Space Flight Mechanics

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Lecture No. #41

Attitude Dynamics (Cont.)

Last time, we have been discussing about the torque free rotation of the rigid body of we derived the relationship.

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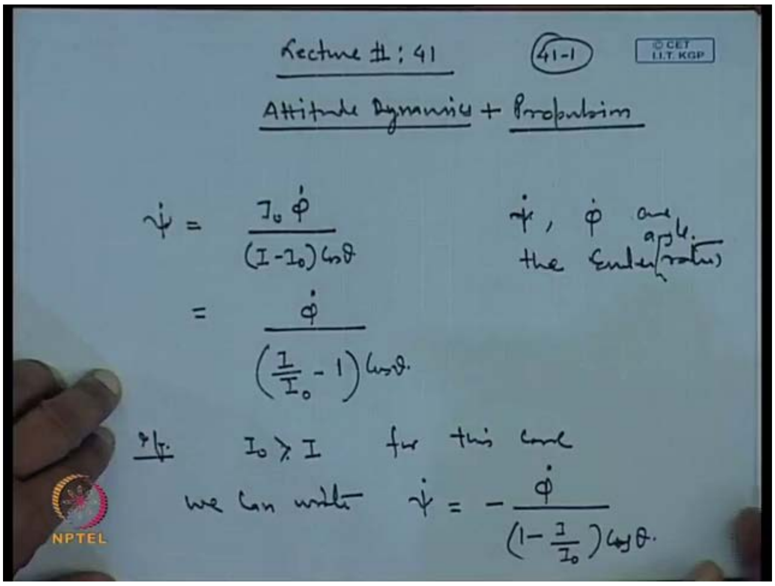

That, psi dot this equal toI 0 phi dot divided by I minus I 0 times cos theta. So, today we will discuss about the attitude dynamic, the remaining topic and this is the last lecture on the attitude dynamics and thereafter, the remaining time we will devote to the propulsion. So, we derived this equation, where psi dot and phi dot these are the Euler rates or the Euler angle rates. So, we can write this as phi dot divided byIby I 0 minus 1cos theta. Now, if I 0 is greater than I.

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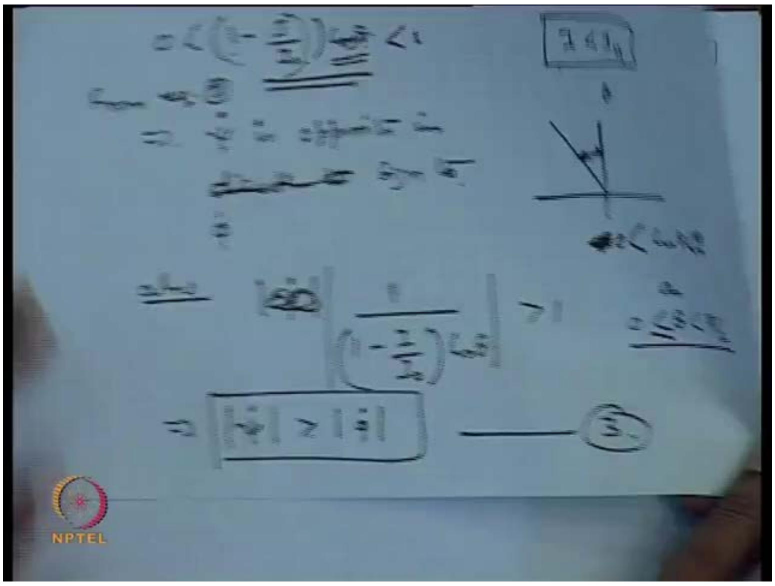

So for this case, we can write psi dot equal to minus phi dot divided by 1 minus I by I 0 cos theta. Now, the quantity 1 minus I by I 0 cos theta, this will be always less than 1.And we can see that I 0 is always a positive quantity I 0 is always a positive quantity and what we are assuming is that, I is less than I 0. So therefore, this is going to be a smaller than 1. So, this quantity will always be positive and theta always it will lie between the theta, we are measuring from the vertical. So, it will lie between minus phi by 2 and plus phi by 2. So therefore, this is going to be always positive.

And this is less than 1 this quantity magnitude of this will be less than1. So therefore, here we can write cos theta is lying between 0 and 1,as theta is bound to lie between on this end, we are not putting the equality sign, which will be quite obvious very soon. So, this quantity will basically turn out to be between 0 and 1 and therefore, what it implies that psi dot is opposite indirection to, opposite in sign to phi dot.

Let it be write this equation as,1 this is 1 and this is 2. So, from equation 2 psi dot is opposite in sign to phi dot also, we can conclude that psi dot magnitude. Now, this quantity is less than 1 and therefore, the quantity 1 by 1 minus I by I 0 cos theta. This magnitude will be greater than 1.

So, for this simply implies, that psi dot magnitude will be greater than phi dot magnitude. And the motion right now, we have considered, so here that phi dot is opposite in sign to the phi dot.

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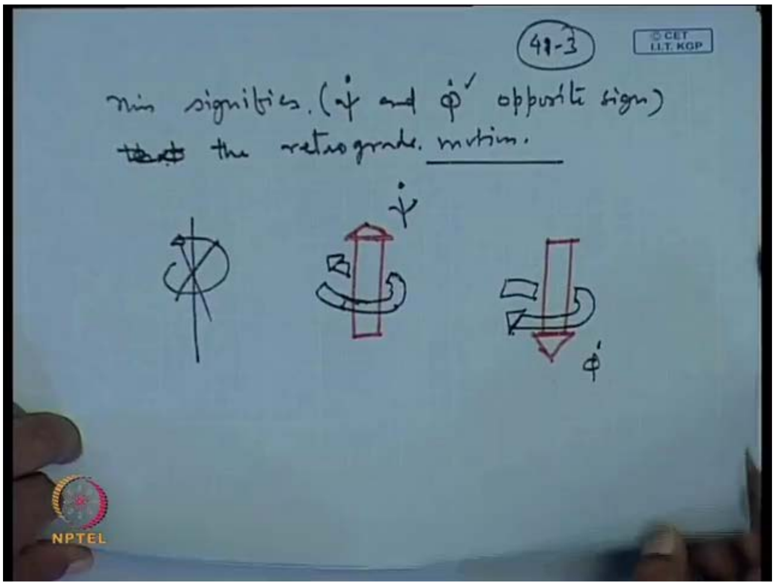

So this signifies, this is the negative sign before the relationship between phi dot psi dot. So this signifies, phi dot opposite sign signifies the retrograde motion. So, if you see if, this is vector suppose psi dot.This is the direction of the psi dot then, phi dot will be opposite in sign. So, that simply implies that, this rotation is being shown anti clockwise. So, phi dot is bound to be clockwise. So, phi dot we can show as with negative sign. So, they are basically not in the same direction. Because the spin vector for the spin about particular axes, we decide it is orientation it is direction according to the way it is spinning.

