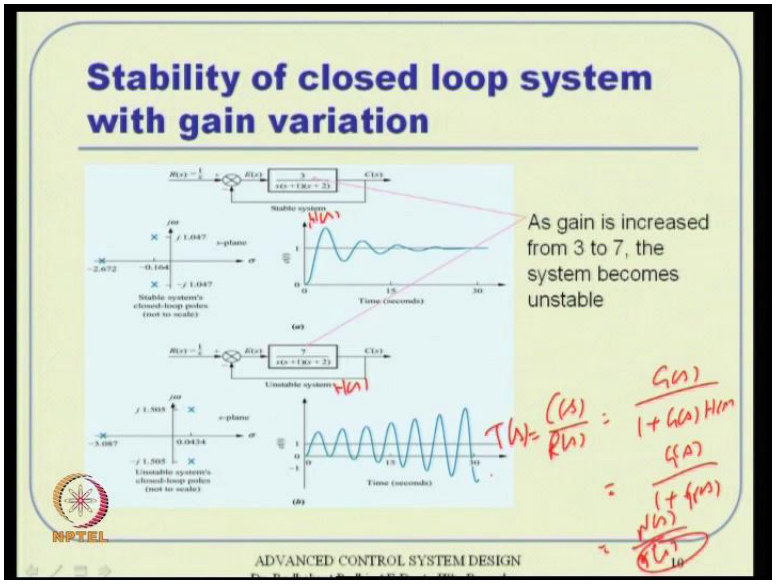

So that is the analysis results sort of thing. And it interestingly, turns out to that if even if you have a same system that is the open loop system remains same. However, something called d c gain that is the number in the numerator as the gain increases 3 to 7. Suddenly the the stable behavior what you see here become somewhat unstable here this is the stable becomes unstable because of the increase of the gains.

So this is essentially you can think of it is a coming from some sort of a control or gain then you have to be careful about that. That control gain cannot be very high it will result in a stability sort of a thing. Why does it happen? Because if you see the closed loop transfer function this is like g i g i phase divided by 1 close G s s s sort of thing that C s by R s that all details will not cover because we will study in class anyway.

So C s by R s in other words if you just talk about that a little in case. So it turns out that C of s by R of s is actually G of s divided by 1 plus G s H s. Assuming it is a negative feedback a negative feedback here, what you see here is negative sign here actually. So that is what and H of h of s is actually is 1, because it is a unity feedback system otherwise you will have a H of s sitting here in the feedback path H of s will be coming here actually.

So that is actually one here, so what you what you essentially have is the G of s divided by 1 plus G of s. So if you do the computation that way and then the numerator the numerator will turn out something and denominator will turn out something and this denominator what you have here, is essentially which will what we are talking here. The poles that you see here these pole locations are essentially the pole locations of this transfer function what you see here. So, that the gain value does play a role in locating the closed loop poles actually, so I mean so then the natural curiosity turns out that without having a solution that without actually solving for that C of s to time domain can we infer something about the stability of the closed loop system really knowing that the closed loop transfer functions.

So this is this is the transfer function that will assume that this is this is something called T of s assuming T of s closed loop transfer known to transfer function known to as s can we enforce stability directly.

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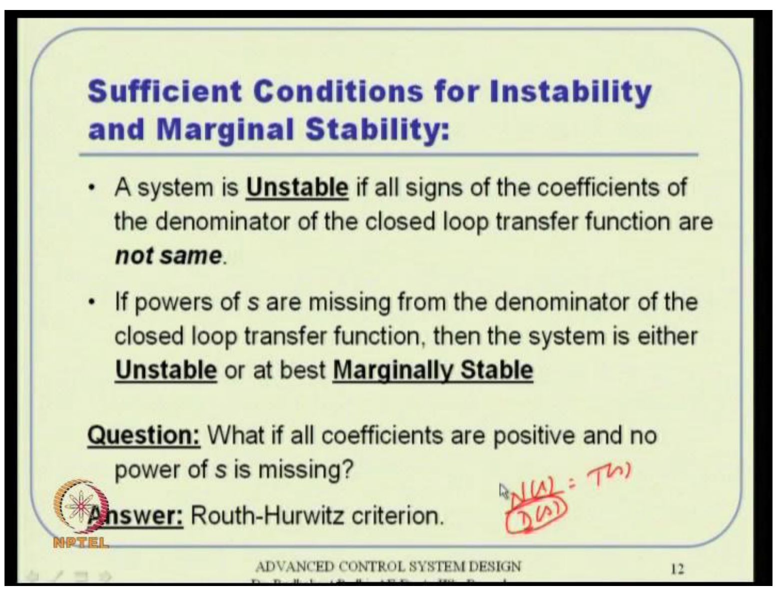

So this essentially leads us to a topic in classical control called as Routh Hurwitz approach for stability Analysis the very standard approach we will see that in a nut shell. And the there are some sufficient conditions. If you if you put those sufficient conditions in perspective then a system is unstable I mean a system is unstable, if all the signs of the coefficients of the denominator of the closed loop transfer functions are not same.

That means you have a transfer function which is a let us say you are talking about transfer function N s by D s. This is about transfer function that you are talking about. So this D of s what you are looking at this particular this particular denominator thing, if this D of s is essentially a polynomial in Laplace variable list and (( )) is also polynomial. So if this polynomial d of s and the signs of its coefficients are not same that means either all of them has to be positive or all them has to be negative.

If there is some signs in like for example, s to the power 5 plus 4 is a 4 minus 3 is q sort of thing then there is a change of sign basically. So in those, if it happens then you do not really need to do further analysis, you can directly say that the system is unstable number 1. Number 2 if the powers of s suppose it is a fifth order polynomial then all powers of s raise to the power 5 by plus some s to the power 4 some coefficient and things like that all of them still exit actually.

If something is missing suppose for example, s to the power 5 is there 4 is there and suddenly square is there and one is there, so essentially you are missing some q term actually that means the coefficient of that is 0 essentially. In those situations the system is either unstable or at the best it can be marginally stable it can never be stable. You get those analysis are they are all in sufficient conditions they are not necessary by the way.

Now it leads us to the question because these are not necessary it leads us to the question that what if all coefficients are positive and no power of s is missing. Suppose is if all are negative then it is also equivalent to having all of them are positive we could minus of that you can take common and make it 0. Essentially you are making it equal to 0 to find the roots. So if all of them are negative it is equivalent of saying that all of them are positive as well actually.

So with that conditions with that in mind we will analyze this in particular case that if that all the coefficients do exist and all of them are positive basically.

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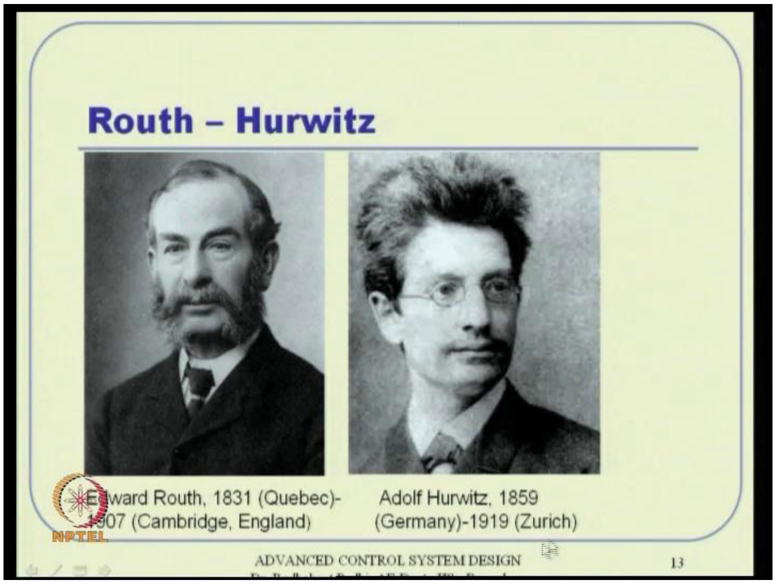

So this is where the critical condition arises and this is where the Routh Hurwitz's criterion he plays a important role. These are the two famous mathematicians you can see them. So one was English mathematician Routh and the another one is German. And he is actually born in German, but last part of his life was spent in Zurich which is in Switzerland.

