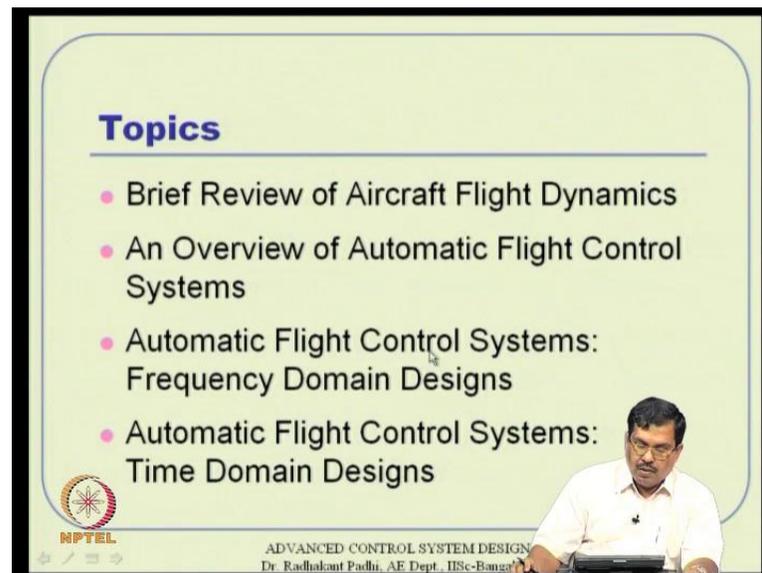


**Advanced Control System Design**  
**Prof. Radhakant Padhi**  
**Department of Aerospace Engineering**  
**Indian Institute of Science, Bangalore**

**Lecture No. # 29**  
**Linear Control Design Techniques in Aircraft Control - I**

Hello everyone, we will continue with our lecture think lecture series now we are at lecture number 29 today, lot of like control synthesis I mean control analysis as well as synthesis techniques we have studied before. Both in the like classical domain and little bit in the beginning as well as modern control domain quite a lot later on. And then we have also seen this flight dynamic concepts actually before. So, now the question is can we combine the two and come up with nice flight control design techniques especially, in the context of aircraft control. So, that is that the topic we are going to discuss today actually.

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**Topics**

- Brief Review of Aircraft Flight Dynamics
- An Overview of Automatic Flight Control Systems
- Automatic Flight Control Systems: Frequency Domain Designs
- Automatic Flight Control Systems: Time Domain Designs

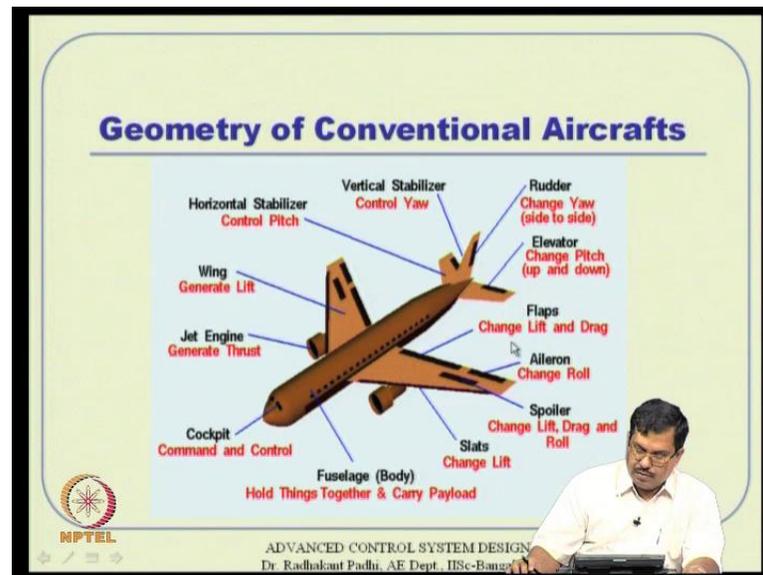
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So, the topics today is something like this, first we will again quickly very quickly rather we will review the aircraft dynamics one more time, just for recapitulating all that we have done before. Then, we will also give an overview of automatic flight control systems, then we will proceed with the frequency domain techniques. Followed by time domain, but probably the we will cover only the up to this today, we will cover like automatic flight control

systems, with frequency domain design essentially using some of the classical control design techniques actually. Next class, we will follow it up with some of this modern control design as well. So, let us see this, a brief review of aircraft flight dynamics – flight dynamics; essentially, we have studied all these things before and just the recapitulation of what we earlier done before.

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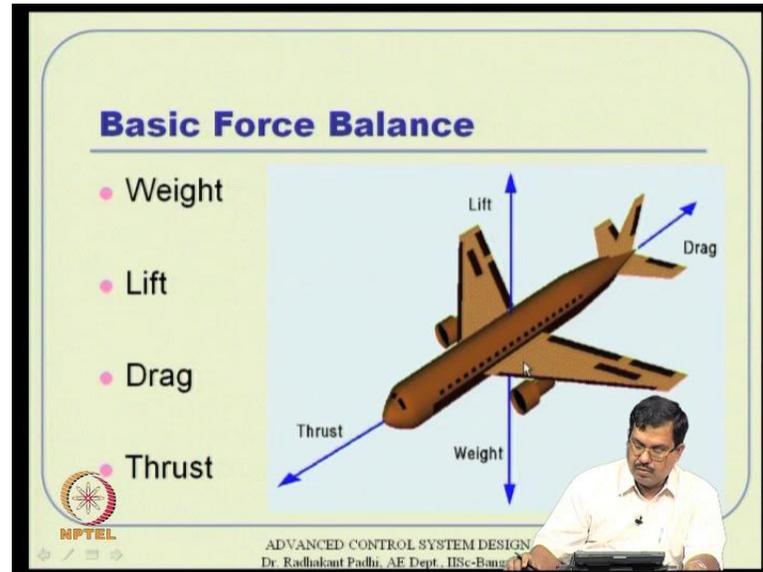


So, this a geometry of a conventional aircraft you will see that there are various **various** sub systems of the I mean on the aircraft and then each of the sub systems have their own role to play and whatever, you see this darken lines and all that these are all essential you can think of a some sort of a movable parts and not fixed part. And by moving these surfaces up and down and think like that you essentially, change the aero dynamics around the around the surface and hence, the generate additional forces movements and thereby we control the aircraft actually.

So, essentially you have this control surfaces like elevators right here and then we have this Ailerons which is here, which is somewhat here as well as here and then there are Rudder actually, somewhat here and there are three primary control surfaces acting on the aircraft. Other things are like spoiler, slats and all that are also there but they are not very frequently

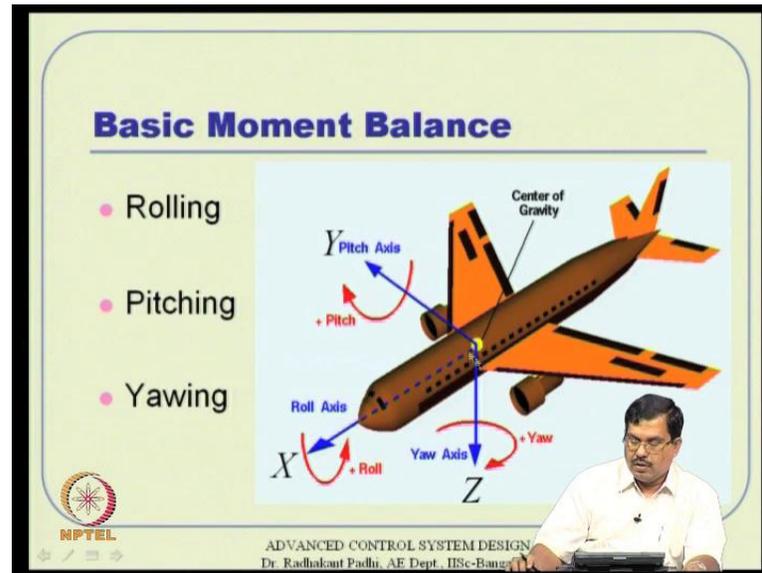
used, there are used primarily by takeoff and landing, but not during flying actually, anyway so these are all various components acting on the system.

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And then basic force balance on a stadium level flight it is obviously, something like this we have weight of the aircraft acting down, so something has to sustain that **that** weight. So, it has to be lift vector and while producing this lift we produce drag as well, and to counter act the drag we have to produce **(( ))** actually. So, this thrust drag and weight balance each other in a stadium level flight. Side forces are typically **(( ))** actually, normally there is no forces acting sideways, then about the moment balance **(( ))** this is actually the force balance what happen there and there is a moment Balance in other words.

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If you want to rotate the aircraft then you have to essentially need excess frame about each unit to rotate, and that is where there is a concept of body access frame, which is centered at this centered gravity of the aero plane and X axis towards front and Y axis to the right side Z axis to the down, all that we have discussed before and any positive motion about the X axis like right hand thermal sort of thing and little positive role actually. So, similarly, if you grab this Y axis right hand and then, your thumb pointing towards the Y axis then the direction in which the aircraft will rotate is actually called it is motion actually and similarly, this is the motion. So, (( )) are about the motion I mean, motions about the X Y Z axis actually and this is what we discussed before.