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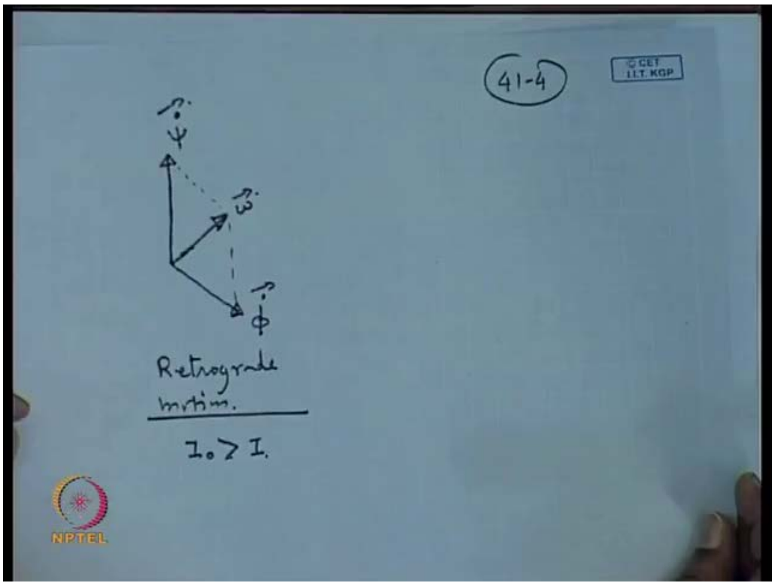

If, it is spinning anti clockwise means, the direction is like this if it is spinning in the clockwise direction. So, we put the direction downward. So, according to the same convention wehave shown here. So now, considering with this convention. So, if the this is psi dot and say if this is the phi dot vector, this will be omega and this is our retro grade motion and this we have written for I 0 greater than I.

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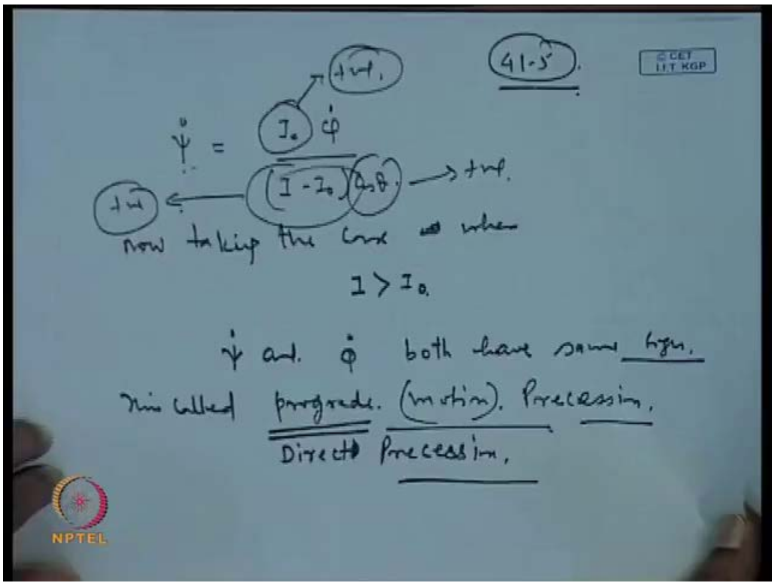

And already, we have described the body cones for the retrograde and the prograde motion. Now, again taking the equation number 1 here, now taking the case when I is greater than I 0. So, if this is the case then, it simply implies that psi dot and phi dot both have same sign, because this quantity is positive, this quantity becomes positive and this quantity is positive. Therefore, the sign of psi dot and phi dot it is the same and this is called prograde.

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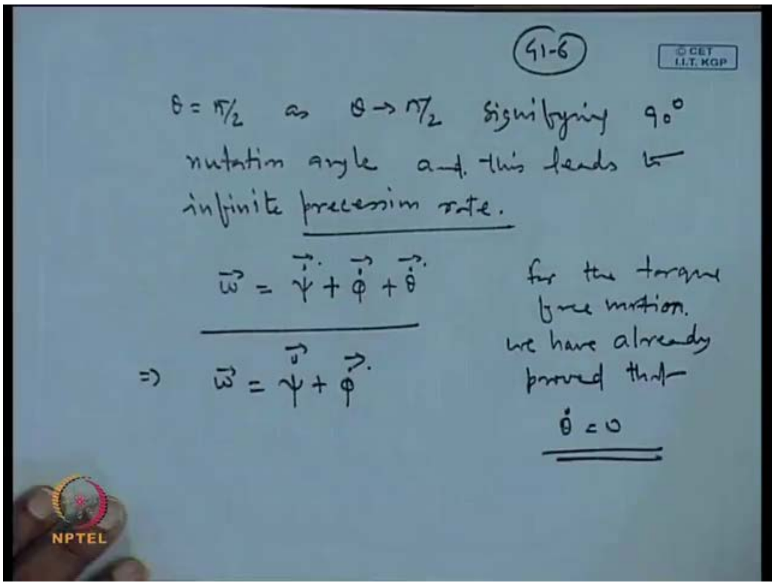

A retrograde motion also and we can turn this as retrograde precession. Retrograde precession and this we write prograde precession or also this is called as direct precession. Moreover from this equation number 1, we can observe that if, theta tends to phi by 2 means this quantity will tend to 0. At that time psi dot becomes infinite if this tends to 0. So, this quantity will tend to the whole quantity on the right hand side will tend to infinity. So, implying that psi dot becomes infinite.

So, if means the precession rate once it becomes infinite, only then you will have the nutation angle, this theta is signifying the nutation angle. So, if we can write here, theta is equal to pi by 2 or as theta tends to pi by 2 signifying 90 degree nutation angle and this leads to infinite precession rate and physically this is not possible obviously.

So, in this case what we can write omega is psi dot, this is a vector plus phi dot vector plus theta dot vector and for the torque free motion for the torque free motion we have already proved, that theta dot equal to 0. So, this implies omega is equal to psi dot plus phi dot and this is what we have shown in this figure. So, this is retrograde precession and now, we can add one more figure. So, we can show it on the same slide may be;

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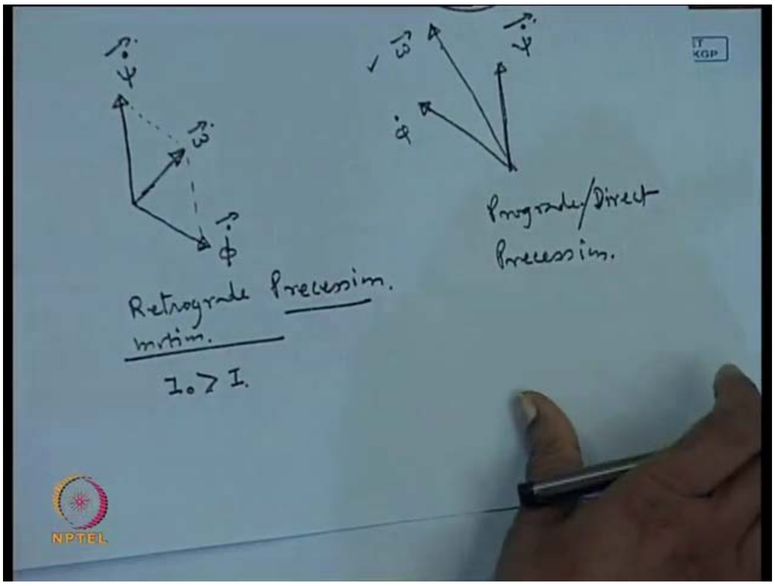

This is your psi dot vector and phi dot vector. So, let us say this is your phi dot vector. So, omega dot vector will point in this direction, omega vector will be pointing here, not omega dot and this is our prograde or direct precession. So, this completes our discussion ofthe rotationnow, we come to the stability part.