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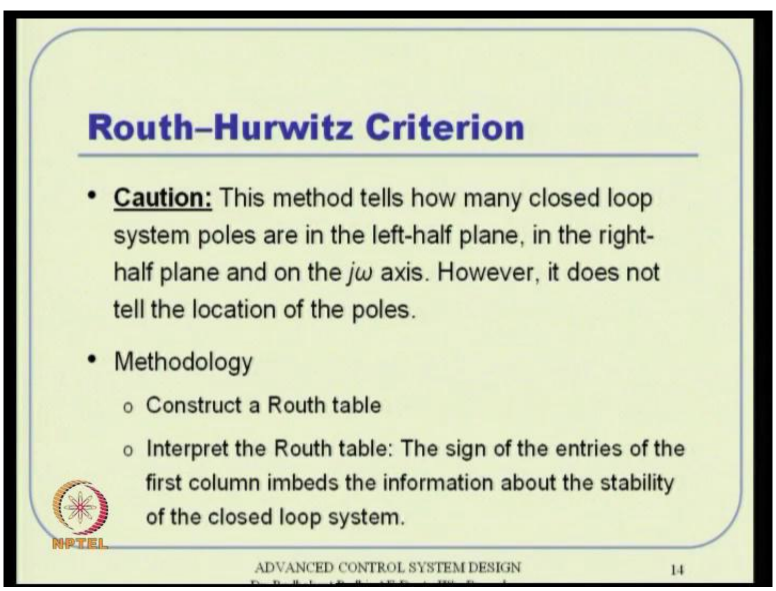

Actually now these two are pioneers in this particular thing they are actually professor. It is what you are going to study here. So before you study the criterion in detail, there is a caution this particular method tells it only tells how many closed loop system poles are in the left half plane, in the right half plane and on the j omega axis. It only tells how many poles are there in left half side half side on the j omega is something like that actually.

It never tells where those poles are located. It just tells these many two poles on the half side 3 poles on the left half side 1 pole is imaginary axis, I mean that kind of that kind of things it will it will tell actually, but that is sufficient for stability analysis you can very clearly tell the system is stable or unstable, so that that becomes a sufficient information for that actually.

So what is the methodology, the methodology consists of two parts one is construct a Routh table and then interpret the Routh table. So, we will see how to construct a Routh table and then you will see how to interpret the Routh table actually. Interpretation is fairly rather fairly easy it only you have to look at only the first column of the table ultimately, and tell the, and observe the sign of the entries of the first column that is all. And if there is no sign sense then you are done the sign sense in the system is unstable sort of thing we will see that actually.

So you construct a table you look at the first column and the first column. All that you do not have to even see the numbers you have to see only the signs actually. Whether that is positive or negative sort of thing and you see that if all of them are positive all of them are negative then also the system remains stable. You have to only see the number of sign changes actually the result comes in the picture.

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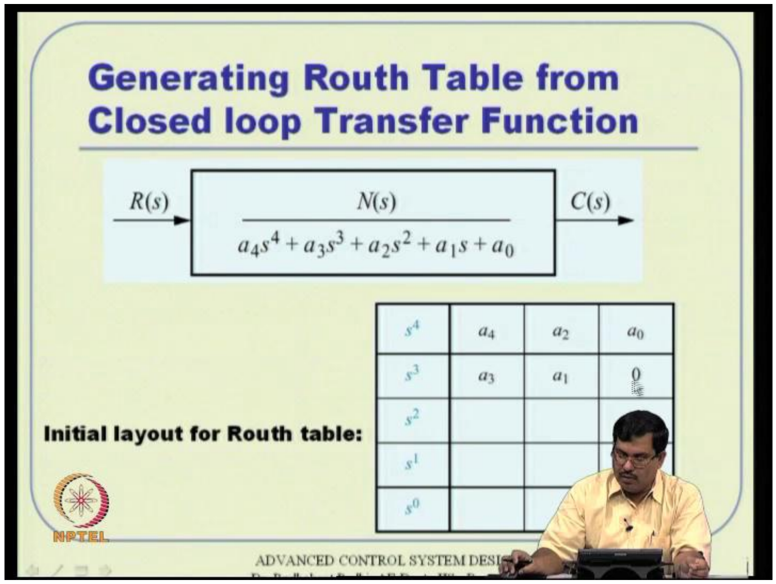

So where is the here is the detail I mean suppose this is our closed loop transfer function this is the commanded and this is the reference input sort of a thing. So this is just the numerator coming polynomial divided by the denominator polynomial D of s and D of s plays a role here. So we are expanding that let us say the D of s is a fourth order polynomial in this form.

So what is this what do you start with you construct a table like that you start with a highest power then the next one then the next one then the next one until s to the power 0 that means it is just one sort of thing. Now first took a lines here find first 2 rows you just fill it up by looking at this polynomial, so s to the power 4 you first start with this coefficient a 4 so put it here then go to the alternative signs actually you skip this will come to the next column anyway I mean next row.

So first you start with a 4 you put it here, then skip that 1 next is a 2, put it here skip that 1 next is a 0 put it here, so that you look that will essentially complete the first row. Then you go to the next row and the next I forgot alternatively again, so s cube there is a coefficient a 3 put it here, then skip this one next one is a 1 put it here and then there is nothing so that is 0. So this is how you construct the first two rows actually.

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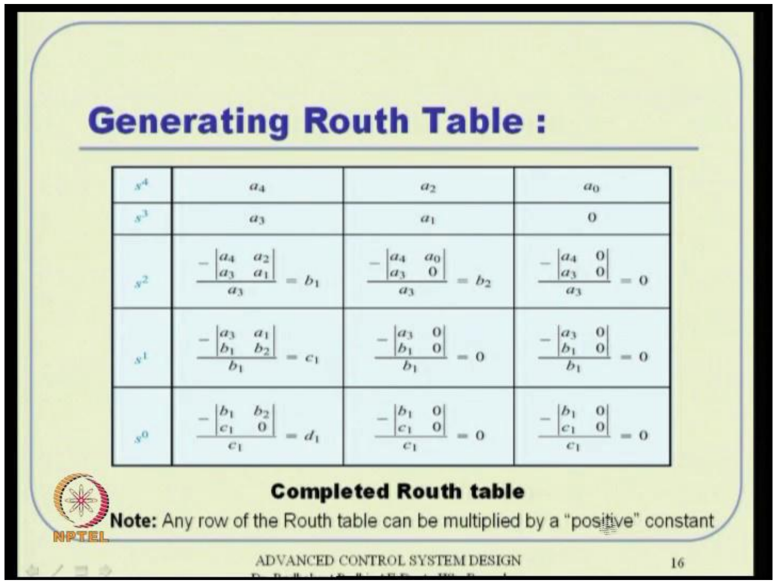

Then, how about the other rows then you carry out the algebra. So, this algebra is given fairly standard basically like using this entries out here, you can fill up all the entries that way this is the standard thing actually. So, this is negative of this determinant divided by a 3 this pivot element sort of a thing, and this is a you construct this two I mean entries a 4 a 3 here and then the next column a 0 here.

And then put it there the next 1 is nothing there, so that will automatically become 0 0 and that the entry is 0. So, you construct all this like this using this formula and you will essentially come up with numbers b 1 c 1 d 1 here b 2 here then these entries will turn out to be 0 in this case actually. So, now you are you have constructed this Routh table. So, now you have to interpret that, but before that also note that any row of the Routh table can be multiplied by a positive constant, and we sometimes see if it if it this entry turns out to be let us say 5 10 and 15 then essentially you can rewrite that using 1 2 and 3 cancel that 5 factor.

So, that you essentially that essentially helps in simplified algebra basically, and you can do that as many times as possible and you want to do it provided that the constant that we are dividing or multiplying is a positive constant remember that, it cannot be negative constant actually. So now that you have constructed this table what the condition?

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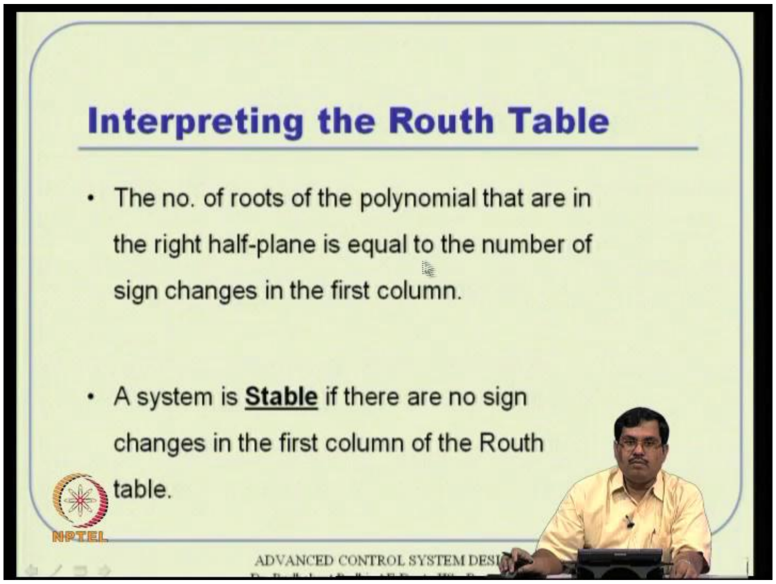

The condition tells where the number of roots in the polynomial that are in the right half-plane is equal to the number of sign changes in the first column. So you have constructed the first column I mean constructed the tables we just see the first columns here in and we come back to this question and we interpret this way this table, so that the number of roots of the polynomial that are in the right half plane remember only one root in the right half plane then system is unstable.