This type of motion is called Roll and primarily these black surfaces that you see here these are Ailerons these will contribute to and they are deflected, I mean they are not symmetrically but asymmetrically that means one is up, another one will be down. So, that the differential force acting on the side of one side of moving about the other, creates a kind of a moment there and for which this aircraft kind of rolls actually. Similarly, it is a when you do this symmetric deflection of the elevator it creates a moment about this in this direction. Essentially it leads to pitching itself and then similarly, it is like if you create that deflection of Rudder then it essentially cross the hearing motion actually. So, all these things we have seen before.

Now, the question is that in a automatic control flight system we have to generate all this control surface deflections properly either directly coming from the pilot command or even coming from a guidance loop actually. That means, autonomous flight and all that, if the guidance thing is also in action then the reference command to follow and all that comes from there and all these control surface deflections, so you have to create (( )) appropriately so, that is the problem of so called control system design for aircraft o auto pilot design actually.

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### Airplane Dynamics: Six Degree-of-Freedom Model

Ref: Roskam J., Airplane Flight Dynamics and Automatic Controls, 1995

$$U = VR - WQ - g \sin \Theta + \frac{1}{m}(X + X_T)$$

$$V = WP - UR + g \sin \Phi \cos \Theta + \frac{1}{m}(Y + Y_T)$$

$$W = UQ - VP + g \cos \Phi \cos \Theta + \frac{1}{m}(Z + Z_T)$$

$$\dot{P} = c_1 QR + c_2 PQ + c_3(L + L_T) + c_4(N + N_T)$$

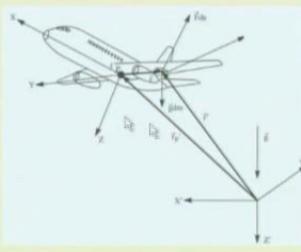
$$\dot{Q} = c_5 PR - c_6(P^2 - R^2) + c_7(M + M_T)$$

$$\dot{R} = c_8 PQ - c_9 QR + c_4(L + L_T) + c_9(N + N_T)$$

$$\dot{\Phi} = P + Q \sin \Phi \tan \Theta + R \cos \Phi \tan \Theta$$

$$\dot{\Theta} = Q \cos \Phi - R \sin \Phi$$

$$\dot{\Psi} = (Q \sin \Phi + R \cos \Phi) \sec \Theta$$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

$$h = -z_1 = U \sin \Theta - V \cos \Theta \sin \Phi - W \cos \Theta \cos \Phi$$



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While doing that we have also derived this what is called the six degree of freedom module of the aircraft where seen each of the variables you have discussed before. These are velocity components and then the P Q R or the role rates pitch rates about the body near X Y Z component. These are other angles pi theta psi and these are like X Y Z as seen from the inertial frame possibly situated on the ground. So, this is this will give you the, I mean the co ordinates of the c c of the vehicle as it moves as seen from the inertial frame and that way. And then there is a may be h dot essentially negative of z I is here the negative of that is height gain actually that way so, anyway these are the what is called six degree of freedom model.

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### Representation of Longitudinal Dynamics in Small Perturbation

State space form:

$$\dot{X} = AX + BU_c$$

$$A = \begin{bmatrix} X_U & X_W & 0 & -g \\ Z_U & Z_W & U_0 & 0 \\ M_U + M_W Z_U & M_W + M_W Z_W & M_Q + M_W U_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} \Delta U \\ \Delta W \\ \Delta Q \\ \Delta \theta \end{bmatrix}$$

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_r} \\ Z_{\delta_e} & Z_{\delta_r} \\ M_{\delta_e} + M_W Z_{\delta_e} & M_{\delta_r} + M_W Z_{\delta_r} \\ 0 & 0 \end{bmatrix}$$

$$U_c = \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_r \end{bmatrix}$$

$$X_U = \frac{1}{m} \left( \frac{\partial Y}{\partial U} \right), \quad X_W = \frac{1}{m} \left( \frac{\partial Y}{\partial W} \right) \quad \text{etc.}$$

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And then we discussed about linearised models that means, this is like too many variables and two couple equations and all that. Especially, when we expand all that force movements which are coming from aerodynamics X Y Z coming from aerodynamics thing and x t y t coming from thrust components and similarly, l m n and l t n t moment components, coming from thrust and aerodynamic components acting on the (( )). Once you expand this coefficient and all that it takes a very complicated firm and becomes difficult to analyze as it is.

So, what the next idea is you have decoupled this motion along the place called longitudinal dynamic and then lateral dynamic so, when you do longitudinal dynamic you consider this perturbation perturbed motions. That U W Q and theta you club them together, that is one side of the thing and in the lateral mode you do that in the reverse delta v p r and delta phi basically.

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### Representation of Lateral Dynamics in Small Perturbation

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State space form:  $\dot{X} = AX + BU_c$

$$A = \begin{bmatrix} Y_p & Y_p & -(U_0 - Y_R) & g \cos \theta_0 \\ L_p^* + \frac{I_{xz}}{I_x} N_p^* & L_p^* + \frac{I_{xz}}{I_x} N_p^* & L_R^* + \frac{I_{xz}}{I_x} N_R^* & 0 \\ N_p^* + \frac{I_{xz}}{I_z} L_p^* & N_p^* + \frac{I_{xz}}{I_z} L_p^* & N_R^* + \frac{I_{xz}}{I_z} L_R^* & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} \Delta V \\ \Delta P \\ \Delta R \\ \Delta \phi \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & Y_{\delta_A} \\ L_{\delta_A}^* + \frac{I_{xz}}{I_x} N_{\delta_A}^* & L_{\delta_A}^* + \frac{I_{xz}}{I_x} N_{\delta_A}^* \\ N_{\delta_A}^* + \frac{I_{xz}}{I_z} L_{\delta_A}^* & N_{\delta_A}^* + \frac{I_{xz}}{I_z} L_{\delta_A}^* \\ 0 & 0 \end{bmatrix} \quad U_c = \begin{bmatrix} \Delta \delta_A \\ \Delta \delta_R \end{bmatrix}$$



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So, these four variables and all and the other four variables are clubbed together that sort of thing. Now, you can decouple these variables from the other side, because the in principle all the variables are coupled with each other, but there are some strong dependence. So, will you club them together in a strong dependency sense and tell this is what is my linearised dynamics, looks about looks like? So, essentially I have a where a b takes this firm x is a definition like that and you see the control vector, which is essentially delta e and delta t that is elevator and thrust deflection angle thrust deflection percentage here.

So, once I have this dynamics these definitions and all are like aerodynamic derivatives and all that you find that way once, I have this model then I can proceed with the design or I have the liberty of going to the transfer function of and then talk about a some sort of transfer function and all.

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**Short Period Mode**  
Reference: R. C. Nelson, *Flight Stability and Automatic Control*, McGraw-Hill, 1989.

- Heavily damped
- Short time period
- Constant velocity

Change in angle of attack

Time

Motion occurs at nearly constant speed

Short period (several seconds)

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So, this dynamic this four state variables we still decouple that in to two two each and this is like what we discussed in some period mode, which is typically there vibrates quickly, it does not go through a long entire vehicle does not go up and down it is just a vehicle motion about its own c c and that will lead to small perturbation of the vehicle trajectory a little bit. So, essentially the picture will be depicted like this you have this change of  $\alpha$ , but it will die down very quickly so, immediately you will have some vibration and then it will go so, that is heavily done this odd time period while it does that it does not really lose its velocity.

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**Short Period Dynamics**

State Space Equation: 
$$\begin{bmatrix} \Delta \dot{\alpha} \\ \Delta \dot{q} \end{bmatrix} = \begin{bmatrix} a_{11} & 1 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta \alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \Delta \delta_e$$

Transfer Function Equations:

*Handwritten notes:*  
 $\dot{X} = AX + BU$   
 $Y = CX$   
 $\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B$

$$\frac{\Delta \alpha(s)}{\Delta \delta_e(s)} = \frac{A_\alpha s + B_\alpha}{As^2 + Bs + C}$$
$$\frac{\Delta q(s)}{\Delta \delta_e(s)} = \frac{A_q s + B_q}{As^2 + Bs + C}$$

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So, that is the motion we are talking about and these motion is typically, given by these two equations alpha and q so, essentially delta alpha dot and delta q dot they are given like this linear equation. And if you really want to, I mean this model getting converted to anything to transfer function thing then you know how to do that so, if it is x dot equal to A X plus B then we know this is, if you give like x dot equal to A X plus B and obviously, y equal to let us say c x that has to be taken care. Then y of x divided by u of s is nothing but this c s I minus A inverse b c s I minus a inverse into B (( )) so, if you do this then you will get a transfer function.