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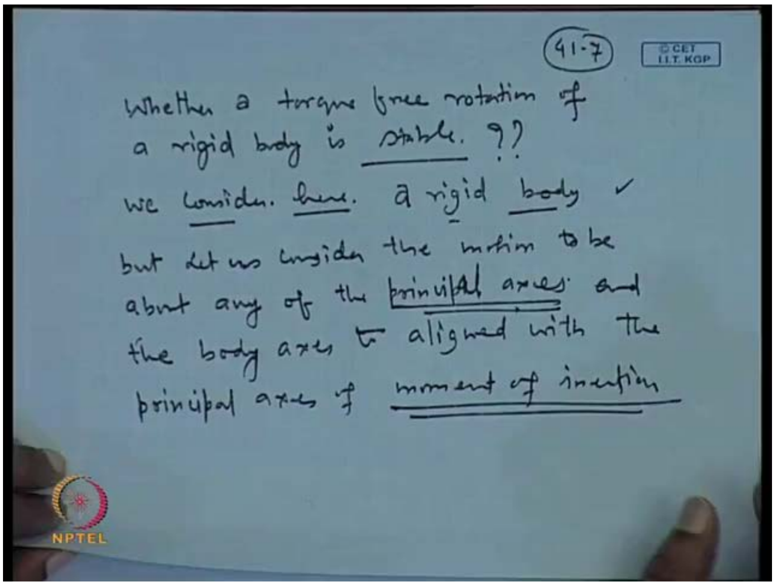

So, our question is here, whether a torque free rotation of a rigid body is stable? So obviously, in this case we have started with the case of a cylinder. So, we will continue with that. So, we consider here a rigid body. So, in general we can choose anybody we have a case of cylinder already. So, either we can take the case of a cylinder or either we can take any rigid body .Let us consider the motion to be about any of the principal axes and the body axes to be aligned with, the principal axes of moment of inertia.

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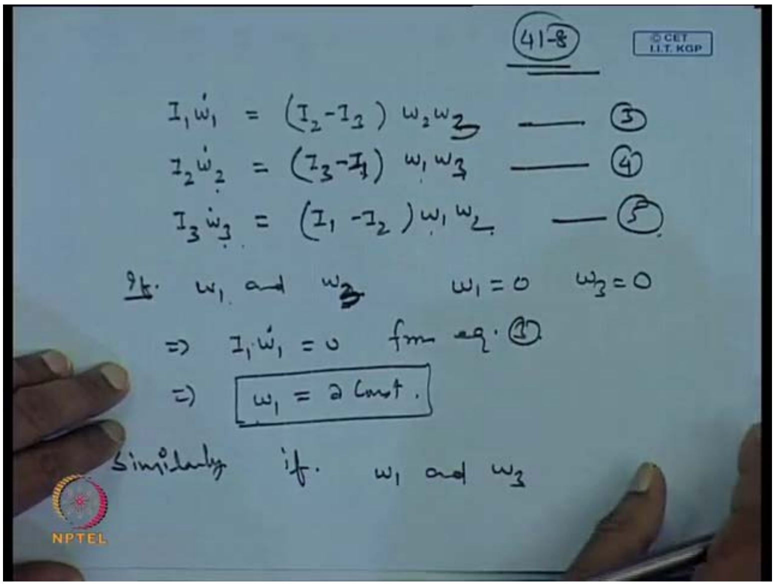

So, here this principal axes implies principal axes moment of inertia it is the same thing and we want to study the stability of this body to start with, we have Euler's dynamical equation we have already written. So, when the half diagonal terms are 0. So, we have this equation earlier I 2 minus I 3 this times omega 2 omega 3 and similarly we can write I 2 times omega 2 dot.

So, we have these equations now in these equations, we can see that if omega 1 and omega 2 equal to 0. So, this implies I 1 times omega 1 dot, this is equal to 0 from equation 3. So, this implies omega 1, this is a constant. Similarly if omega 1 and omega 3. So, here let us put like omega 2 and let us correct this here omega 2 and omega 3 omega 2 omega 1 and omega 3 omega 3 omega 1 and omega 2. So, if we have omega 1 and omega 3, these are 0 that is omega 1 is equal to 0 and omega 3 equal to 0.

So, at that time we will have I 1 times omega1 dot, this is equal to 0 from equation 3 and this implies omega1 is a constant similarly if omega 1 and omega 3 are 0. That is omega 1 equal to 0, omega 3 equal to 0. So, this implies I 2 times omega 2 dot this is equal to 0.

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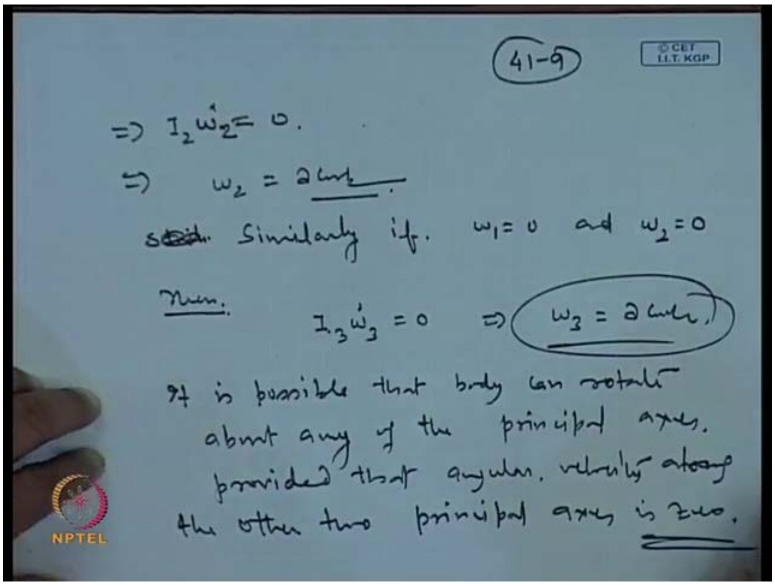

Implying that omega 2 this is a constant. Similarly, if omega 1 equal to 0 and omega 2 equal to 0 then, we will have omega 3 times I 3 times omega 3 dot equal to 0 implying that, omega 3 this is a constant. So, the conclusion is that, it is possible body, that body can rotate about any of the principal axes provided that; angular velocity along the other two principal axis is zero.