So, if there is only one sign change then system is anyway unstable what did it tells little more than that there is a number of roots of the polynomial that are in the half plane is equal to the number of sign changes in the first column and if there is no sign changes in the first column, that mean the system is stable. So that is the that is the condition that it turns out further analysis and all you can see a classical control book and all this material for this particular lecture everything is taken from Norman Nise actually.

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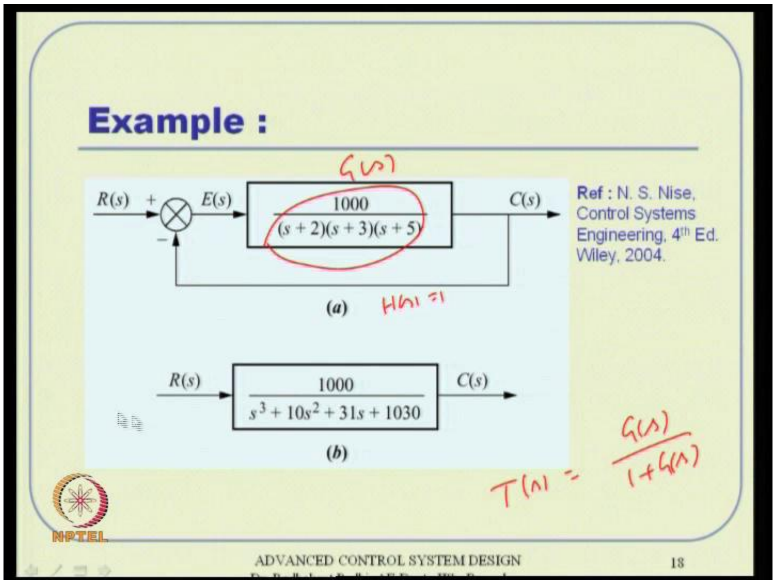

So you can see all more details in that that book all now going to an example, very quickly this let us start with some sort of an example thoroughly on polynomial. So first I have to do is this closed loop transfer function starting from this open loop transfer functions. So, I mean again that that formula is something like this T of s is please remember this is a negative feedback, so it will be something like seeing that this is E of s H of s is 1.

So, you will have this formula s G of s divided by 1 plus G of s if you carry out this instead of G of s you take in this one this particular transfer function and then it will turn out to be something like this actually. So, now you are ready with a closed loop transfer function. So, essentially we are more worried about the denominator, so we will see the denominator here, and then we will construct this Routh table actually.

First of all before even if you before constructing the Routh table observe that none of the powers of s are missing here that means all the powers of s s cube s square s n s to the power 0 all are there and all of this coefficients of the same sign they are all positive anyway. So, you cannot directly inbuilt the sufficient condition until the system is unstable well whether the system is stable or not we still do not know in case of because of that we have go to the Routh table and infer from there actually.

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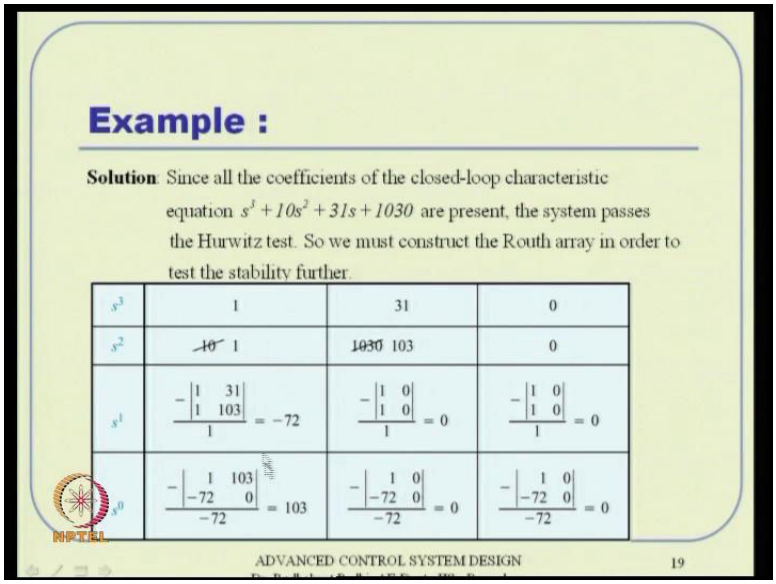

Let us so constructing the Routh table so first you start with s cube and then s that is the first row and then s square and 0 that is the second row and your these columns are 0 0, so I think they are after that so s cube and 31 will come here and 10 and this 10 30 will come here remember this 10 and 1030 you know 1030 is actually multiple of 10 so I can cancel by 10 10 is a positive number.

So I will essentially have one and one and 1 0 3 this will simplify my algebra little more easily. And then I will construct the Routh table construct the Routh table for rest of the two rows, and then ultimately see that here is a sign change actually. So it is a positive to negative and then negative to positive again because there are two sign changes here actually.

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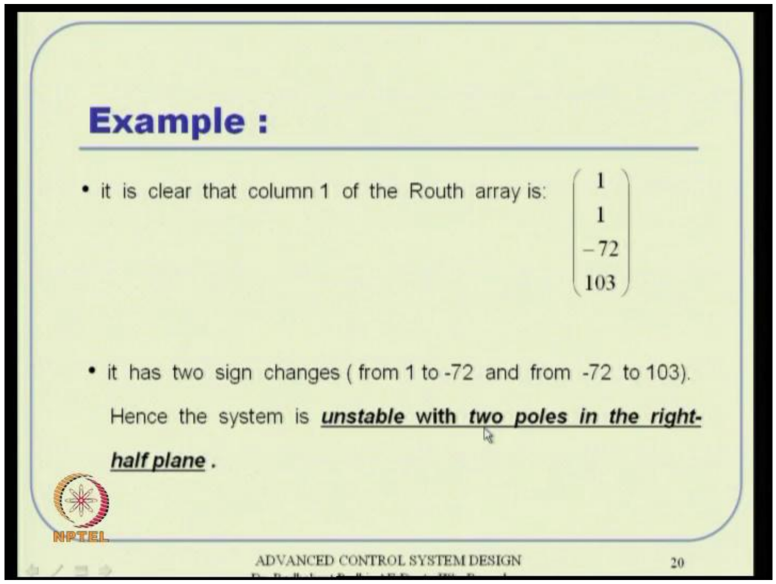

So and hence the system is unstable obviously and not only unstable it is 2 poles in the half plane and you can very quickly verify this in fact using the Routh function of mat lab I mean I see all of you are comfortable with mat lab. So there are there is a function called roots.

And if you use that actually, I think with a function something like this mat lab roots and then this is the polynomial that you have to give something 1 then 10 then 31 and then 1 0 3 0, if you do this there will be this brackets and all that and then enter then I can tell you the roots actually of any other polynomial number.

So you have to keep on giving the coefficients of that polynomial in descending order and then it will find out the roots actually. You can verify that the 2 roots will be in the right half side of this particular polynomial.

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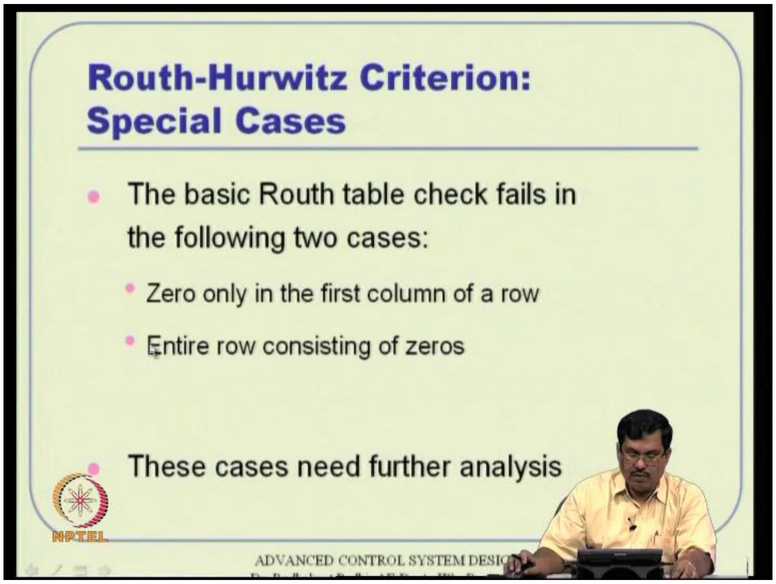

And now there are specific cases they are all regular cases what happen so far. So this is something called basic Routh table alnow special cases arise in one of the following cases.

One is something sometimes it may so happen, that there is a 0 in the first column of the row in there when you construct the Routh table you are always dividing some by a 3 here then b 1 here then c 1 here. So, if any of these entries turns out to be 0 then there is a problem number 1. Number 2 it can also, happen that all these entries of a row can also turn out to be 0 in some special cases then you are stopped there. You cannot construct till this regular manner actually. So how do you handle those issues basically? So first case you can I mean if it turns out that there is a 0 in the first column that we consider that is a case number one this is 0 in the first column then we will consider the entire row consisting of 0.