So, that is what is done here and you get this delta alpha by delta e transfer function that way and delta q by delta e transfer function typically these are second order transfer functions. And that is also evident from the second order normally, we have this is another reason why we wanted to get back from four dimensional to two dimensional (( )) transfer function for using which we can do the design very easily rather compared to fourth order polynomial.

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**Long Period (Phugoid) Dynamics**  
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Lightly damped
- Changes in pitch attitude, altitude, velocity
- Constant angle of attack

Motion occurs at constant angle of attack

Change in altitude

Time

Lightly damped oscillation

Maximum speed

Minimum speed

Long period (order of 30 or more seconds)

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That is what it is done here, then long period actually is a reverse way of doing that, I mean looking at the thing does not very short period of oscillation and all it is essentially goes up and down up and down like that in a relatively longer timer. Essentially, this the entire vehicle goes up and slowly comes down then again goes up and comes down like that. So, in that process it changes the pitch altitude and velocity and all this happens with some sort of a constant angle of attack, angle of attack does not change I mean the angle between the attitude of the vehicle with respect to the so that remains a kind of (( )).

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**Long Period (Phugoid) Dynamics**

State Space Equation:

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} a_{11} & -g \\ a_{21} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_E \\ \Delta \delta_T \end{bmatrix}$$

Transfer Function Equations:

Assumption:  $\Delta \delta_T = 0$

$$\frac{\Delta u(s)}{\Delta \delta_E(s)} = \frac{A_u s + B_u}{As^2 + Bs + C}$$
$$\frac{\Delta \theta(s)}{\Delta \delta_E(s)} = \frac{A_\theta s + B_\theta}{As^2 + Bs + C}$$

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So, if you want to approximate this Phugoid dynamics or long period dynamics then this is the way to do that delta u and delta theta you try to club together. Then this is the transfer function they are telling and here this delta t that is the engine deflection, I mean engine thrust level percentage it actually plays a role here. So, this is the elevator deflection angle differential angle of course, this is differential thrust magnitude and this is seen like a percentage of maximum thrust. So, once again if you remember control actions and this one very fast and the compared to this delta t, so you can assume that delta t remains 0, so if you assume 0 it becomes like single control input again.

And again you can come up with this transfer function both delta u with respect to delta e and delta theta with respect to again these are second order transfer functions. This is all about longitudinal dynamic what about lateral dynamics again, we can do almost the similar way again these are second order transfer functions this is all about longitudinal dynamic what about lateral dynamics again we can do almost the similar way. We club this b p r and phi together all in the sense of small angle small perturbation and then you discuss about the various linear or rather linearised system equation. And where this control actually two controls now, and both are of similar time scale they are both fast and equal impacts so, we cannot take away one or we cannot assume that one is 0 and all that.

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**Lateral Dynamic Instabilities**  
Reference: R. C. Nelson, *Flight Stability and Automatic Control*, McGraw-Hill, 1989.

**Directional divergence**

- Do not possess directional stability
- Tend towards ever-increasing angle of sideslip
- Largely controlled by rudder

**Spiral divergence**

- Spiral divergence tends to gradual spiraling motion & leads to high speed spiral dive
- Non-oscillatory divergent motion
- Largely controlled by ailerons

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So, we have live with that and then this particular motion where the analysis of this particular thing leads to several phenomenon's and one of that is directional divergence that means, it does not possess directional stability, so slowly it will deviate from its intended path and that is equal to directional divergence. And then you have spiral divergence that means, if it is gradual spiraling motion and also in other words the angle of attack keeps building very fast. As well as the it is not just side sleep angle the aircraft also rolls so, it kind of goes through the spiral divergence in a fast way.

So, this is slow divergence it will keep on going slowly so, aircraft does not turn, but because of the side sleep angle it is just keeps on going away, but here the aircraft also turns and all that. So, it gives you a large I mean this large spiraling motion and all that typically this is the this motion is the is controlled by second order. Whereas the spiraling divergence is controlled by  $(( ))$  typically produce much more moment, because I h even though, because the inner say about  $x \times x$  is small is with respect to  $x \times x$  is the moment of inertia. So, moment of inertia about  $x$  axis small mass distribution along  $x$  axis is typically small, from the  $x$  axis is typically small and is typically small actually that is why this  $(( ))$  will have larger impact on the aircraft.

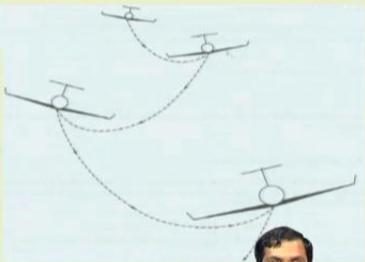
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## Lateral Dynamic Instabilities

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

### Dutch roll oscillation

- Coupled directional-spiral oscillation
- Combination of rolling and yawing oscillation of same frequency but out of phase each other
- Controlled by using both ailerons and rudders




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And then there is an interesting phenomena couples both of that and that is something called **(C)** that means so, it will first turn towards the left I mean it will roll towards the left so it will also diverge. By that time it turns to the right it will again come that side and again then go that way typically, it is like how the play and it is typically controlled by both **(C)** as well as.

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## Dutch Roll Dynamics

State Space Equation:

$$\begin{bmatrix} \Delta \dot{\beta} \\ \Delta \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta \beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta_A \\ \Delta \delta_R \end{bmatrix}$$

Transfer Function Equations:

$$\frac{\Delta \beta(s)}{\Delta \delta_A(s)} = \frac{A_\beta s + B_\beta}{As^2 + Bs + C} \quad \frac{\Delta \beta(s)}{\Delta \delta_R(s)} = \frac{\hat{A}_\beta s + \hat{B}_\beta}{As^2 + Bs + C}$$

$$\frac{\Delta r(s)}{\Delta \delta_A(s)} = \frac{A_r s + B_r}{As^2 + Bs + C} \quad \frac{\Delta r(s)}{\Delta \delta_R(s)} = \frac{\hat{A}_r s + \hat{B}_r}{As^2 + Bs + C}$$

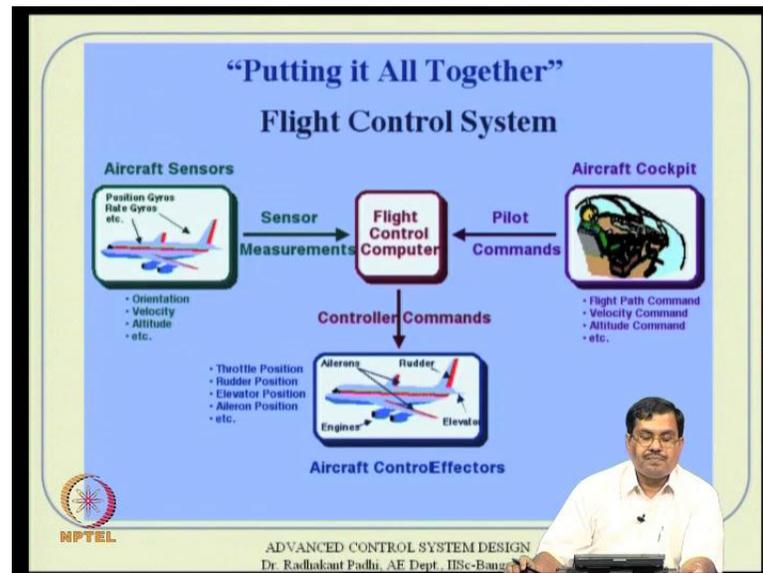

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So, if you really want to have (( )) I mean this transfer function and all that this is how typically all that takes the form, remember that this is like every input every input in the sense both (( )) will have impact on both the variables. So, if you want to visualize what is happening then for each input each output then, you have to construct this transfer functions with respect to that. So, we have this  $\frac{\Delta \beta}{\Delta a}$  and  $\frac{\Delta \beta}{\Delta r}$  as well similarly,  $\frac{\Delta r}{\Delta a}$  and so, all these things are like second order transfer functions and using that you can design a control system.

If you rely on classical control system typically, you need these single output single input modules based on transfer function this is what you use. So, that is like a peak overview of what we have already done before, I mean we have derived we have derived linearised equation, we have discussed about this lateral instability I mean this sub period and long period.

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That is the role of that thing now, coming to the flight control system of our view what happens there is (( )) if we put all together this is what happens in a flight control application. So, aircraft typically have a number of sensors, where the unit consist of they will also have this air delta system (( )) all sort of thing many sensors I will list them out in next slide or like that. So, this sensor information will keep on going to the flight control

computer and each aircraft is a good flight control computer basically and if you commercialize (( )). So, that information keeps coming and what the aircraft pilot wants to do and he can do a variety of commands actually, he can give velocity command all sort of things and those information are also fed to that.