So, if it happens then, it is a possible that it will the body can rotate and that is what we said the question. So, we assume that here, let us consider that the motion to be about any of the principal axes, not about all the principal axes; obviously, any angular velocity vector can be broken along all the body axes, we can choose along the principal axes.

So, in that case it implies that the motion is taking place, along all the principal axes. So, that situation, we are adding we have taken a simple case,where the body we have chosen to rotate about any of the principal axes.

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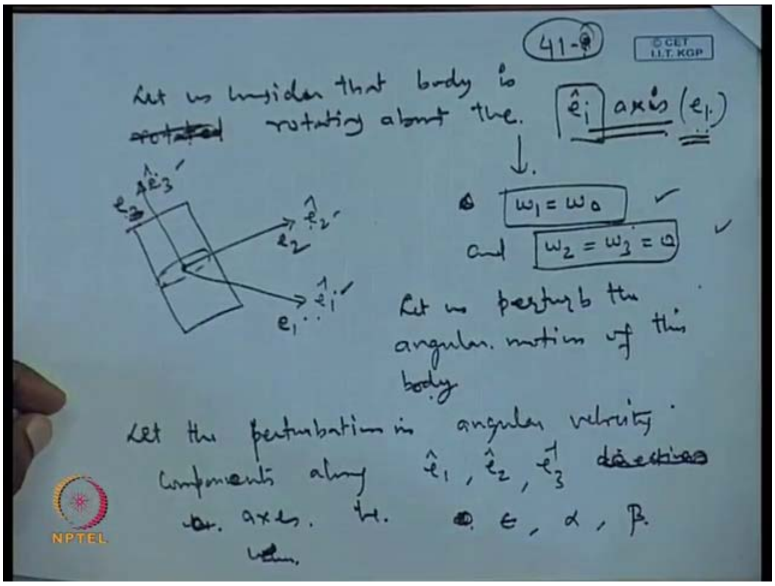

So now, let us consider body is rotated is rotating body is rotating about the E 1 axes. If, you remember we choose the body like this. So, this was your E 3 cap here in this direction and this was E2 cap and here it was E 1cap. So, these are the unit vectors in the E 1 E 2 and E 3 direction. So, let us consider that the body is rotating about the E 1 axis or simply we can put as here E 1 notation, because mostly notation we choose as for the different axis as E 1 E 2 and E 3, while these are indicating the unit vectors. So, we will write here that omega 1 is equal to omega 0 and omega 2 omega 3 this is equal to 0.

So, with this assumption let us, perturb the angular motion of this body. So, what we are assuming that omega 2 and omega 3, they are 0. Let the perturbation in angular velocity components along e1, e2, e3 directions or along the e1, e2, e3 axes be epsilon alpha beta. So, if we are taking this omega2 omega3 omega and all of them to be 0. Similarly, here quantity omega3 is 0, where in this place also omega2 is 0, except omega1.

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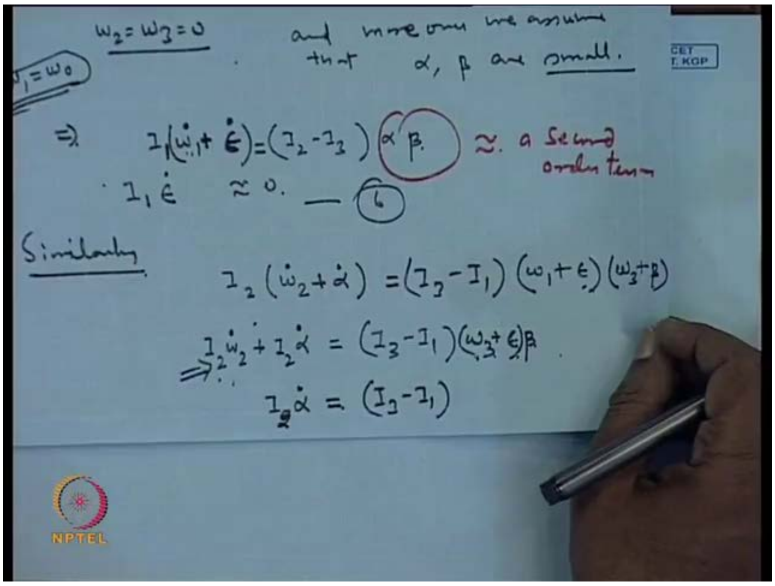

So, what we get I 1 times omega1 dot.We can write this as I 2 minus I 3 times, omega2 plus and we are assuming that there is the perturbation, we are giving is alpha beta and gamma. So, these are correspondingly given to in the omega2, we are giving alpha and in the omega3, we are giving the perturbation beta.

Omega2 is equal to omega3 equal to0.This we have already assumed and more over we assume that, alpha beta are small. So, this implies I 1 times omega1 dot is equal to I 2 minus I 3 times alpha beta, because omega 2 omega3 are 0 and the quantity which is alpha beta. This is a second order term and therefore, this will be nearly equal to 0.

Similarly, I 2 times omega 2 dot plus alpha dot, this will be I 3 minus I 1 times, omega1 plus epsilon.This is the perturbation in omega1 and in omega3.We put as omega3 plus epsilon alpha and gamma, we have taken beta. So, here omega3 is 0. So, this gets reduced to I 3 minus I 1 times omega3,omega3 plus epsilon times beta.

Now, we have already chosen that,one thing here in this equation moreover we can write, as we have written here in this place on this side the perturbation we have given to be epsilon. So, here we can write the epsilon dot. Now, instead of writing in this way now, omega1 we have assumed to be constant. So, omega1 we have written as omega0. So, derivative of this quantity we can set it to 0 and so omega1 dot will be 0 and simply, we can write I 1 times epsilon dot.

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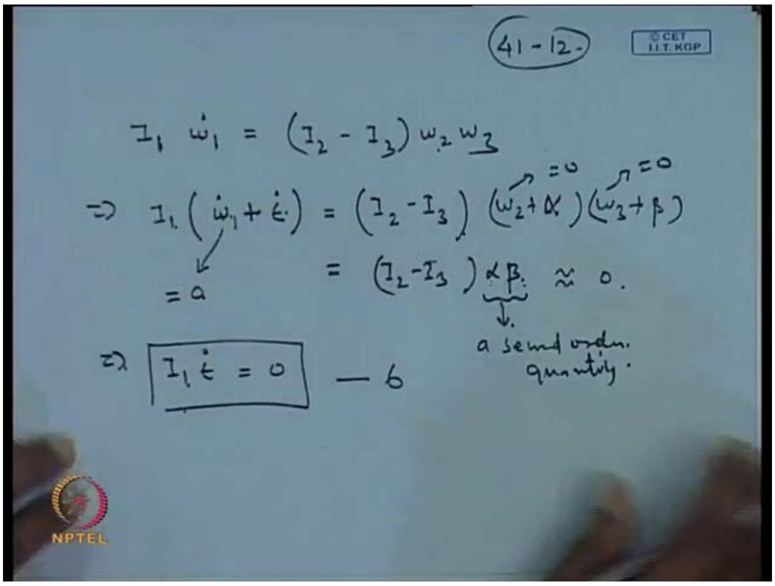

This is our equation number 6 and here in this place similarly, omega2 we have to assume to be 0. So, this quantity vanishes, leaving us with I 2 times alpha dot. This is called 2 by3 minus I 1 times and omega 3 is already 0. So, this will vanish now, I am writing fresh the whole thing, I 2 minus I 3 times omega2 omega3.