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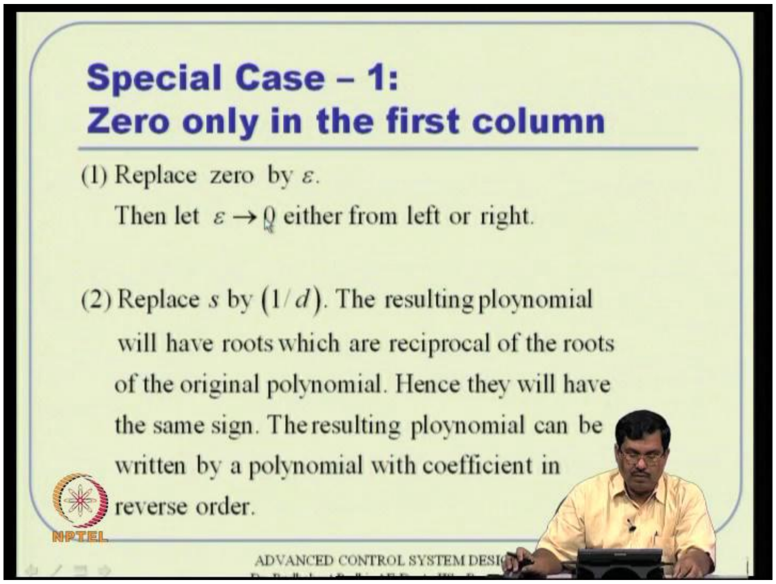

These cases need further analysis and I am just going to tell you the results actually. So, in first case if 0 happens in a first column then there are two ways of handling this one way is to replace that 0, wherever 0 exists by a number epsilon carry out the algebra with respect to psi epsilon then analyze the sign things and all in a limiting sense and in the sense that you allow epsilon to tend to 0 either from left or from right actually. So I mean either epsilon tends to 0 plus or 0 minus in this in that particular limiting case what happens actually.

So that is one way of handling that another case is this observation is something like that something like this. See,, if there is a pole location somewhere in this plane if I just take one by that particular number, then the sign of that particular number remains in the same side actually, that means if there is a positive pole, let us say is equal to some 2 plus 3 j sort of thing and I just take a reciprocal of that 1 by s sort of a thing actually.

Then, it will be different number however, it will not travel from left side of the plane to side or vice versa it will remain on the same side actually. So, exploiting that what you can do is you can replace s by 1 over d, then it will result in a polynomial in the in d essentially and that means it will have a polynomial in the reverse order basically, like s to the power n is there that will turn out to be 1 one by d to the power whole n basically. So s to the power suppose it is I mean I i there is no example here because of time distinction probably but, then I will just give you simple example.

See suppose you have something like s square plus let us say 3 s plus 2 actually that is the polynomial then I replace that by 1 by d square plus 3 1 by d plus 2. So, this will result in something like one I mean 1 by d square plus 3 by d plus 2 and it will result in something like one plus 3 d plus 2 d square. That means if you just interpret this polynomial it will it will take the reverse order it will be b square this is essentially 2 d square plus 3 d plus 1 essentially.

So, that means that this two has become come here and then this 3 has come here, and then this one is come here the polynomial results in a reverse order actually. So, that reverse order polynomial need not have a need not result in a first column being 0 actually, first column entry. So these are two tricks that you can play one thing is, you replace that 0 by a small value epsilon carry out the algebra and ultimately see what is it is I mean when you take epsilon tends to 0 and all that that is one way of looking at. And the other one is the reverse the whole number and see whether you can evolve this 0 element in the first column actually. And method two has some computational advantage, about the method one about method 2 may or may not work in all cases, usually when you reverse the polynomial order it may still result in a 0 element, so in that case you do not have any choice other than going to method one sort of thing.

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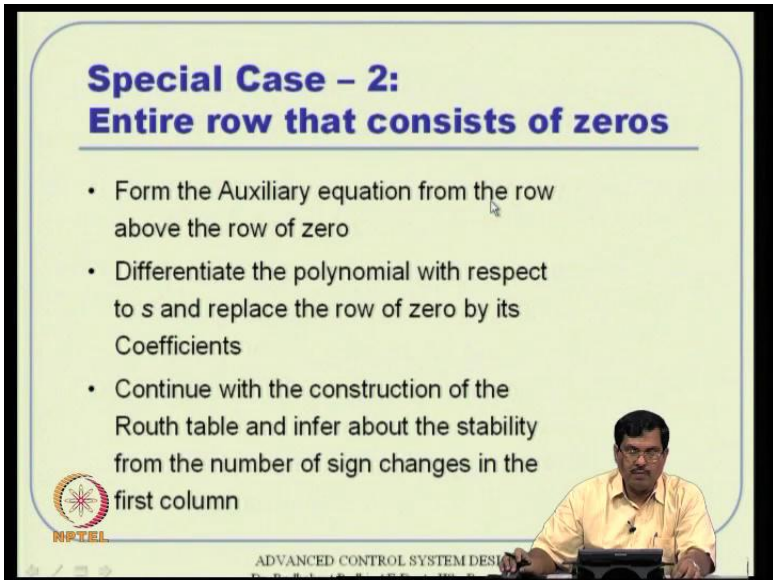

Number 2 in a special case that what if the entire row consist of zero's actually, in some particular case. In that case what you will do is form an auxiliary equation from the row above the row of zero's. And then differentiate the polynomial with respect to s, and replace that row of 0 by its coefficients itself. So, if I go back to this generic table, let us say this entire column is I mean this entire row is become 0 here that means b 1 is also 0 b 2 is also 0 what you do with that? Now suppose I go back to this row and then just take a differentiation of that then s cube will result in s square term and then s will result in 0 polynomial, 0 term actually.

So if I just state this polynomial that means a 3 times s cube plus o 1 s basically then I differentiate that it will turn out to be something like well let us do that a little bit this is something like a 3 s cube plus a 1 s so if I just take difference a d by d s of that it turns out to be 3 a 3 a square plus a 1, so I will take this co efficient and then carry on with the work actually that will result in the same analysis essentially.

So, instead of 0 0 I can replace that this one I can replace here that one1 I can replace there and then proceed further actually. So, that is the trick to follow and all this why it happens and all are there in the book you can see that actually. So, this is one way of I mean I mean handling this special cases essentially. There is also some analysis that you can do from

Routh Hurwitz condition, for the design. Suppose this k let us say this is your control loop I mean this is your closed loop system in unity feedback sense and this K essentially comes from a block which is you can put that k here take something like one here if you want to.

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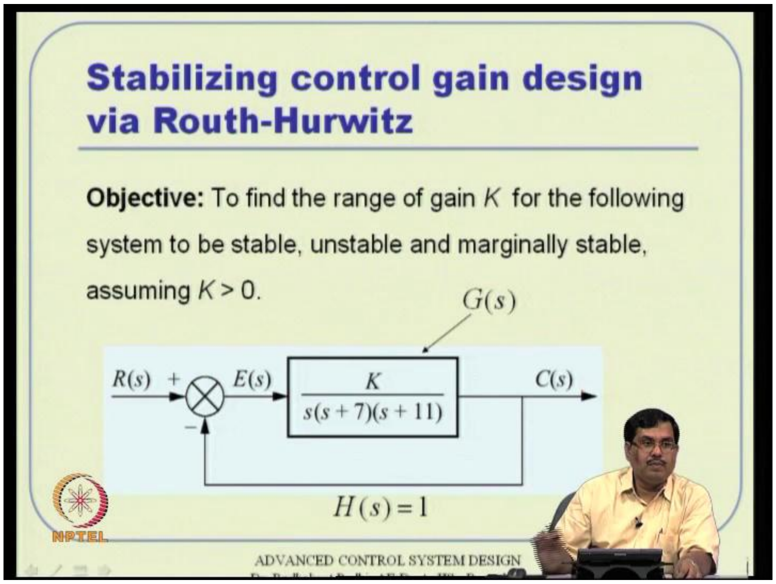

So this K will essentially cater for these gain things and all that. So that is a neat as that now the question is the range of this gain K so that the system to remain stable, unstable, marginally stable and all that if you know a range of l is K that will result in stable stability of the system then within that we will tell it I mean within that will be able to select a value for that that means it gives us a tuning range of this gain K basically.

You really do not have to start from minus infinity go up to plus infinity you know that within this values the system remains stable. So, within that value I will kind of change this gain value and then see what is what results in the best performance actually. So that is the motivation here can we do that, so this is my G of s this is my H of s again that is equal to 1 and this is where the system that we want to analyze it actually.