And then the slide control computer takes the help of flight dynamics into picture like it is own amount of inertia mass, I mean all sort of parameters actually (( )). And then by appropriate computing I mean computation of the control command it is used (( )). So, these actuators will essentially (( )) and that is how the entire aircraft control, surface I mean controls system is activated. So, that is all what happens in atypical flight control system now, the question here is this like the driving a very rough car, which does not have any automatic control support and then driving that for a long duration is obviously, straining so, it cannot be done.

And in a high risk game like a (( )) in all things like that, we need certainly some support of automatic control system for the pilot. However, every dimension (( )) where a commercial aircraft pilot will keep on flying for let us say ten or (( )) hours, from beginning to end he has to be consistently giving input command to the aircraft, because this is not feasible. So, that is why we talked about augmenting this augmenting this control system as much as possible with automatic control loss so, that the pilot can relax on the way, many of the things can be done in automatic sense.

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**Sensors**

- **Altimeter:** Height above sea level
- **Air Data System:** Airspeed, Angle of Attack, Mach No., Air Temperature etc.
- **Magnetometer:** Heading
- **Inertial Navigation System (INS)**
  - **Accelerometers:** Translational motion of the aircraft in the three axes
  - **Gyroscopes:** Rotational motion of the aircraft in the three axes
- **GPS:** Accurate position, ground speed

*The transfer function for most sensors can be approximated by a gain  $k$*

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So, that is what auto pilots do, as I told sensors aircrafts can typically take lot of sensors and one sensor is altimeter gives height above the sea level and then the air data system gives most of the information like  $(\ddot{y})$ . Then we have magnetometers sometimes it gives you heading angle compared to the remember magnets are typically aligned with respect to north south. So, if your base is aligned somewhere else, then it will give you some sort of an angle how the aircraft is oriented with respect to north south direction. So, that is heading it will be inertial navigation system it is a big system of course, consists of  $(\ddot{y})$  depending on the application of what you are talking here and then here also supported with something called global positioning system nowadays, which are essentially set  $(\ddot{y})$ . So, they have their own positive negative issues here I mean for example,  $(\ddot{y})$  helps unit.

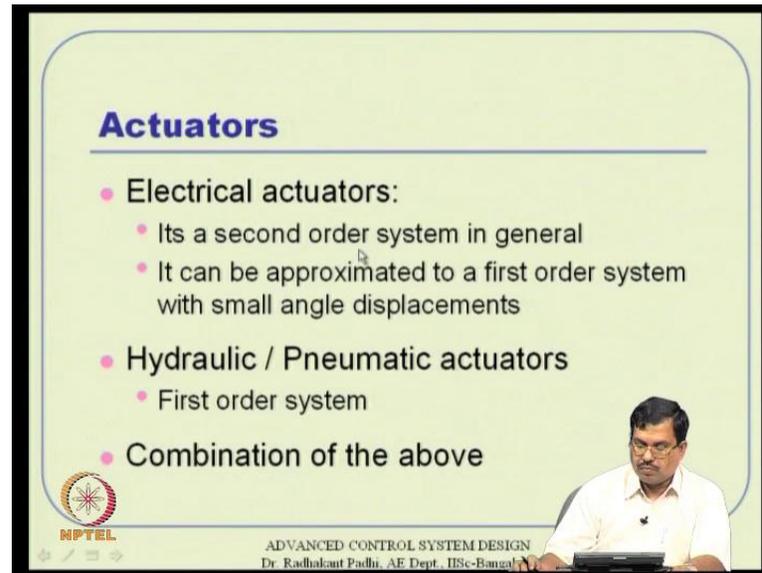
It does not need an external support once, you initialize it properly and then it acts on the then you can integrate the  $(\ddot{y})$ , then once you know this  $x$  double dot information integrate that we get  $x$  dot and then integrate one more time you get  $x$  provided you know the initial conditions properly. And similarly, with respect to  $(\ddot{y})$  you if the rotational dynamics into picture  $(\ddot{y})$  so, integrate that double dot and all that. So, if your initial information is good then it does not lead to too much of inaccuracies, but they have their own sensing units and their own filters in their loop and it is a complete unit essentially.

And nevertheless it does not require any additional support actually so, depending on the level of sophistication we can give you very good information. But the duration goes high and all that they have this problem of drifting and all that (( )) leads to large error like that many issues are there. Now, people talk about position fixing so, there is like a twenty four satellite constellation and all that like those four satellites based on which you kind of find your own position. By computing some unknown variables with some geometric constraints and all that so, if you do that then essentially it leads to very accurate position and also gives you what is your ground speed, speed with respect to ground.

Remember air speed is coming from here air speed is with respect to air, but air can have some relative velocity with respect to ground so, this ground speed can be measured using (( ))). Another constraint is like g p s can be very accurate, but the frequency of update cannot be very fast. So, it is much slower than any system now, people talk about can you really choose these two systems that means, (( )) and all this (( )). Anyway so, these are our host of sensors acting on the flight vehicle essentially a good commercial aircraft and then many of the transfer functions like many of the transfer function for most of the sensors can be approximated simply by a gain k.

Remember each of the sensors are dynamic systems by itself, they sense not immediately or instantaneously, but their own sensing mechanism, but the point here their settling time is very fast compared to a settling time of the aircraft itself. So, anytime you want an information that way so, that means most of the transfer functions can be approximated with just a instead of a dynamic system.

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**Actuators**

- **Electrical actuators:**
  - Its a second order system in general
  - It can be approximated to a first order system with small angle displacements
- **Hydraulic / Pneumatic actuators**
  - First order system
- **Combination of the above**

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Now, how about actuators that is ultimately what lectured the control systems and these actuators consist of variety of things, one is electric actuators other one is hydraulic actuators sometimes (( )) are also used. Typically these are fast rated systems where as electrical they can also be approximated (( )) with a small angle displacement and all that, if you do not demand too much of deflection like pin deflections from the system, then no need to kind of approximate that second order thing we can very well go with first order system. And then there are ideas like you can combine this actuator benefits and all you can talk about electro mechanical (( )) there are combinations of that.

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**Applications of Automatic Flight Control Systems**

- Cruise Control Systems
  - Attitude control (to maintain pitch, roll and heading)
  - Altitude hold (to maintain a desired altitude)
  - Speed control (to maintain constant speed or Mach no.)
- Stability Augmentation Systems
  - Stability enhancement
  - Handling quality enhancement
- Landing Aids
  - Alignment control (to align wrt. runway centre line)
  - Glideslope control
  - Flare control

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Handwritten annotations: 'ALIGNMENT', 'GLIDE SLOPE', and 'FLARE' with arrows pointing to the corresponding items in the list.

So, these will play a role in the automatic control system design actually for the (( )). Now, applications of flight control system now how do we design this automatic system for the aircraft they can talk about variety of things. And first thing that comes to mind is cruise control system, that means once the pilot takes the aircraft and sets the direction and all that for a large duration, I mean it has to rest to continue flying like that and that is what is called as cruise control and in that is the segment where pilot needs to be really relaxed. So, that means that is the segment where we design variety of cruise control systems and one thing is what we call as attitude hold that means, the angular positions of the vehicle and all (( )).

So, that is called attitude hold essentially this is to maintain the pitch roll and angles so, any dispersion of those angles needs to be nullified suppose you want certain (( )) let us hold it actually that is the easiest thing that you can visualize. Next comes to mileage altitude hold that means you really need to maintain a certain height so, these are all regulatory systems some value I want any perturbation around that I want to nullify.

And then there is a speed control mechanism and this one has come to cars also, speed control is essentially to maintain a constant speed or a mach number irrespective of whatever, local phenomena local phenomenon of (( )). Then you can the automatic speed control and either it increases, the power of the engine or decreases the power of the engine

so the ultimately speed is maintained. That has essentially come to cars actually lot of commercial cars has speed control essentially they call it cruise control (()). So, it is nothing but it maintains the speed no matter whether you are climbing a hill or you are going down the hill or your going on the plane highway and obviously, for the safety purpose and all the speed control once you put manual brake and think like that after that you it also note be (()).

But again once you take out the break and you see that nobody is there on the path for kilometers away and this (()) cruise control for the speed again we do not really need to kind of control the speed of the vehicle by changing the accelerator. The very same concept is there in the aircraft essentially the idea started from the aircraft anyway, the next level is what is called stability output augmentation system this is essentially, cruise control system fall under variety of other things also, this is just one thing altitude hold and all that. The other thing what is called is stability augmentation the stability augmentation system is essentially done for two reasons.

One is first of all stability enhancements that means let us say your aircraft is very marginally stable or really it is unstable like (()). Then many times you want to make the aircraft behave as if vehicle is under the influence of continuous control, if the control system is actively taking part it is actively excited then under that the part of the system behaves as if. And over that stable system you can with further input that is also there, but in general the stability augmentation system means that you compute some sort of a component of the control along with which the plant will behave as, if it is a stable plant. So, that is the first thing and normally the second thing which is related to the first one is something called handling quality.