So, this implies I 1 times let us perturb this by epsilon. So, this becomes epsilon dot and omega2 plus alpha and omega3 plus beta. Omega2 this is equal to 0, omega3 this is equal to0. So, we have I 2 minus I 3 times alpha times beta and this is a second order quantity as we have stated earlier second order quantity or it is magnitude of second order both are small. So, the product will be further small.

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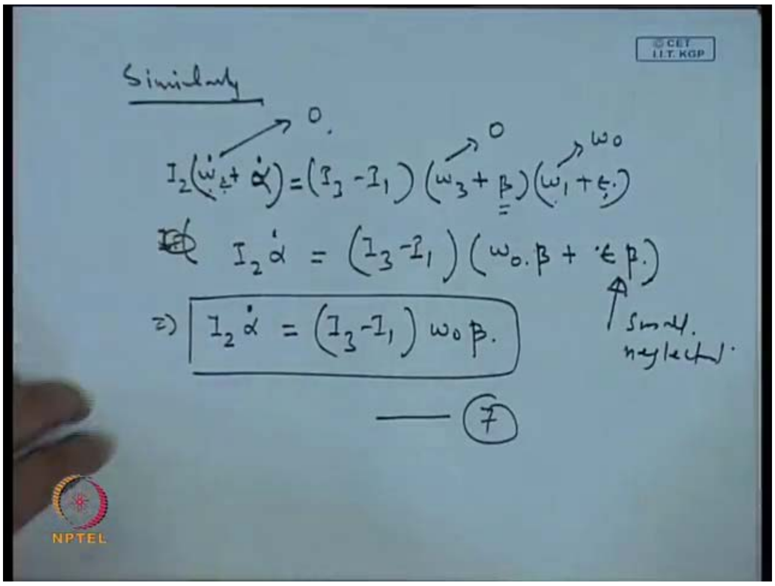

So, this quantity we can set it0. So, this simply impliesI 1 times and moreover this quantity is0 and this we will write the equation number 6. Similarly, we have I 2 times omega2 dot.This is equal to I 3 minus I 1 times here, I 3 minus I 1 times omega3 plus omega3 plus, the perturbation we are giving as beta.

We wrongly copied here. So, I cancelled this page. This is omega3 plus beta times omega1 plus epsilon. Now, omega2 dot this will be 0, but the perturbation we can write here let us start the perturbation here in this place, omega 2 dot plus alpha dot. So, from here we have omega2 this quantity is again 2 dot 0. So, I 2 times alpha dot this becomes I 3 minus I 1.

Omega3 this we have set to 0 beta, we can multiply here omega1 we have set 0 this is omega0 and times beta plus epsilon times beta. This quantity is small, so neglected. So, this implies I 2 times alpha dot this is equal to I 3 minus alpha times omega0 beta.This is our equation number 7.

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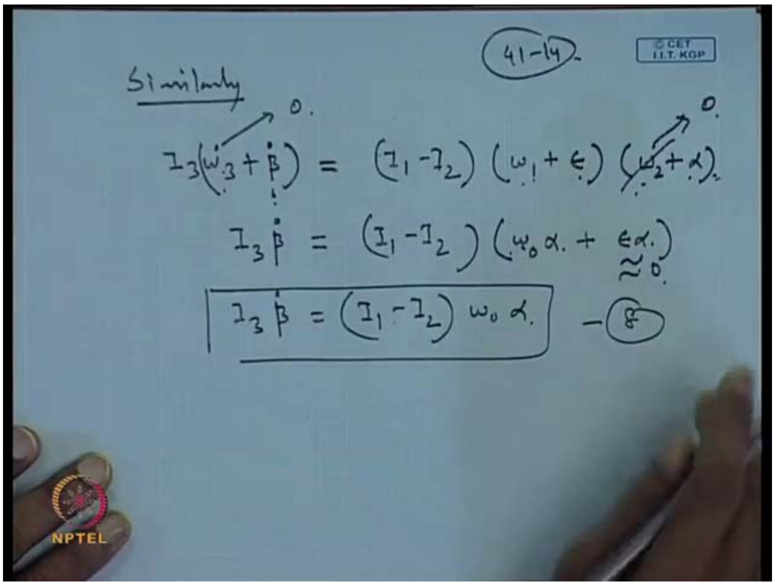

Similarly, I 3 times omega3 dot, we have written earlier. So, omega3 with this we give the perturbation beta. So, this becomes I 3 times omega3 plus beta dot. This will be equal toI 1 minus I 2 times omega1 plus epsilon and then, omega2 plus alpha. This quantity is 0 beta dot; we get here from this place. So, I 3 times beta dot. This is equal to I 1 minus I 2 and if you look into this we get here omega1 which is omega0 omega2 this quantity is 0.

So, this we can cancel from this place. So, this becomes omega0 times alpha plus epsilon times alpha again, this quantity is small. So, we drop it out and what we get is I 3 beta dots equals I 1 minus I 2 times omega0 alpha.This is our equation number 8. Now, equation number 7 and 8 as we can see from this place. This is our equation number 7,this is in written in terms of on the left hand side the rate of alpha and right hand side beta is appearing. Similarly, here we have the beta dot and on the right hand side alpha is appearing. So, this is basically coupled equation. So, this simply implies that one motion will be affecting theother motion. So therefore, now we can differentiate the equation number 7.