So, first thing is closed loop transfer function again and this is again G of s by 1 plus G of s H s and it result in some sort of a problem here like this. So, the question is I mean will the system remain stable and is clearly a function of K because K comes in the denominator. So, now we will go back to this Routh table and construct this table, so first we start with s cube and then that is one here and 77 here that comes here.

Then, this s square and s 0 that means 18 here and k here, now you construct the rest of the table it turns out that this values are somewhere here. Now according to the Routh condition the signs of this things should remain same for the system to be stable and these are already positive. So these two must also remain positive actually. So that gives us a condition that k must be greater than 0 here from the fourth entry and how long can you do K should be less than this value.

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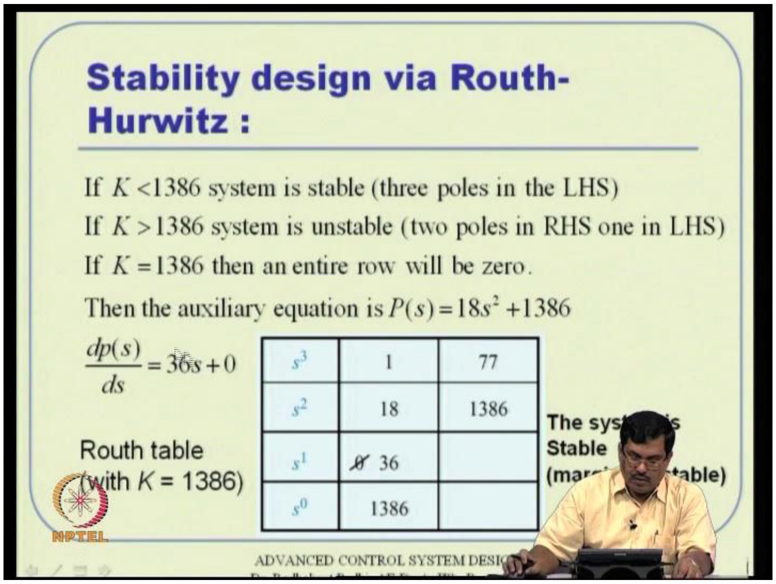

As long as K is greater than 0 and less than that values system signs will all remain same and hence will have stability. Now, if it is greater than that obviously, there will be signs change here where 2 signs are rather, and hence it will remain unstable actually negative values of case are not allowed I mean not allowed in the sense if we if we pick up the systems becomes unstable sort of t hing.

And the question is what if it is exactly equal to that value that means there will be a 0 element here. Then, you go back to that previous row and then talk about like differentiate this polynomial whatever polynomial you have and put it back there, and then it will result in something similar actually. So, then it will again turn out the system is stable but, in this particular case further analysis will show that system will remain marginally stable.

So, essentially we would like to avoid this particular number in a control design sense, we will be able to varied that the between 0 and this number somewhere. So, this analysis will give us a method of tuning this the n K so that is all about this stability analysis from Routh Hurwitz conditions further details are there in the book and it becomes very handy when you do not have computer power in them.

And essentially it can tackle about any other polynomial does not have to have any second order thought about that you can talk about one third polynomial still you will be very quickly able to infer whether the system is stable or unstable and moreover how many poles are in the left half side and how many poles the right half side all those information can be derived rather easily from just constructing a table basically that is the power of this.

Now next topic we will in a classical control sense, the great deal of interest for steady state error analysis as well actually. So, if the system remains stable then the next question is what is its performance? So in that sense we will be able to talk a little bit about the steady state error density. So the definition steady state error is the difference of difference between the input and the output for a prescribed test input as t goes to infinity.

Remember, when you talk about the input this is the reference input signal it is nothing to do with control input. There is one thing that I want to emphasize here many people think about the difference between the real output and the control input whether that is not the case when you talk about R of s that is the reference input that you want to track actually using the control.

So, if your if your reference input is something and your actual output of the system is something else even when t goes to infinity then there is a finite amount of error sitting there that is called steady state error actually. Ideally speaking we do not want steady state errors, because if we there is a reference command we want to track it in a perfect sense, that means steady state error is actually 0. So, that will whether that will happen in what condition that will happen and things like that and that is all and that is the analysis that we are going to talk in brief here. And usual state to test inputs that are used for steady error analysis as well as design we know that these are the step input, ramp input and parabola input and a justification of this essentially comes from Taylor series analysis. Sometimes we also test the signal using sinusoidal terms and that justification comes from Taylor series

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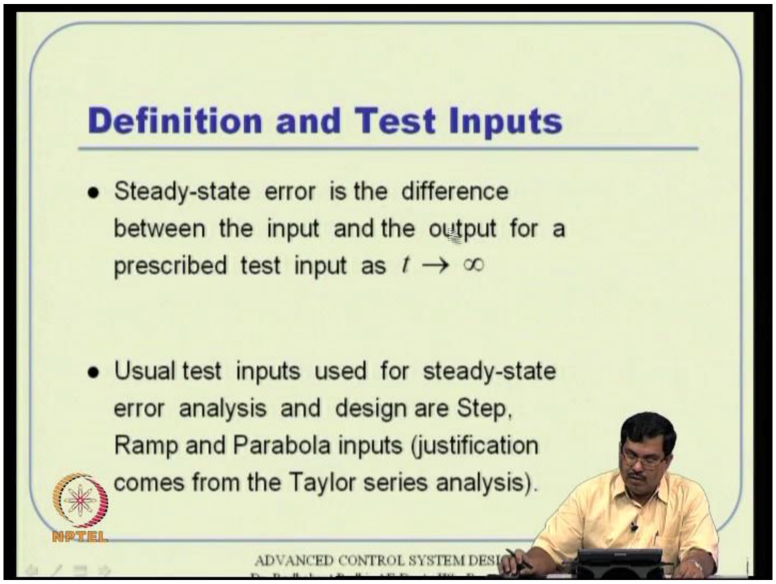

For most of the time the justification I mean the test signals used are step ramp and parabola to begin with essentially most of the time it is step and ramp and some specific case we will talk about parabola as well. And if you are testing the your system with respect to these three inputs that means essentially you are testing your system with respect to first three terms of the Taylor series which is usually sufficient. So that is that is the motivation why I do that and this is the test form.

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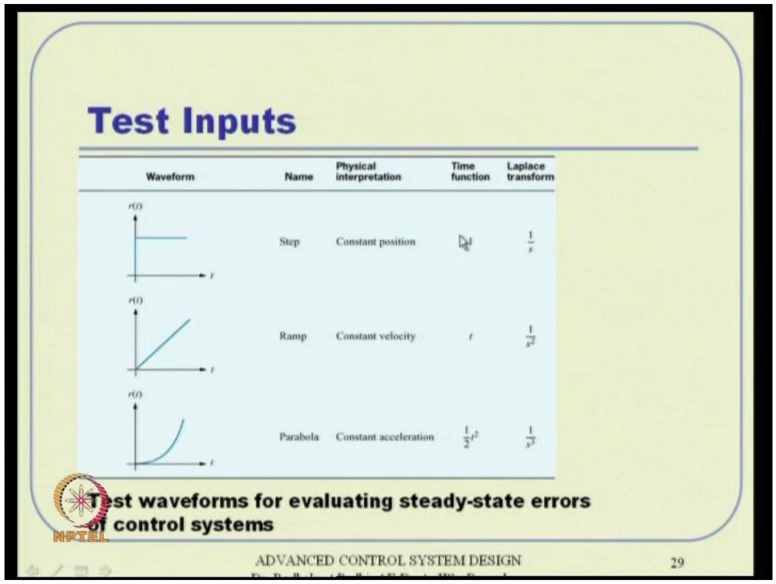

The signal waveform at a constant step mean there is a constant value and most of the time we will consider that some sort of a normalized input that means the we take it as input being just one, unit step input, and there is a ramp input again is just a t again with a slope one just keep on increasing with time and the parabola input is something like this you can associate a constant value along with this.

It can be any arbitrary constant K it can, may be K t it can be K by 2 t square as well. So that product constant does not play such much of a role here so that the signals are standard forms in standard form of 1 t and t square by 2. So with respect to these 3 signals we will see what is the steady state errors of our system so the input is let us say we talk about steady state error with respect to step input. Then, either the output can very nicely converse to that the reference input or the output can stabilize somewhere else if it commences to that value then the steady state error is 0 otherwise there is a finite value E 2 of infinity for output 2 and E 1 of infinity is actually 0 here. Similar things can also happen here in case so you are talking about output one which nicely converse just to that so E 1 of infinity is 0 and E 2 can have some finite value which is and E 3 can also have a different finite value.