Handling quality is like the ease of driving a car I mean you can visualize it that way so, that is like suppose you want to drive a car you feel it is easy to drive and all there are variety of reasons, the thing is if it is too responsive (( )). So, you have to have some sort of good response coming from that and that kind of a idea is called handling quality. Now, in other words he really wants to go up he wants to give a command of say change altitude from one

to two, then the aircrafts will really change it to within that time. And if you visualize this let us say (( )) without too much of oscillatory think like that.

So, those qualities are called handling qualities and has stability enhancement (( )) are coupled with each other that is one thing (( )). And then it is also, something called flare control I will touch up in a little bit as we go along, but roughly speaking this align after alignment happens suppose, this is runway let us say this is run way then it the aircraft should come roughly in a straight like that is called glide slope. And then it is should flare of in the sense they after that it cannot be just closed in a straight line and if it goes then it will hit by ground in a very bad way.

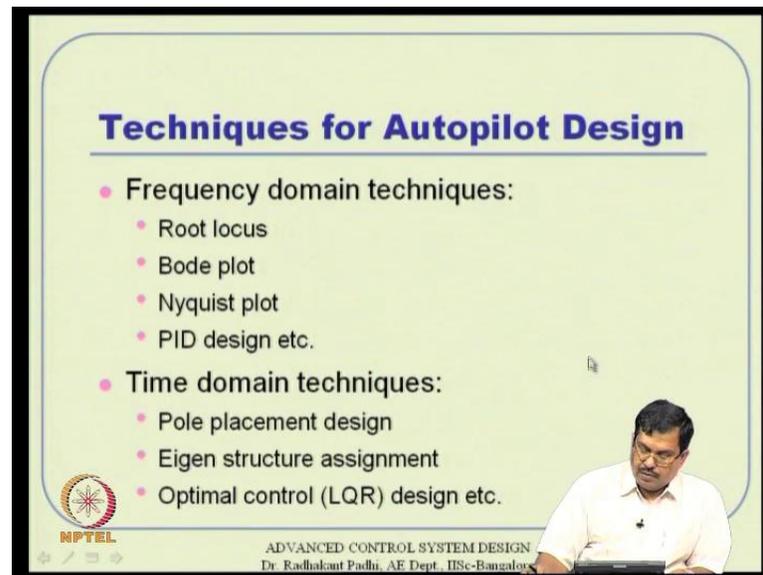
So, what is done? If after I mean very once the aircraft is very close to the runway they excite some sort of a exponential path. So, that while it touches the runway really and the vertical component of the velocity is manageable so this part is called flare and this part is called glide slope, this is flare and this is glide slope. And before that it has to align actually so that is also like alignment so this is like alignment glide slope control that is where that is where your control system also comes into picture. And in a reverse way it is also you can visualize that automatic take off also has a reverse sequence basically so, that is when you talk about landing automatic take off is also kind of included in that.

So, there are various levels of automation that we can visualize as far as application of automatic control system, power flight control applications are concerned. The most popular is of course, cruise control as a regulatory mechanism and then these are design side of the story, I mean this is what a what happens as the aircraft flies a in each of the flying condition. And this is happens without pilots knowledge probably other than the fact that may be test failure to know that or something, but regular failure may not even know that. So, that is that is going inside the flight computer in a built in manner and landing aids are anyway there and take off aids are also part of the (( )).

And there are secondary level flight control applications which I am not talking here and these are like path design guidance design essentially, where the aircraft will go what direction it will go what time it will reach and things like that. So, that is guidance level which is outer loop control sort of things that is also a control loop and if you want to use

control theory for that you can very well use it. And initial guidance application which is used heavily anyway so, similar concepts are there for aircrafts also. That means the mission of the aircraft also become certain amounts and that is that concept is very popular in a v domain where the pilot is not there it is on (( )) vehicle so, it needs to be guided in a automatic sense in that sense it comes very close to misguidance.

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The slide is titled "Techniques for Autopilot Design" in a blue font. It lists two main categories of techniques:

- Frequency domain techniques:
  - Root locus
  - Bode plot
  - Nyquist plot
  - PID design etc.
- Time domain techniques:
  - Pole placement design
  - Eigen structure assignment
  - Optimal control (LQR) design etc.

The slide also features the NPTEL logo in the bottom left corner and the text "ADVANCED CONTROL SYSTEM DESIGN" and "Dr. Radhakant Padhu, AE Dept., IISc-Bangalore" in the bottom right corner. A small inset image of a man in a white shirt is visible in the bottom right corner of the slide frame.

So, there are various levels that we discussed here and then this is like that as far as techniques the we have studied we have studied rout locus bode plots nypuist plots PID design and time domain techniques, we have studied pole placement Eigen structure we dint study and the, but optimal control like l q r design we have studied and things like that. So, all these techniques can become quite handy for addressing those issues that we discussed. Now, further rest of the class I will give you some example and the some part we will not talk each and every system around that, we will just take you through a couple of situations and couple of examples like that, which will give you some sort of a good feeling of what is going on.

So, let us see even this frequency domain design approach that is what I was I am talking here and remember this is a value talk here the frequency domain means, we will largely confine ourselves to this and I will take the help of route locus and to some extant PID

design concept. Bode and Nyquist analysis techniques and I think they are also useful we are not going to discuss here. And here we will discuss like some attitude control and the speed control things like that and next class probably we will go through this some of this time domain techniques and then, we will follow it off with different set of Eigen scheduling and things like that that way.

Anyway so, we will this is where we are some techniques we know some problems are at hand and so, we want to kind of use them together want to apply those techniques for these problems.

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**Velocity Hold Control System**  
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- The forward speed of an airplane can be controlled by changing the thrust produced by propulsion system.
- The function of the speed control system is to maintain the some desired flight speed.

The diagram shows a control loop. The input is  $\Delta u_{ref}$  entering a summing junction with a plus sign. The output of the summing junction goes to a block labeled 'Forward path compensator'. This is followed by 'Engine throttle control', 'Engine lag', and 'Aircraft dynamics'. The output of 'Aircraft dynamics' is  $\Delta u$ . A feedback path labeled 'Feedback elements' connects  $\Delta u$  back to the summing junction with a minus sign. The slide also features the NPTEL logo and the text 'ADVANCED CONTROL SYSTEM DESIGN Dr. Radhakant Padhi, AE Dept., IISc-Bangalore'.

So, let us see that and the first thing that comes to mind is velocity hold control system so that is speed control system basically. So, I mean anyway this entire class material is taken from this book say this is a nice book as far as automatic control application for flight control is concerned is R. C Nelson from MCGraw hill and the 1989, but I think there is a newer version also available so, if you want you can buy the book also it is a good book itself to buy. Anyway so this is like velocity whole control system what you what you do the obviously, in any type of control system and these are all based on this PID ideas and things like that.

So, you have some sort of a reference command that means that is that much speed they want to maintain and their actual speed can be something different so, you give the throttle command as a kind of a some constraint gain times the error in the output. Where the p part and that, if you really want talk about PID control, then p control is like k times e basically or minus k times e whatever, if  $(( ))$ . So, you have the output there and you know that this output that you are talking this delta u that means, the error in the speed is actually a function of this error in this reference command and things like that, which is typically compensated by engine control engine throttle control ultimately.

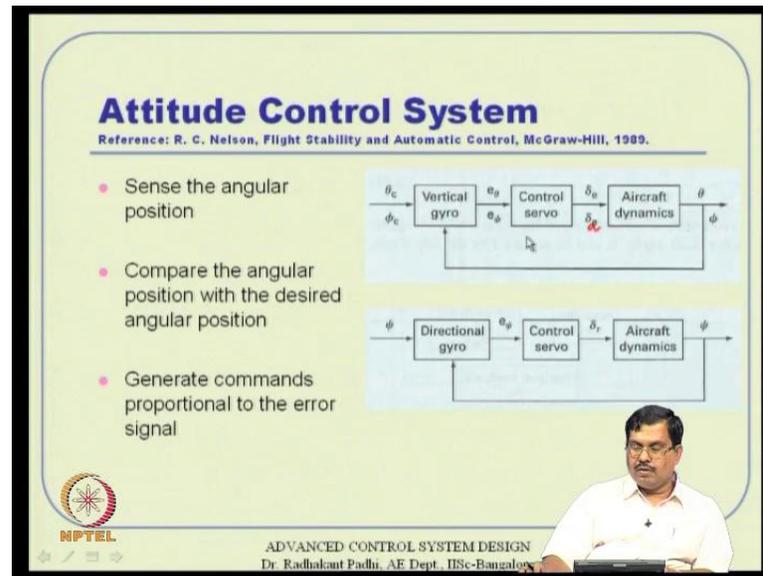
So, here we have to have this error signal known to us first and for knowing the error signal we need to know what is my current u and what is my reference u, typically this is actually, if you put delta u delta u is deviation and things like that that is 0 delta u that, I mean desired delta u is all is typically 0 in delta u is a difference of speed basically reference speed. So, anyway so you have this actual information which you can sense it or using this let us say at data system or u p a system and things like that. So, depending on what ever whatever, is your required value is generate a kind of an error command and then you multiply that with some gain or if necessary you kind of derive this rate so that you can talk about PD control or do this integral manipulation here.