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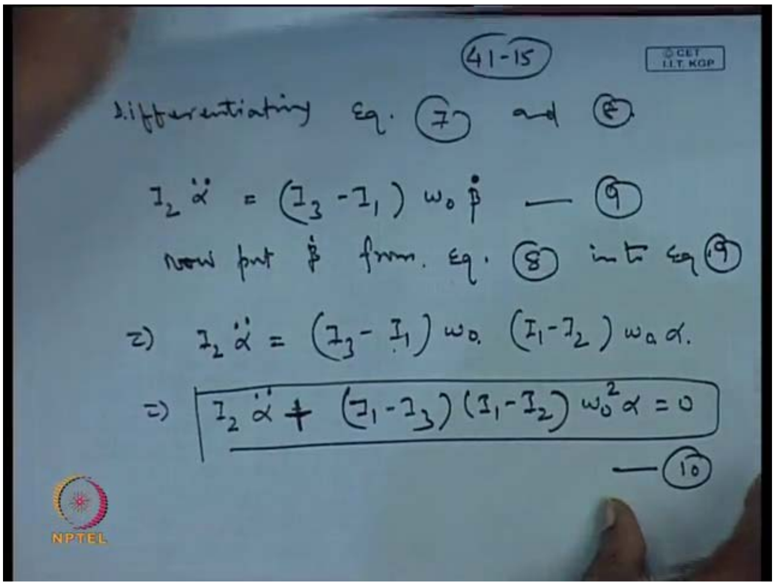

So differentiating, equation 7 and 8. So, what we get I 2 time alpha double dot this is equal to I 3 minus I 1 times omega 0 times beta dot. Now, put beta dot from put this as equation number 9 from equation 8 into equation 9. So, this will imply your I 2 times alpha double dot is equal to I 3 minus I 1 times and from here, we have the beta dot available. So, this is I 1 minus I 2 times omega 0 alpha. This we can write as, we can bring it on the left hand side and can be written as I 1 minus I 3 times I 1 minus I 2times, omega times0 square and alpha. This is equal to 0. This is our equation number 10.

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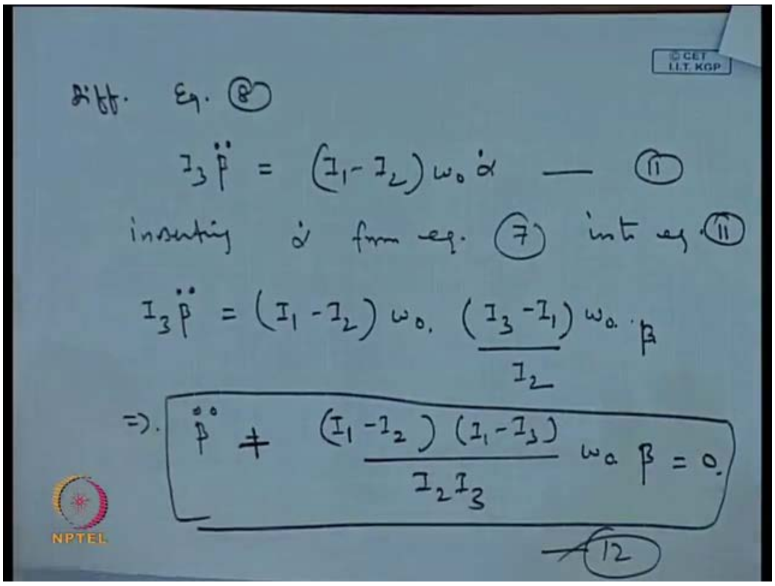

Similarly, for if we differentiate the equation number 8. So, differentiating equation 8, we get I 3 times beta double dot omega 0 times alpha dot. So, alpha dot we have already available in equation number 7. So, this is our alpha dot present here. So, we insert this. So, inserting alpha dot from equation 7 and these, we write as equation number 7 into equation 11.

This gives us, I 3 times beta double dot equal to I 1 minus I 2 here, we have missed 1 term. We put here the term I 1 minus I 2, because we inserted for beta dot and for beta dot we have the multiplied by I 3. So, we put here I 3, write it by I 3 and this is divided by I 3. So, for now we are inserting for alpha dot in this place and in the alpha dot equation, we have I 2 present here and we divide it by I 2.

So, this equation, let us further simplify and this can be written as alpha double dot plus I 1 minus I 3 times I 1 minus I 2 divided by I 2 times I 3 times omega 0 square times alpha equal to 0 and we will write this as equation number 10. So, this implies beta double dot equal to I 1 minus I you can add to bring it the whole thing to the left hand side,omega0 square.

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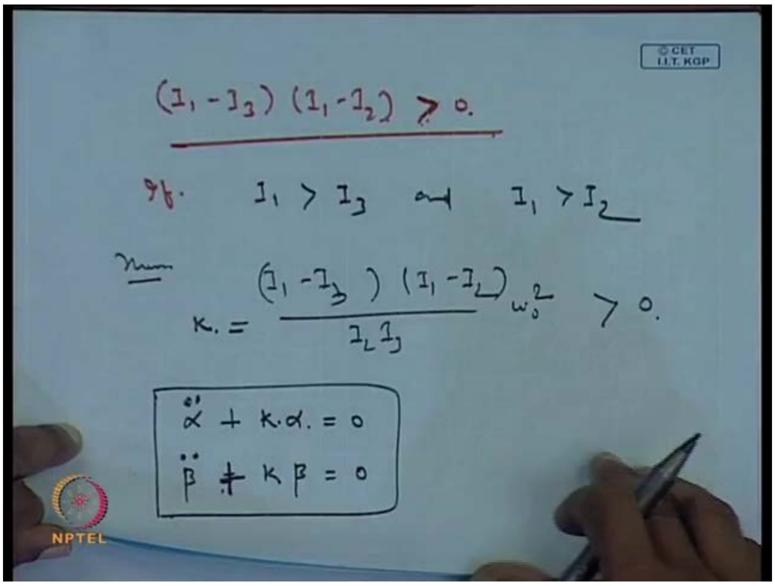

And this beta is changing this place and this beta isand this is our equation number 12. Now, from the equation 10 and 12,these are the two equations we have got.we can see that this equation is a simple harmonic motion equation provided the quantity I 1 minus I

3 times I 1 minus I 2 this is greater than 0 otherwise, if you have a, if this quantity is less than0.

So, obviously, this will not result in a simple harmonic motion equation, but it will have 2 real roots and1 of them will be negative and that will lead to the Ernest table solution. So, what we are looking into the stability of this motion,whether this motion rotational motion is stable or not. So, if now we can write here if,I 1 is greater than I 3and I 1 is greater than I 2 then, quantity I 1 minus I 3 times I 1 minus I 2divided by I 2 I 3 times omega0 square. This will be greater than 0.

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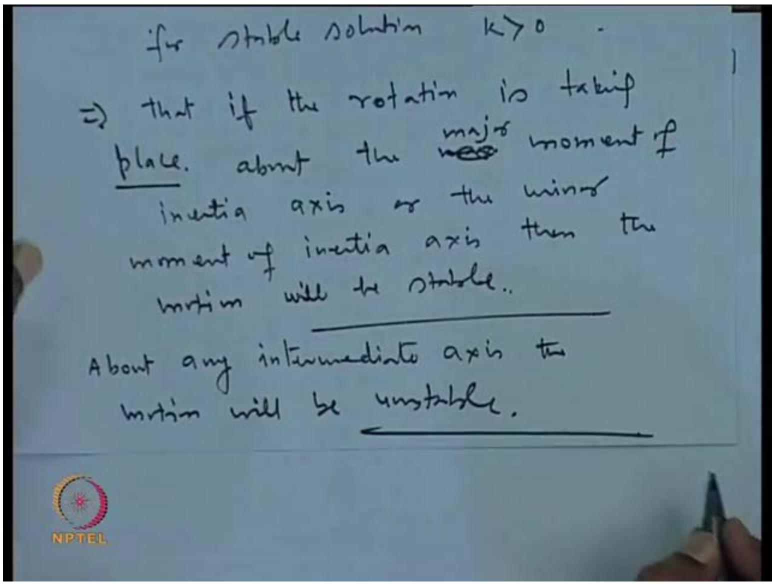

So, let us put this quantity as k. So, if we write it in this ways alpha double dot, we can write as k times alpha equal to 0 and similarly, we can write for beta double dot equals to k times, beta double dot plus k times, beta equal to 0. Now, for a stable solution k must be greater than0, as we have shown here this quantity must be greater than 0 and that will be satisfied if this quantity is greater than 0 or I 1 is less than I 3and I 2 I 1 is less than I 2.