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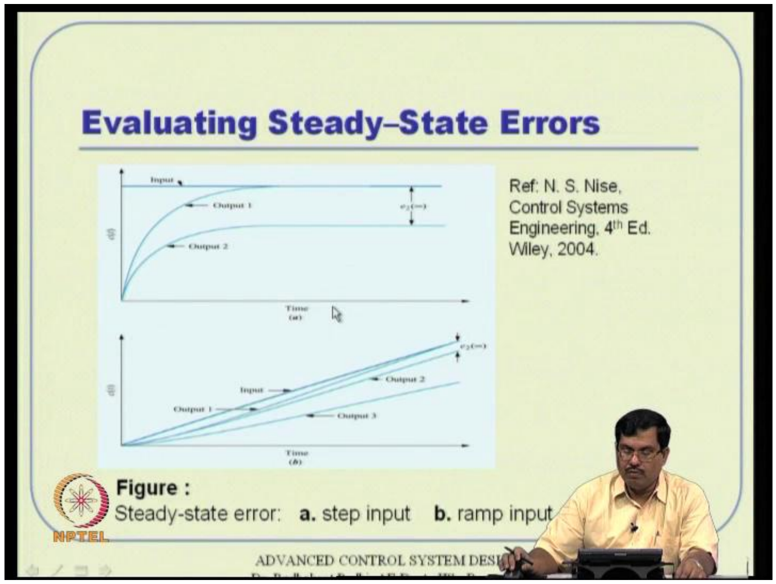

Essentially, it can also grow if it grows then there is a system leads to output leads to some sort of instability however if it remains at a finite value it is parallel to that line, but it is not 0 actually, so that is the finite error that you are talking about here. So, we want to study in which condition will lead to what and can be informed a little more about that without again going to the solution and analyzing the solution as it turns to infinity that is not our motivation here.

So, how do you go using the transfer function how do you go about analyzing this steady state error there are two representations essentially, one you can one you can think of this is my closed loop transfer function already, so this is my reference input this is my actual output of the closed loop, I mean system. So, obviously if I take reference input minus the actual output of the of the closed loop system then that is my error.

That is what I can interpret it one way other way to interpret is that is let me convert this the entire system to the unity feedback to negative feedback system unity negative feedback system. In that sense what will happen if this E of s is (( )) same R of s minus E of s what i will interpret that as G of s and there is unity of feedback H of s equal to 1. This unity feedback structure has some little more advantage in analyzing and interpreting our systems.

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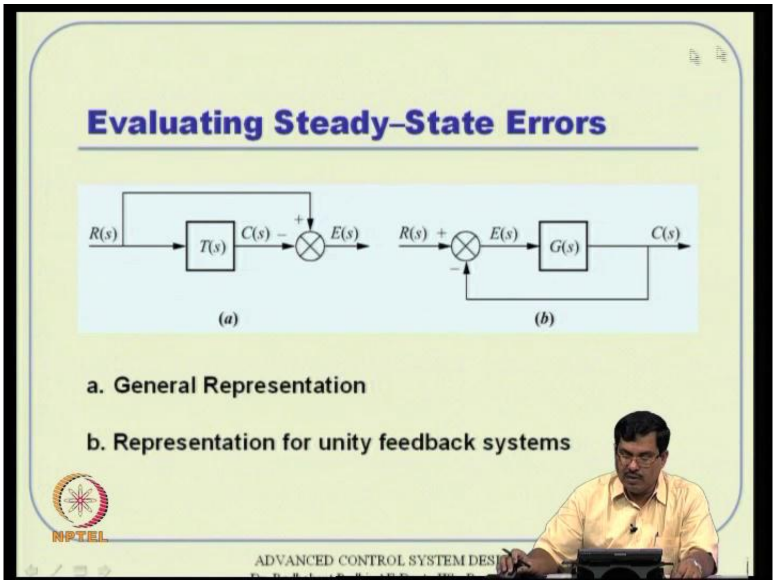

So we will see that particular case in detail, later we will also see that if there is a non unity feedback system we can essentially equivalently convert it to unity feedback system then interpret the results that way. So, that is not this is not going to be too restrictive in that sense actually. So in unity feedback system then you can talk about E of s is nothing, but R of s is minus C of s however this C of s is essentially E of s mean R of s into this entire system that is the T of s.

So, you can talk about E of s is essentially my R of s into, I mean if we use this 2 results if you put this C of s feedback in here and take the R of s common then you can we can read essentially lead to that actually. So, applying the final value theorem, remember all this thing we are talking about this stable system the system is unstable, there is no concept of steady state or it goes in infinity in anyway.

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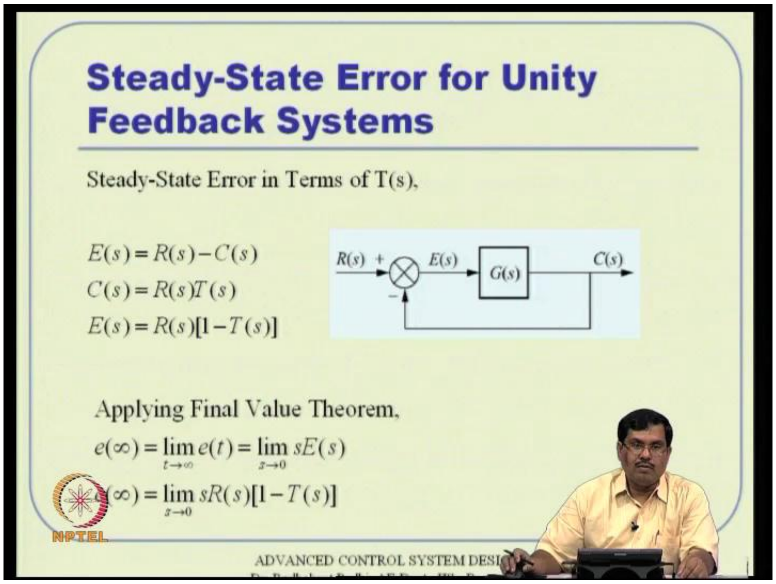

So for stable systems, we can use final value theorem and e infinity turns out to be limiting limit s tends to 0 s into E of s that we know from final value theorem. So using that we can conclude that this e of infinity is actually that so, if we know the closed loop transform function directly we can go for that actually. We can actually evaluate without going for the solution part of it. So, there is an example you can see this is the transfer function.

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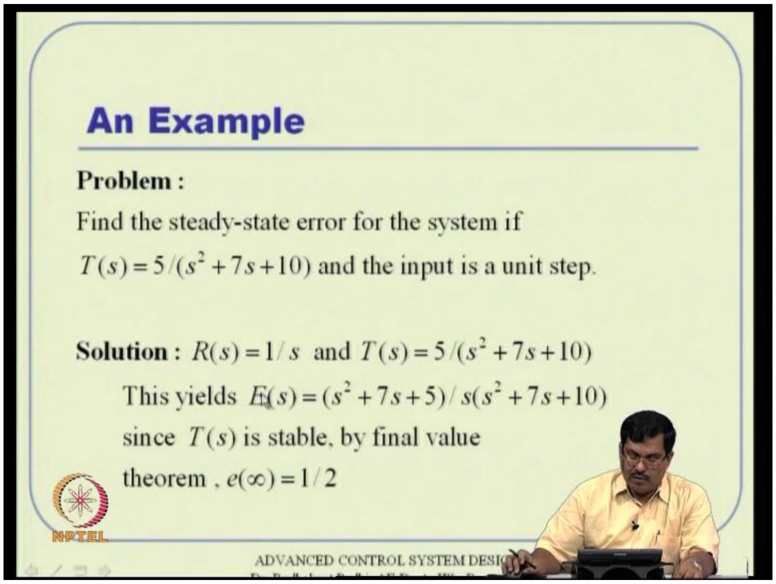

Let us say then unity is I mean step input then reference I mean R of s is essentially 1 over s T of s essentially turns is that and it yields to something like if you carry out this algebra E of s it turns out to be something like this. So, if you see this pole locations and all that it will turn out to be stable system only one only one pole on the 0 I mean net 0 and the other things will be in the left half side the system is stable, so using the final value theorem and all that you can talk about well, when we talk about system being stable you do not talk about E of s primarily you can even talk about T of s. You can simply talk about pole locations here. Now you can write this E of s is like that. So if T of s is stable then you can use the final value theorem you can just put it here and do whatever you have it here even the E of s is available now so carry out this algebra this s will go.

So s E this s e is not there s into E of s that is what you are talking then you lead all s to be 0. So that means all these coefficients are not there, so essentially results in 5 by 10 5 by 10 is essentially half. So E of e infinity will turn out to be half here. Another way of interpreting this unity feedback sort of idea here that E of s is essentially, R of s minus E of s. However, C of s is essentially E of s into G of s here so you can carry out you put it back here and then tell E of s is nothing but R of s divided by G s actually. The closed loop transfer function, remember is G of s divided by 1 plus G of s what E of s is essentially R of s divided by 1 plus G of s. So, by again using the final value theorem you can turn e of infinity limit test s 0 s into all these actually what you have here. So, if you know this input R of s reference input R of s and the system G of s in a unity feedback form. Then essentially you can use this final value theorem this formula and compute a value for infinity. And now the coming to little more analysis this is let us say this is you talk about step input. So R of s is essentially 1 by s, so infinity turns out to be, you just put it 1 by s here, whatever here R of s is 1 by s s and r s and 1 by s will cancel out, so you will be left out with this actually.