So, we can talk about PID control things like that and ultimately this command will go to that and then it will pass through this engine lag also. This is what you can visualize that as an  $(( ))$  system passes engine lag and ultimately that will excite the system. So, as long as this error is there and then this engine control throttle control is excited, that way it will take it I mean it will keep on getting excited until it is kind of 0 and it the correction happens in the proper direction with this negative feed feedback in the loop that way. So, this is how this is recognized and once you know the each of the subsystem transfer function gain values things like that, there will be some sort of a gain tuning mechanism for this engine control total control.

So, those can you can you can tune based on time domain specification based on like let us say dumping specification and things like that there are variety of specification that you can

talk about and based on that you can tune this gains and then visualize or kind of realize this control system.

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So, that is essentially the idea of velocity hold control system now, coming to attitude control system what you do here is say very similar anyway so, instead of velocity you kind of sense the real attitude either pitch angle theta or roll angle phi or kind of angle psi either way. And then depending on what is your kind of intention so, you take the corresponding error signal either  $e_\theta$  or  $e_\phi$  or  $e_\psi$  so, you and then excite the servo control mechanism to get a valid value for this deltas. Either delta deflection or kind of a rodor deflection or deflection things like that, this will be deflection this is deflection and this is deflection so, this a typically small type of mistake probable this is deflection delta a alright so that is how it is actually.

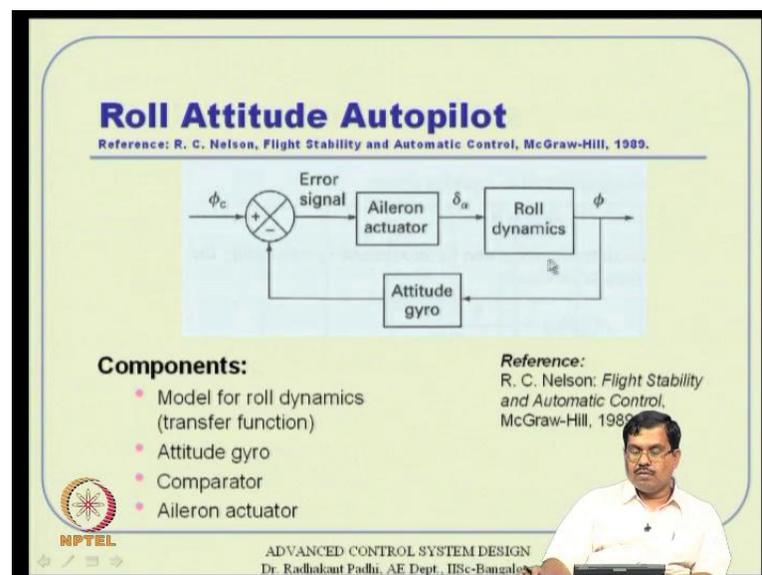
So, essentially what you are doing your sensing the angular position whatever, position there then there is a desired angular position let us say the pilot wants to pitch out. So, that means he is given little bit theta increase and things like that or he wants to roll so he is give phi c command and things like that so, that way this command is actually compared with this actual command which is coming from the sensor in the, I mean whatever sensor you imply. And then western this error signal you excite this control mechanism largely done

through p I d control anyway so, then generate this  $(())$  commands, then it will pass through this  $(())$  system and what you see is servo control and all is like  $(())$  also, in the loop actually that way.

So, ultimately this will be kind of a coming out of the  $(())$  output of the  $(())$  and it goes to the aircraft and excite the system that way. So, it is not very much different from what you are studied in feedback control you have something some think like a set point regulation problem, we have some we have got some command this actual command and there is sort of a error signal. And then based on this error signal, if I was just multiplied with a gain then I got p control, if I do this algebraic manipulation of like let us say differential or differentiation or integration and things like that ,then in can I can realize a p I d controller so that is how it operates.

Now, let us instead of l s o field let us talk little more detail of what is called role attitude autopilot and similarly, you can talk about let us say pitch attitude and  $(())$  attitude as well. So, I will pick up a role attitude autopilot and try to let us say explain what all things goes there through an example probably.

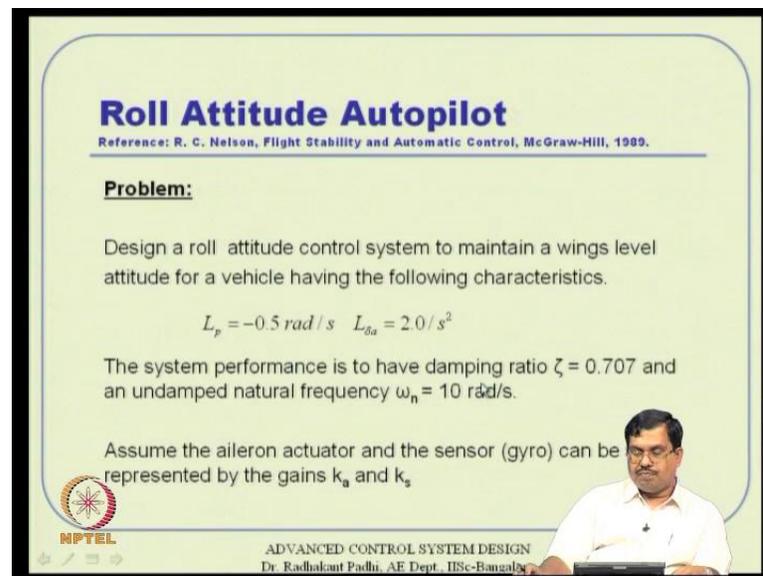
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So, if you really see the role attitude autopilots this is the mechanism you have got some sort of a role comma and coming from let us say pilot. And if those attitudes gyro which will sense the sense of phi is phi, the real phi so obviously you have an error signal and it has to pass through certain control design to get this delta a basically. So, this delta is aileron deflections that will that will that will be given to the role dynamics and I mean that will pass through this system model that we are talking about. Now, the components of this as you see it here in the block diagram, we have a model for the role dynamics which is kind of important here, based on that model we do several things so you can analyze the things.

And we can tune the gains actually these gain tuning will depend on these transfer function what you have there because close loop transfer function will be like this transfer function into that transfer function so, obviously this transfer function plays a role while tuning the gains. And we obviously, we need attitude gyro to find out what is our actual phi and we need a comparator thing to realize this comparison signal like plus minus and all that we get a error signal. And obviously, we need an aileron actuator to finally, execute the command so, these are all the various components that will go through the roll attitude autopilot and similar thing will happen for pitch attitude as well as gyro attitude.

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**Roll Attitude Autopilot**  
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

**Problem:**

Design a roll attitude control system to maintain a wings level attitude for a vehicle having the following characteristics.

$$L_p = -0.5 \text{ rad/s} \quad L_{\dot{\alpha}} = 2.0 / \text{s}^2$$

The system performance is to have damping ratio  $\zeta = 0.707$  and an undamped natural frequency  $\omega_n = 10 \text{ rad/s}$ .

Assume the aileron actuator and the sensor (gyro) can be represented by the gains  $k_a$  and  $k_s$ .

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Now, with this let us say you pick up some sort of a problem here now, we are all parameters of the aircraft and all there where it goes and all it will go to that that modeling part of it will not concentrate so much on that right. Now, we just assume these numbers will go somewhere in a model that way somewhere in the transfer function. So, what we are telling here is the aircraft has this characteristics and the our job is to design a roll attitude control system to maintain a wings level attitude that means, phi has to be 0 basically and for the vehicle has this kind of things. And then not only that while assuring this kind of this wings level conditions and all from any initial condition suppose, the aircraft is already rolled you want to make it wings level that is the problem.

And while doing that we want to assume that this damping ratio remains at point seven point seven 0 seven and the natural frequency happens, to be ten radians per second, I mean that is the desired  $\zeta$  width specification and all that. We have a damping ratio the for that and you remember the in that under close loop system. So, can we do that I mean that is what is a problem and you can assume that the aileron actuator and the sensor gyro can be represent it by some sort of constant gains only. So, there is do not have this transfer function is coming into picture additional transfer function we assume that these are simply gains.

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### Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1999.