So, what does it say? This implies,that if the rotation is taking place, about the maximum moment of the inertia or the major moment of inertia axis or the minor moment of inertia axis then, the motion will be stable. About any intermediate axis,the motion will be unstable.

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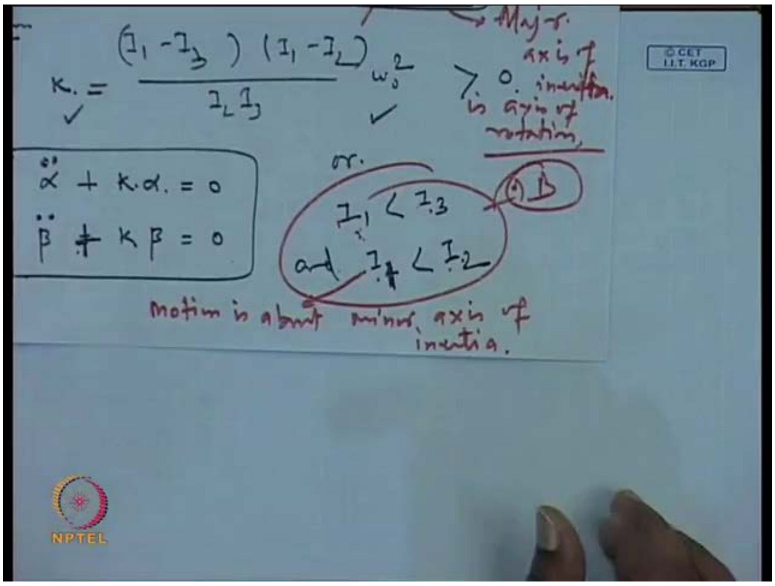

So, we have two conclusions, this was our conclusion number A, this is conclusion number B.That I 1 which is the we have chosen as the axis of rotation. So, this should be I 1 is greater than I 3 and I 1 is greater than I 2 implying that this is the major axis of inertia. So, this is major axis of inertia. Inertia is axis of rotation and what does this implies this imply that I 1 is less than I 3 and also I 1 is less than I 2. So, we have only 3 moment of inertia axis. So, the I 1 is less than both of them I 2 and I 3 implying that.

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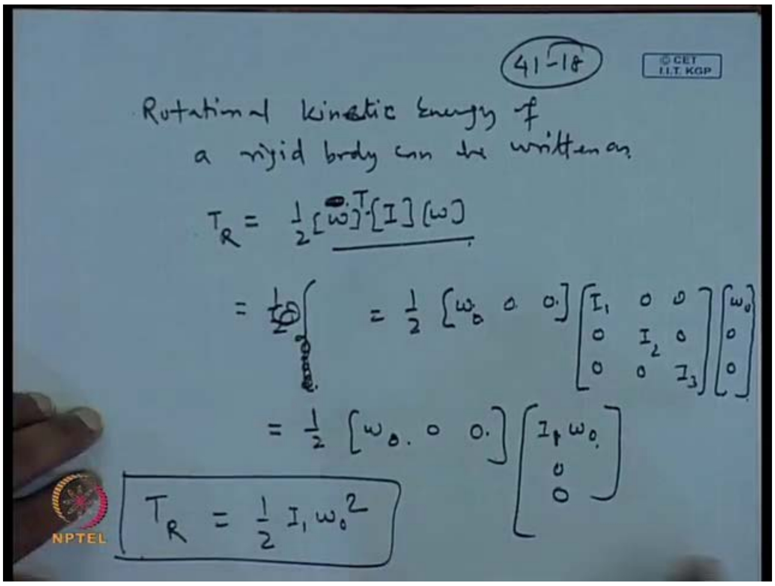

Now, the motion is about minor axis of inertia. So, if it takes place the rotational motion is taking place either about the major axis of inertia or the minor of a axis of inertia. The motion is going to be stable otherwise, it will be unstable. Now, we have done the stability part. Now, we can look into one very important conclusion and that, we derive based on the energy results and this is not a very rigorous statement.

But we are going to do a something adopt treatment. So, let us say, we know that the rotational kinetic energy,of a rigid body can be written as let us say T Rotational this is equal to1 by 2 times omega transpose times I or we can remove this indicating these are the vectors and matrix.

So, here in this case we have omega 11 by 2 omega transpose will be omega1, which we are putting as omega 0 and just we are omega2 omega3 we are putting as 0 moment of inertia I 1 I 2 I 3 and all diagonal terms being 0 and omega here again, omega000. So, this implies omega00.

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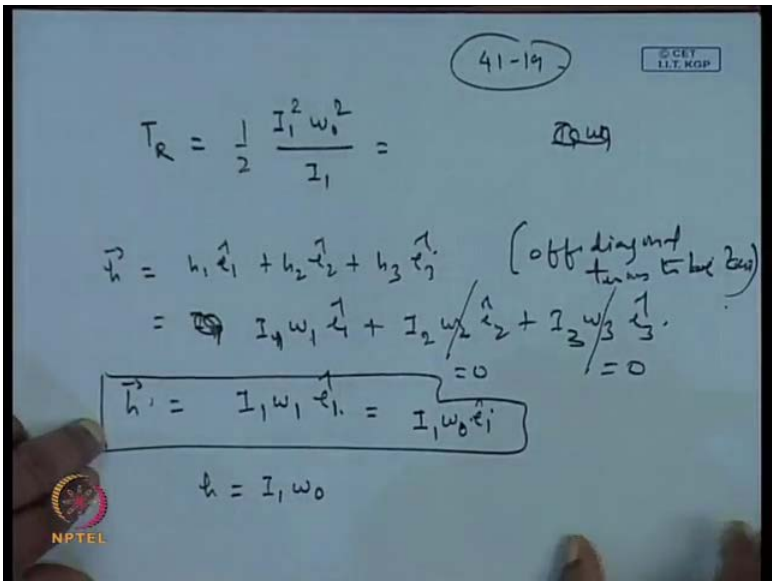

Now, multiply this will be I 1 times omega0. So, this gives you1 by 2 times I 1 omega0 square. So, this is the rotational energy of a rigid body. Now, this T R can be written as I 1 square times omega0 square by I 11 by 2 and we know that, if the rotation is taking place at about a particular axis earlier.