So, for 0 steady state error that is what we will look forward to whether the system is really 0 steady state error, what will happen when the system is 0, this is 0 I mean when this expression is 0 provided this term turns out to be infinity this will turn out to be one over infinity sort of thing actually. So, we will look forward to some cases where this G of s is actually in a limiting sense when s goes to 0 then this particular terms leads to infinity actually.

That means this is G of s needs to be in this column structure. Where this n is at least 1 1 of higher, so that means this particular thing s to the power 10 must appear. That means if you have one integrator one integrator in G of s one pole exactly sitting on the on the 1 pole exactly sitting at 0 the origin then you have the steady state error 0 actually. That is why the integral feedback is essentially helps us in eliminating the steady state roles actually.

So if you see this entire term has to go to 0 this value I mean this particular term has to go to infinity. And this particular term will go to infinity provided if this is in this form where s to the power n appears where n is at least 1 then it will be ok. When s tends to 0 then this entire term will go to infinity and this entire term will go to 0.

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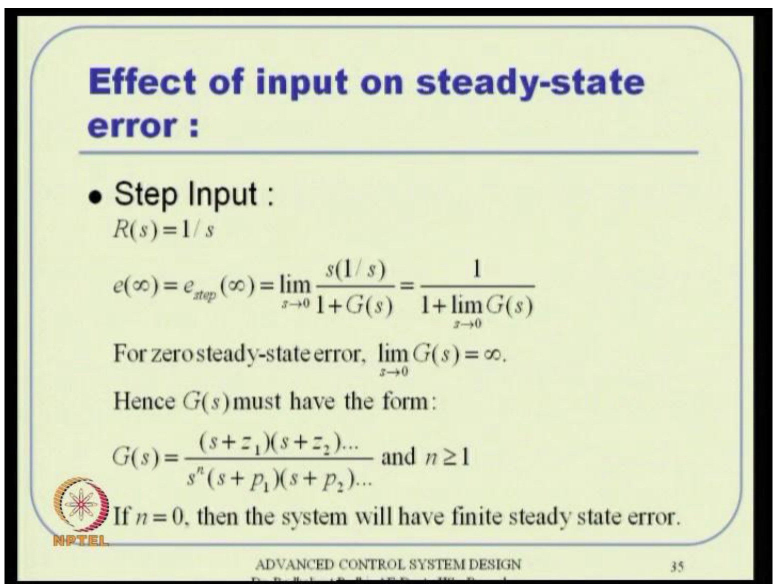

So the integral feedback will help us in that sense actually. I mean integral control of essentially not that means if you have some sort of a I mean if this fellow does not contain any pole at the origin, then one way of doing that is probably you can just put a controller block here k by s actually then these 2 will multiply and then s will appear here this that is that is a whole idea of b I d control when you discuss a little bit later actually.

So, in a similar way you can talk about ramp input as well in case, so for ramp input you carry out this same exercise again, now you now you will realize that if this error has to go to 0, then this still be there from this form but in this particular case you need n has to be at least 2 that means you still have 2 poles sitting on the I mean say sitting at 0 basically, so if you have to have a double integrator essentially then even with respect to the ramp input the steady state error is going to be 0 basically.

So, similar thing with parabolic input it will turn out that if n is at least 3 then the similar thing will also happen here. So, essentially this how many integrators you have in the forward path that that will decide whether your steady state input I mean steady state error is approaching 0 or not. If you have no integrator obviously you have to live with the fact that you have to have some sort of a finite error essentially steady state error.

Well in the integral in the loop is also slightly dangerous we will see that in the p i d control part of it there is sincerely that had its own problem of control wind up and other things actually. So, which is not a very neat idea to put an integrator all the time actually.

So example, sense if you can you have this transfer function here is your phase then h of s is one again then you can carry out the algebra and you can tell this is my steady state error for step input, this is finite value however, you want this steady state error to a step input itself is a finite value to a ramp in parabola input they will sincerely go to infinity. So, remember this ramp and parabola inputs are not really bounded input these are unbounded inputs.

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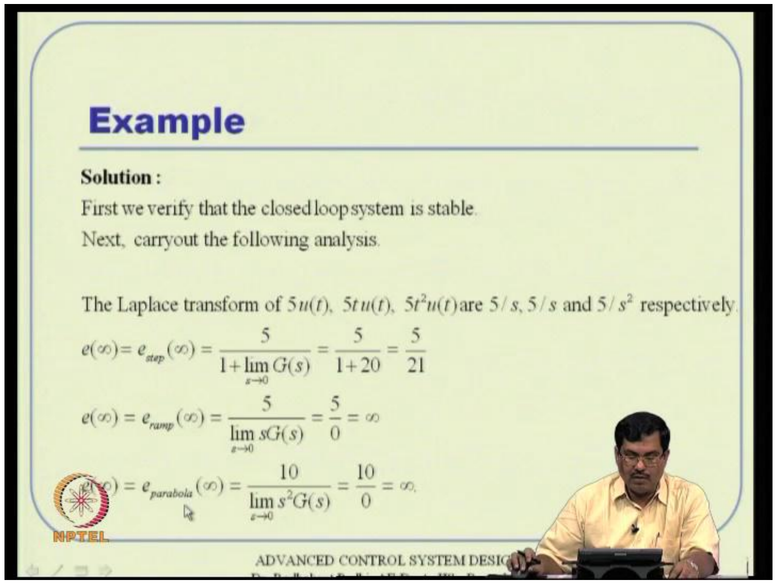

When t goes to infinity the value itself goes to infinity. So the system stable the system can still be stable in that sense because the b I b o sense boundary input boundary output sense we have not confined ourselves to bounded inputs you put a number of bounded input here. So but the steady state error saying they can still go to like infinity and thing like that the difference between these two actually.

Now, many times it is convenient to the define this something called the position constant velocity constant and thing like that because this essentially come in to picture when you when you do this analysis, if you talk like if you talk about this these essentially dictate the steady state error actually. So whatever terms you have here these terms will essentially we will be able to let us say we define these terms.

Some sort like position constant then velocity constant then things like that then steady state error we can essentially like we can talk about steady state error we assess in this particular case will term out to be 1 by 1 plus K p, and in this particular case this in this particular case you assess it 1 by K v like that. So this is this is because of that and system type is a essentially the value of n by definition.

System type is a value of n in the denominator of G of s that means what we discussed here all over this particular value of n dictates the system type, if it is 0 the type is system is 0, if it is one type 1, if it is 2 type 2 like that. Actually, and this is summary of a the entire results of the entire results of and analysis and all that this table is there in the Norman Nise book anyway.

So, if it is a type 0 type 1 type 2 and if you have a step input ramp input variable input there are nice formula like steady state error formula is 1 by 1 plus K p 1 by K v 1 by K a all that that is all. Then, there are like what is the error value essentially you can see for type 0 system this is a like that steady state error value type 1 it will turn out to be 0 here you know that.

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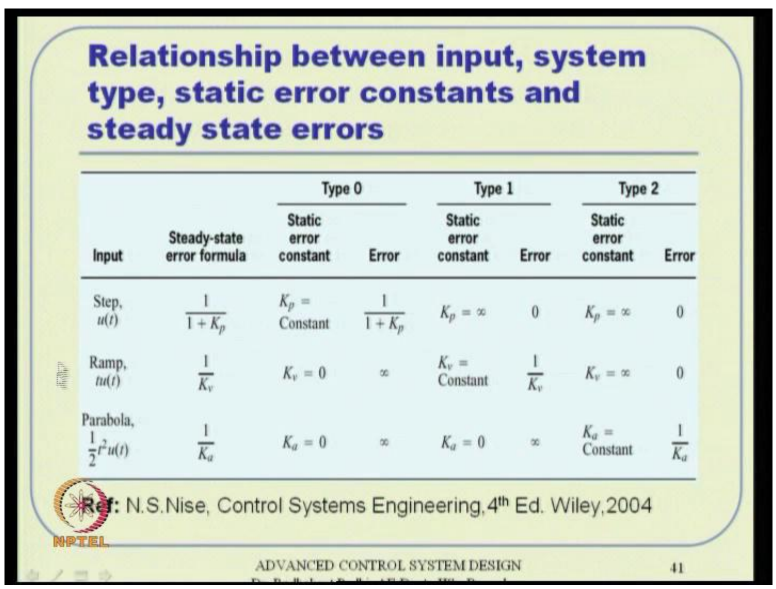

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

So if your system is of type 1 or type 2 then for a step input the steady state error is guaranteed to be 0 say if your system is type 2 even for a ramp input the system steady state error is going to be 0 for probably we can put is going to be a finite value so like that. so it is in steady state error sense is good to have a system of type 2 for studies for having lesser and lesser steady state values for all sort of inputs actually. So however for other things it may not be desirable actually. So we will see that again so what do you infer let us say somebody gives us a value say K p is 1000, so what do you infer from there?