The system transfer function

$$\frac{\Delta\phi(s)}{\Delta\delta_a(s)} = \frac{L_{\delta a}}{s(s-L_p)}$$

Forward path transfer function

$$G(s) = \left( \frac{\Delta\delta_a(s)}{e(s)} \right) \left( \frac{\Delta\phi(s)}{\Delta\delta_a(s)} \right) = k_a \frac{L_{\delta a}}{s(s-L_p)}$$

$H(s) = k_s = 1$  (unity feedback assumption)

The loop transfer function

$$H(s) = \frac{k}{s(s+0.5)}, \text{ where } k = k_a L_{\delta a}$$

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So, can we design a control system to do the job, this is what you can do now, if you see this  $\Delta \phi / \Delta a$  this transfer function that we are talking here this takes the one of the this one, that means this is like a second order pole the two poles are there and the when numerator is not 0 is actually so it is only a two pole system. So, for airport transfer function if you see this transfer function this is nothing but this to this transfer function,  $\Delta a / e^{\phi}$  this is  $e^{\phi}$  probably put that this is  $e^{\phi} \Delta a / e^{\phi}$  and multiplied with that fellow  $\Delta \phi / \Delta a$  this is like that right. This two the transfer function  $\Delta a / \Delta e^{\phi}$  and then  $\Delta \phi / \Delta a$  basically so, that that transfer function, if you multiply these two transfer function you will get it.

And we were assuming that the case is won so we essentially, land up with unity feedback kind of situation with analysis becomes easier any other gain is also fine that way. Now, the open loop transfer function, if you see this is a  $g(s)$  is known to us  $h(s)$  is known to us so  $g(s) \times h(s)$  is nothing but the article so  $h(s)$  is being one is nothing but simply  $g(s)$ . Now, these two gains I can multiply together and I can define something called  $k$  actually, if I define these two multiplication  $k \times l$   $\Delta a$  is nothing but  $k$  then my open loop transfer function takes the form something like  $k / (s(s + 0.5))$ . So, obviously you have a pole at 0 I mean 0 location 0 0 and you have a pole at minus 0.50.

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### Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

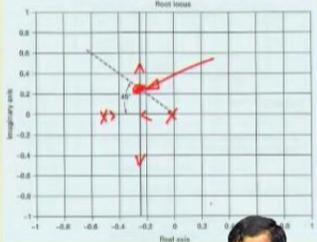
The desired damping needed is  $\zeta = 0.707$

We know  $\zeta = \cos \theta$ . Hence,  
draw a line of  $45^\circ$  from the origin.  
Any root intersecting this line  
will have  $\zeta = 0.707$ .

Gain is determined from:

$$\frac{|k|}{|s||s + 0.5|} = 1,$$

This leads to  $k = 0.0139$ . However,  
 $\omega_n = 0.35 \text{ rad/s}$  (much lower than desired!)







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So, let us draw the route locus diagram now and if you do the route locus diagram this is one pole here and one pole here this is one pole in the origin and one pole here so, this minus point five location. So, the poles I mean if you apply the rules of the route locus or you simply draw it using  $(\cdot)$  and things like that they will start from the pole they will go that way and then ultimately they will meet here and then will diverge this way, I mean that is the pole. So, once you have this kind of a route locus you have to find out gains from there which gain will satisfy wherever you want to operate. Now, I remember that one of the problem thing I mean what demands is that  $(\cdot)$  is to be 0.707 when  $(\cdot)$  is nothing but  $\cos$  theta of where theta is this angle so for  $\cos$  theta is that 45 degrees remember 0.707 will be one over root two sort of thing and 1 over root 2 is  $\cos 45$  is one over root 2.

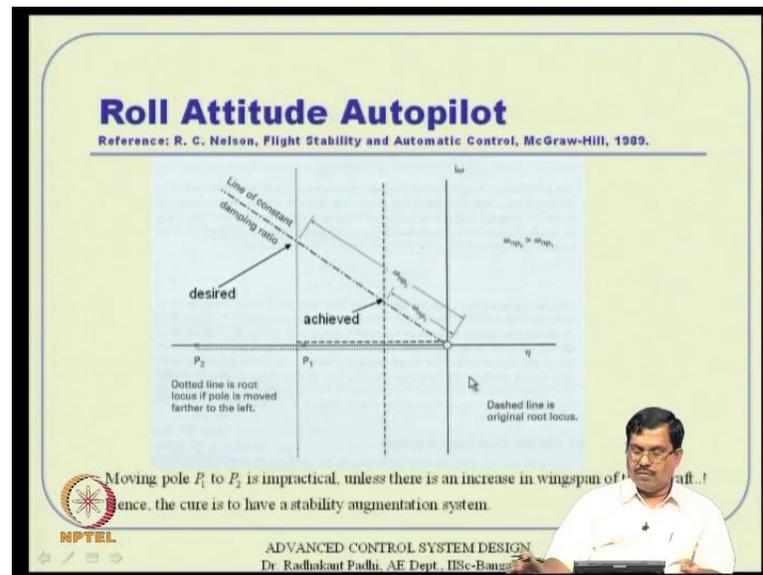
That means, if I draw a 45 degree line whatever, line is that and wherever my route locus crosses through I have to select a gain for that way. Now, if I select a gain that means I have to select a gain which is which corresponds to this location really this is what I need to select. Now, this gain can be found out from because this lies on the route locus obviously, it satisfies the gain condition sort of thing I mean the magnitude conditions that means, if I take magnitude of the route locus it has to be I mean the open loop transfer function it also satisfy one. Open loop transfer function is this so, if I simply take magnitude of that it will satisfy one so that is why I do that and essentially it leads to like what well you have for this when the that came all that it leads to this thing.

Because that point you know the you know the s value s value at that point is known to us so you substitute that and then find out the gain value so, gain turns out to be like that. Now, the problem with this so means by selecting this gain k and k is essential at this that means you are able to select a k a, because we know this parameter l into delta and all that this is parameter this is part of this transfer function. So, once you know this into that is that values so obviously, you divide by that value to get k a value that means that will have to operate there.

But once you know the k a value that is not sufficient in the sense that it satisfies this  $(\cdot)$  condition, because we have selected that way, but what about omega n condition? An omega n in this particular case, if you if you go back and compute then it turns out it is nothing but

point three five radiant per second, way less than the desired value of ten radiant per second that is what we wanted. So, this cannot be simply by gain tuning we cannot do the job that is the thing that is the message one thing can be achieved not both now, how do you do both and that is where this stability augmentation system comes to picture and that is observe this condition now, let us say we somehow extend this line.

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Now, some point of time see remember this is nothing but omega the magnitude of this length gives us omega n that means, if I keep on moving along the direction obviously my  $\zeta$  remains constant  $\omega_n$  angle remains constant. And some point somewhere I will get the omega the desired omega n, but this point happens to be that desired omega n. Now, the problem is route locus has to pass through that now if the route locus has to pass through that, then this four location p one what was earlier a at minus point five has to be shifted to point p 2 somewhere it is left I mean further left to this p one. But this location poles p one is coming from the transfer function of the aircraft dynamics which you do not have a direct hold on.

Really if you want to put the pole there, then the only way to do that is go back to the designer stage and have a larger wings plan, like about that is not an option whether  $\zeta$  I mean aircraft is already been designed and we was always to go the control system loop in

that. Can we do the thing by augmenting this system in a artificial with a artificial control system that is the problem there. That is what it tell here moving the pole p 1 to p 2 is impractical unless there is a increase in wings one of the aircraft so, that is not an option with us and hence the (( )) is to have a stability augmentation system so, we augment the stabilities so that it will see a the omega n that we require.

Ultimately the close loop route loop has to pass through this point otherwise there is no I mean no hope at all this point is the point, which will give us the value of (( )) anomaly both. So, close loop hole close loop route locus has to I mean the route locus of the close loop one ahs to pass through that anyway.

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### Roll Attitude Autopilot

Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Compensator is added in form of rate feedback loop to meet desired damping and natural frequency.
- The inner loop transfer function can be expressed as follows:

$$\frac{\Delta p(s)}{\Delta \delta_a(s)} = \frac{L_{\delta a}}{(s - L_p)}$$

- Transfer function of inner loop

$$G(s)_{IL} = \frac{k_{IL}}{s + 0.5}, \quad H(s)_{IL} = 1(k_{rg}), \quad k_{IL} = k_{am} L_{\delta a}$$

$$M(s)_{IL} = \frac{G(s)_{IL}}{1 + G(s)_{IL} H(s)_{IL}}$$

$$= \frac{k_{IL}}{s + 0.5 + k_{IL}}$$

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So, that way it has to be decided on that, I mean the anyway so that is we will see that and how do you do that so, what do you do here is instead of simply having this kind of a realization what you see here directed realization with simply an actuator I mean k a sort of thing. Now, you have some sort of a two loop trans I mean two loop some sort of a realization here where that not only I will give theta feedback that is what I was doing here the only I was giving some only angle feedback. So, instead of that I will give you give something like a I do not know this is not theta is phi, instead of giving only phi feed

feedback I will give some sort of a  $\dot{\phi}$  feedback  $\dot{\phi}$  is the slightly coupled with  $p$  any way  $\dot{\phi}$  is  $p$  plus some small components and all that.