If you remember that the h vector, we wrote as h1 times e1 plus h2 times e2plus h3 times e3. Where h1 wasI 1 times omega1, this was I 2 times omega2 and I 3 times omega3 E3 cap.

While, assuming the off diagonal terms to be 0 that is the rotation taking place about the principal moment of inertia axis. Now, if omega2 omega3 these are 0. So, this simply whirls down to I 1 omega1e 1 cap and this is nothing, but I times in our case I 0 times e1 cap.

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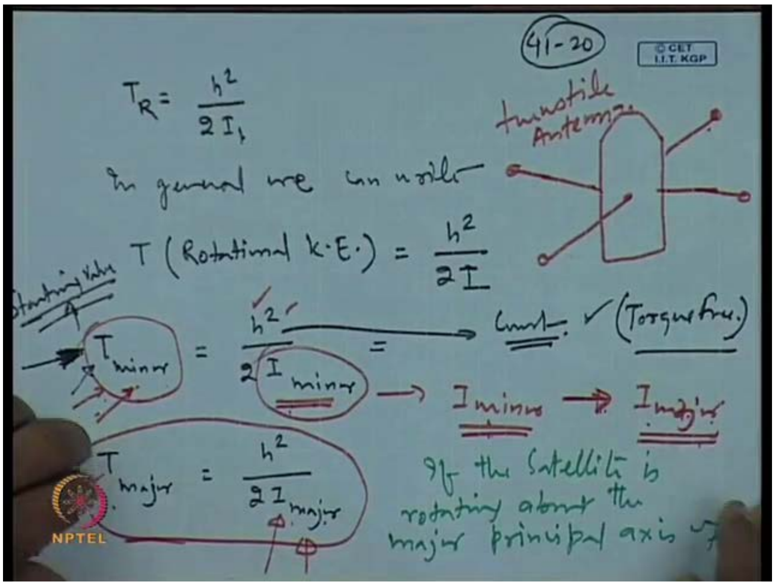

So, the magnitude wise we can write h is equal to I 1 times omega0. So, till now we calculated T R is equal to h square by 2 I 1. So, in general we can write, T the rotational kinetic energy this is equal to h square by2 I. So now, we can have various situations in which let us say,T we write as T minor this is equal to h square by 2 I minor.

And similarly we write T major.This is equal to h square by 2 I major. Now, until and unless the external torque is acting, the T on the left hand side, we are we are starting with certain value of T minor suppose. Now, until and unless the external torque is acting h will be constant this constant, will remain constantand also suppose in the beginning with the, for whatever the value for the h we had the T minor here. So, this we start with a certain value.

So, the starting value is give. So, h is a constant if, it is torque free situation. Now, there are situation in which, this kinetic rotational kinetic energy it can die out. So, we can have some, vibrating part mounted on the satellite. Suppose, this is our satellite, we can have antenna mounted on this satellite. So, this antenna it is a flexible and it is turnstile antenna.

So, if this vibrates. So, this will absorb energy. So, your it is a torque free situation. So, this part is constant. Now, the energy the rotational energy will be absorbed continuously, because this antenna may be oscillating. If, inside the satellite there are some moving parts or there are some dampers there. So, that may also add to absorb the energy of the satellite rotational energy of the satellite. So, this energy will start decaying.

So, this part is constant. So, this can be compensated only if this part changes. So, in this case what happens? So, if this is decreasing kinetic energy is decreasing. So, this will happen only if I is increases. So, I minor, this will get changed to I major means, the if the satellite is rotating about the minor principal axis of inertia. So, it will slowly deviate from this motion will get perturbed,the start satellite will start tumbling and thereafter it will resort to rotation about the major principal axis of inertia. This is major while it will look into this situation. So, here there is no scope of change of this high major. Here, there was a scope that high major can change high minor here there is no scope.

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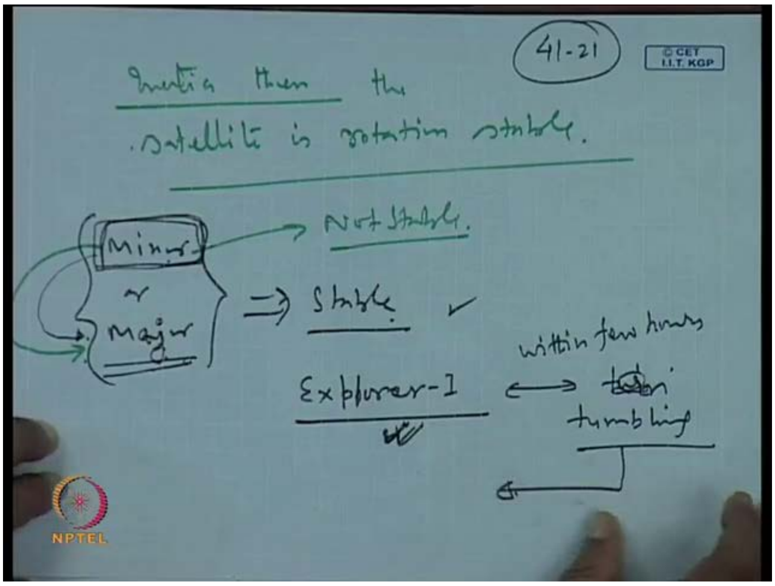

So, we say that if the satellite is rotating about the if, the satellite is rotating about the major principal axis, of inertia then, the satellite rotation is stable. So, then we concluded earlier theoretically, if the satellite is either rotating about the minor axis or the major axis. So, the motional the rotational motion will be stable, but we see that rotationally, the energy wise this part is not stable, because as the energy dies out that satellite deviates from the minor axis and comes to the major axis rotation.

So therefore, the satellite will remain stable only if it is rotated about the major moment of inertia not about the minor one major moment of inertia axis. So, we have completed this part now, to quote as an example, this American satellite explorer 1. So, after the launch within few hours, it has started tumbling. within few hours it has started tumbling

And this tumbling was, because of the presence of this turnstile antenna. So, because of this antenna the satellite, this vibration of this as it vibrates. So, it will absorb the energy the vibration of this some energy will get dissipated as the heat. So, because of the absorption of the energy. So, slowly it has started it was set initially in theit is rotation was set on the minor principal axis. So, as this vibration has started.

The energy has started getting dissipated. So, it moved from the minor principal axis to the major principal axis and therefore, in this case the satellite gets lost. If, you do not have the control system, on board the satellite then, you cannot restore the orientation of the satellite again. So, say if launched a satellite, which was put by using just the rotation it was rotationally stabilized and you did not do anything for this.

Now, you do not have control inside to change it is orientation. So, you will find that this satellite it has changed its orientation and its functionality is lost. So therefore, the attitude control is a necessary part to reorient the satellite. So, anything can go with the satellite, some unfrocking can also be there. So, even if you have not designed for that, those unfrocking things bit if your controls are (( )) then, you will be able to reorient the satellite and restore its functionality. So, we stop here and next lecture we will start with propulsion. Thank you very much.