First thing is the system is stable because K p is a finite value and hence e s is also a finite value so system is also stable and the system is type 0 because k p is finite if k p is finite system is to because K p is turns out to be kind of infinity for other two. So this is a system of type 0 and you can also eventually conclude to infinity steady state or infinity in using this formula it will turn out to be that way.

So all those information is embedded in K p being a number and if you know that number actually. So you can there are concepts called disturbance inputs as well. So when there is a control input which is here control design sort of thing, the input comes like an error between input and output and the disturbance input can also come to plant a some sort of a model uncertainty thing like that actually.

And if you have disturbance input there then it essentially there the error or signal whatever coming here it can decompose you can carry out the algebra like this, and you can see that E of s consist of two parts, first part comes because of R of s second part comes because of D of s. So essentially it is a combination between r and d sort of thing. So if you really want to decrease this E of s this particularly this E d you do not want this component wield off actually coming from disturbance input.

So you really want to reduce this disturbance input effect then you have to think about how to reduce this actually. Now this particular thing I can divide numerator and denominator by G 2 of s. So essentially it will turn out to be one by 1 by G 2 plus G 1. So this particular term what I am talking here, in this particular term well if I divide by G 2 then it will turn of 1 by 1 by G 2 plus G 1. So essentially you can reduce this by either increasing G 1.

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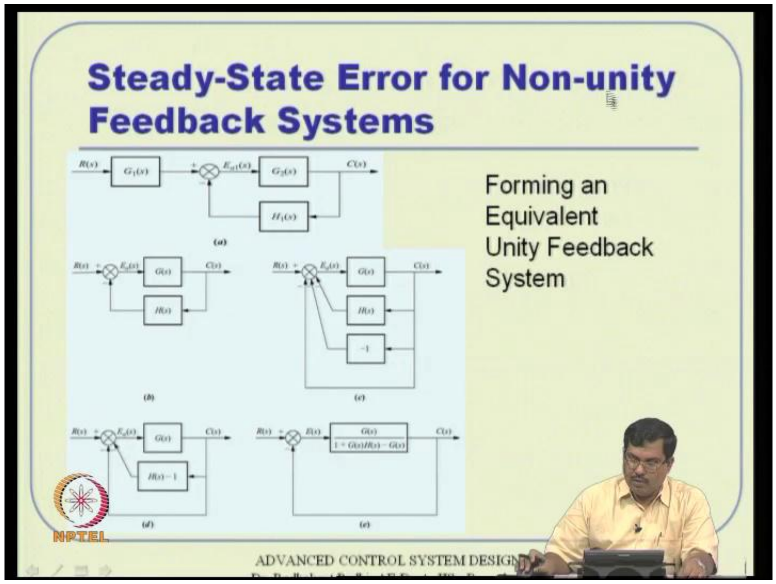

If I increase this G 1 it will go to infinity or I decrease it to it will also go to kind of infinity basically. So this will turn this entire term will become 0. So I can say one way of the mechanism here is either I talk about some sort of a high value of G 1 or a low value of G 2. G 2 normally I do not have a hand on but, I have a hand on G 1 because that is a control loop that is the 1 I want I can design actually. So you can increase this control increase means what I can increase the gain sort of thing here.

Then you can have a reduced effect of the disturbance essentially it also makes sense because if you are having high gain control or the correction becomes faster. So even if the disturbance takes you to somewhere else some of the initial condition then the gain will make sure that you come back quickly sort of thing what is. So finally, now how to handle this non unity feedback system in all that thing we discussed is about unity feedback system.

So if it a non unity feedback system in general there is a block here and there is a block also here, then you can do this transformation convert this entire system to a unity feedback system something like this actually. So I mean you can interpret sequence you can carry out the sequence of operations like that and once you are having here this H of s I mean this H 1 and G 1 first you convert it to something like this G 1 H.

So this H of s will contain the effect of G 1 and H 1 together and in block diagram reduction is something like is all of you know or it is easy to read in the text book also it is not very difficult and this particular course is all about modern controls I will not about talk about too much details about that. So if there is a block say like something here is possible to get something like that, but still it is not unity.

So what you do you add 1 add 1 and subtract 1 now it will become parallel loop from here this 3 and you know parallel things you can adopt and serial things you do with the multiplication that is the rule actually. So if there is a parallel thing I can I can add this two basically in case so turn out to H s minus 1. Then I will dump this two and this will go something like this now it operates in some sort of unity feedback sense. So all the thing that I discussed in unity feedback sense is still valid even if there is a general system actually so that is the that is the way to approach it actually.

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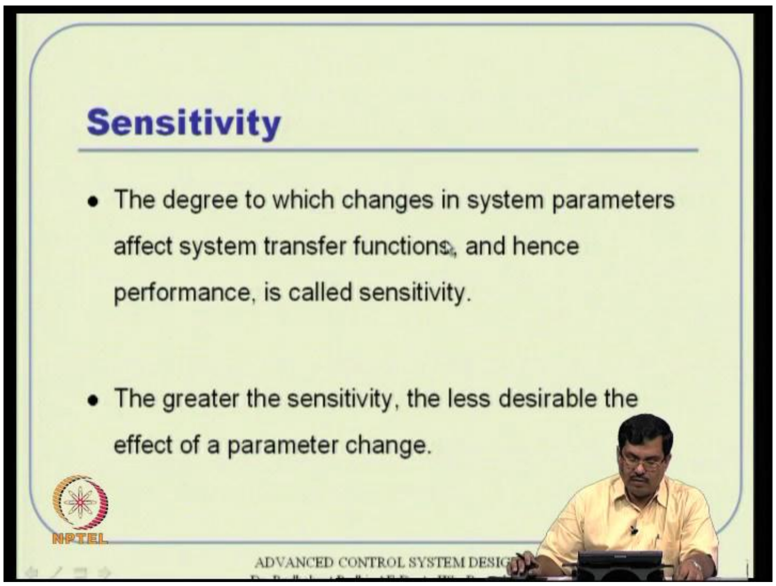

Now finally, there is another concept something called Sensitivity and this is essentially the degree to which changes in the system parameter affect the system transfer function and hence the system performance actually that is called sensitivity. So greater the sensitivity, less desirable is the effect of parameter change. So this is just mathematically something like this sensitivity of F with respect to p that this is if you take limit delta, if it tends to 0 that the fractional change in the function F divided by fractional change in the parameter P.

So that is the fractional change in function F divided by divided by fractional change in parameter P and then you can carry out this algebra it turns out to be something like this P by F partial differential of F with respect to P example, if you have something like this and there is let us consider a ramp input is a K v something like that infinity is like this infinity is 1 by K v anyway so that be like that.

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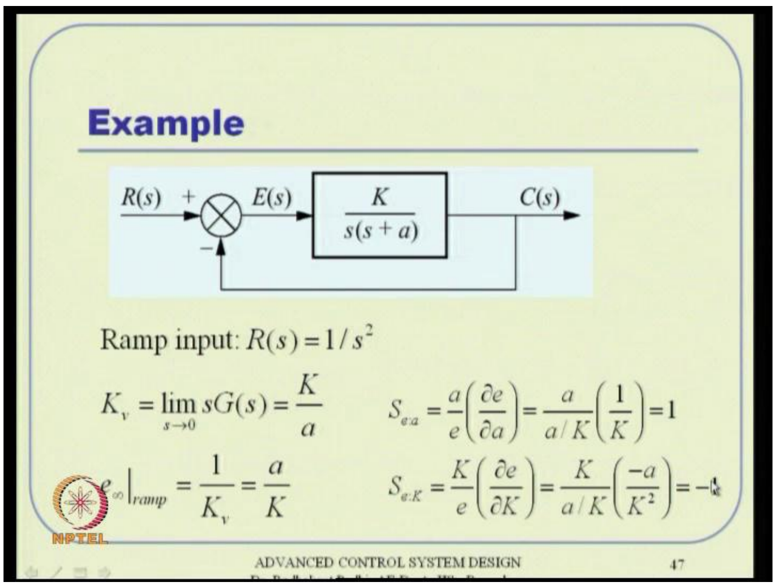

Now it turns out what is the sensitivity of this e with respect to a and what is the sensitivity of this e with respect to K and we have the formula e with respect to a is a by e this partial derivative and e with respect to K is K by e this partial derivative. So you can see that actually. So one is plus 1 this is minus 1 so that means if I have some change in parameter a a is going to increase actually. So if I have a positive increase in a e is going to increase in other words if I increase a e infinity is going to increase.

If I increase K e infinity is going to decrease because the sensitivity is minus 1 this is very clear from here that is how we carry out the analysis in sensitivity sense that is all for this lecture and further things we will continue in the next one thank you.