So, I take  $p$  which is rate now,  $\theta$  angle and all that so this rate feedback I have to sense now so that means I can do that only using a rate gyro not as position gyro only. So, let me sense that with a rate gyro and then, that to that information I will get it I will find out the I mean this particular loop will close it first. So, if I close this loop class with respect to this loop, if I put a big box out here then this outer loop is still there the outer loop is what you discussed here already physically. Now, this particular loop I want to put one more loop which will which will kind of give me the desired properties that I want that we call as inner loop I I so, this inner loop transfer function and the and things like that.

Now, if you do this ideas this inner loop transfer function can be expressed as something like this  $\Delta p$  to  $\Delta a$  so, that is that is given like a first order system. So, once you have that kind of a thing then in a that transfer function of the inner loop becomes like that for this example, let us say and ultimately you can see that this is the transfer function of the kind of inner loop and things like that. So, what happens now if you see this close loop transfer function for the inner loop and all it is no more this just point five you do not have to live with that, this is a gain fellow which is coming here and this gain can now, be selected according to our wish, because that is on our hand now.

So, we select again in such a way that the pole are shifted far away actually what whatever, we wanted here this pole has to be shifted that way around that that one we can do by selecting again here. So, that because we have a rest into this 0.5 location was coming from this factor only now, this is not 0.5, 0.5 plus something that means  $s$  value is minus 0.5 minus  $k/l$ , so it will go minus point five minus  $k/l$  that way it will it will get shifted that way. So, that means this close loop pole and I mean this root location all for this augmented system it locates for this augmented system will not be like this dotted line now it will be that dotted line basically.

So, that way you can select in appropriate gain is correspond to this value of the root locus. So, that is how it is done so we have we have constructed some sort of inner loop system by doing some sort of a rate feedback, which is altered this loop, I mean loop behavior

previously it was nothing this loop was open it was there, but the feedback loop was open. So, once you put a feedback path this entire system behavior is something different so that is the property that is helping you for shifting this four locations  $p$  one to  $p$  two that is how you will be able to do the job.

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**Roll Attitude Autopilot**  
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Inner loop gain is selected to move the augmented root farther on the negative real axis.

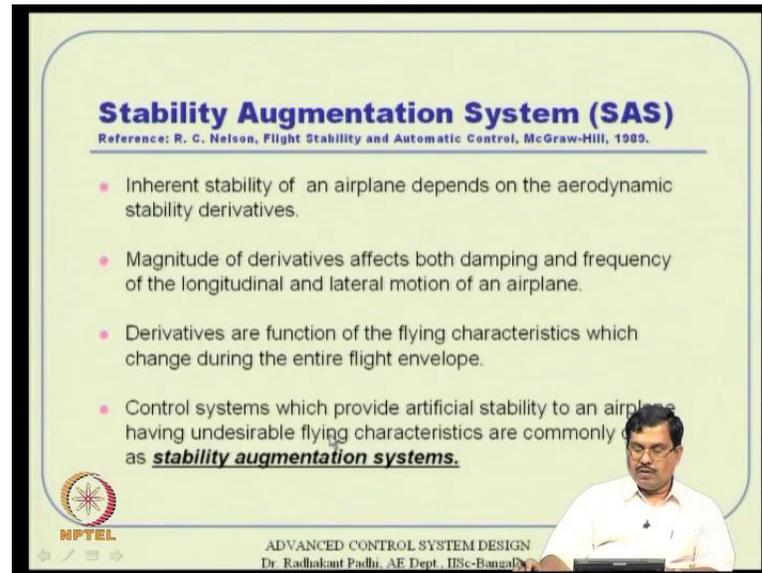
If the inner loop is located at  $s = -14.14$ , then the root locus will shift to the left and desired  $\zeta_n$  and  $\omega_n$  can be achieved.

The graph shows a roll response over time (0 to 3 seconds). The response starts at 16 degrees, drops to 0 degrees by 0.5 seconds, and then exhibits a small undershoot before settling back to 0 degrees by 1.5 seconds.

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Now, if you see that it satisfies all the properties it has nice behavior its settles quickly it is a less  $\zeta_n$  percentage of suitable things like that so, that means both fast response as well as less percentage of  $\omega_n$  and all that can be can be achieved. So, that for that you need to have a stability augmentation system and along with that you have to have a regular control mechanism regular feedback of the position actually, position feedback is essential as well as at red feedback is also essential that way.

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**Stability Augmentation System (SAS)**  
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

- Inherent stability of an airplane depends on the aerodynamic stability derivatives.
- Magnitude of derivatives affects both damping and frequency of the longitudinal and lateral motion of an airplane.
- Derivatives are function of the flying characteristics which change during the entire flight envelope.
- Control systems which provide artificial stability to an airplane having undesirable flying characteristics are commonly as **stability augmentation systems**.

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So, stability augmentation in general does this kind of a thing inherent stability of an airplane depends on aerodynamic stability derivatives, which you do not have too much control on that any way. Then magnitude of the derivatives affects both damping and frequency of both longitudinal and lateral motion that is all types. So, the and also remember the derivatives or function of flying characteristics which changed due to the I mean during the entire flight envelope they do not remain constant also. They keep on changing, because remember aerodynamic coefficients certifiically functions of  $(\alpha)$  number functions of  $(\omega)$  functions of I mean  $\alpha$  and things like that they keep changing anyway.

Now, what will happen is what we are doing here is the control system which will provide this artificial stability to an airplane having undesirable plane characteristics or commonly stability augmentation system. So, that means part of the control is built in basically and then another part you excite through different point this is the t that is what is the idea of stability of maintaining system.

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### Example: SAS

Reference: R. C. Nelson, *Flight Stability and Automatic Control*, McGraw-Hill, 1989.

Consider an aircraft with poor short period dynamic characteristics. Assume one degree of freedom (only pitching motion about CG) to demonstrate the SAS.

Short period dynamics:  $\ddot{\theta} - (M_q + M_{\dot{\alpha}})\dot{\theta} + M_{\alpha}\theta = M_{\delta}\delta$

Substituting the numerical values

$$\ddot{\theta} - 0.071\dot{\theta} + 5.49\theta = -6.71\delta$$

This leads to  $\zeta_{sp} = 0.015$   $\omega_{sp} = 2.34 \text{ rad/s}$

It is seen that the airplane has poor damping (flying quality).

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So, we give an let us say another example, before we windup so let us say short period dynamics is something like this where let us say this dynamics has poor short period dynamics and we want to augment that. And then get a better short of period dynamics how do you do that? So, numerical values if you substitute let us say it is having something like that and this value being very small remember this is like two theta omega and all that this is at most so,  $\zeta$  is very small so it almost says no damping. Then some oscillation happens then it will keep on oscillating obviously, we do not want that so we want to have faster  $\omega$  and things like that. So, then what you do let us say delta e this is your actual command that is what is coming from the pilot.

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**Example: SAS**  
Reference: R. C. Nelson, *Flight Stability and Automatic Control*, McGraw-Hill, 1989.

One way to improve damping is to provide rate feedback. Artificial damping is provided by producing an elevator deflection in proportion to pitch rate and then adding it to the pilot's input, i.e.

$$\delta_e = \underbrace{\delta_{ep}}_{\text{Pilot input}} + \underbrace{k\dot{\theta}}_{\text{Artificial command}}$$

Rate gyro measures the  $\dot{\theta}$  and creates an electrical signal to provide  $k\dot{\theta}$  in addition to  $\delta_e$ .

With this, the modified dynamics becomes:

$$\ddot{\theta} - (0.071 + 6.71k)\dot{\theta} + 5.49\theta = -6.71\delta_e$$

$2\zeta\omega_n = 0.071 + 6.71k, \quad \omega_n^2 = 5.49$

Hence, by varying  $k$ , the desired damping is achieved.

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This delta is we decompose that as one is pilot input directly the other one is some artificial command which is like a theta dot. Once, you put that in this then you put k theta directly here so this entire term part of that goes this side. Whether you k theta dot component and here is a theta dot component so that can get that gets combined and hence, you will get some sort of a close loop dynamics like that, that means 2 theta omega n is not only this one, but it is this 1 plus this one now which is a larger number. So, we can have some sort of a I mean like if you saw omega m square is that most till that is there, that means theta value is quite high now.

And by varying k value this is your tuning parameter here, if you vary k value that means various components how much you want to kind of artificially give input the movement pilot gives an input you are any additional input is automatically generated now how much of that is generate that is a  $(k)$  quantity. So, by doing that you can augment this  $(k)$  you can manipulate this  $(k)$  and that will essentially lead to  $(k)$  so, that means these are all stability augmentation system.

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**Landing System**  
Reference: R. C. Nelson, Flight Stability and Automatic Control, McGraw-Hill, 1989.

Key Components:

- Alignment control (to align wrt. runway centre line)
- Glideslope control
- Flare control

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So, then as I told other applications will include landing system which is like Alignment control Glideslope control we have discussed all that and more details we can find the book probably I do not have too much time to discuss on all these details and things like that. So, with this exposure we will probably stop here thanks for the attention continue further with the modern control applications and all in next class. Thank you.