Space Flight Mechanics

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Lecture No. # 02
Particle Kinematics

If, today we are starting with, the particle kinematics. So, in the Newtonian mechanics, as we know, the time acts as an independent parameter actually in the backdrop of time, all the events take place. While in the relativistic mechanics, the time is not an independent parameter, it gets mixed up with the other three dimensions. So, if as a whole here, the time universally in the Newtonian mechanics, as a whole per time universally applies to all the events, that is one single time will apply to all the events, while relativistic mechanics the same thing is not true. So, therefore, while describing the coordinates of any particle in a reference frame, we always have to tag it with parameter t in the Newtonian mechanics to indicate that, this is the coordinate of the particle at time t.

Now, the radius of this particle, this can be indicated as r, where the x y are the coordinates of this particle, so this is all about two dimension. In two dimensions, while the same thing, if we have to write in the three dimension then, another coordinate z has to appear. So, in two dimension, we can write r as x say along this direction, let us say e 1 is the unit vector and along this direction e 2 is the unit vector. So, x e 1 cap plus y e 2 cap, this will be the coordinate of this particle written in vectorial terms.

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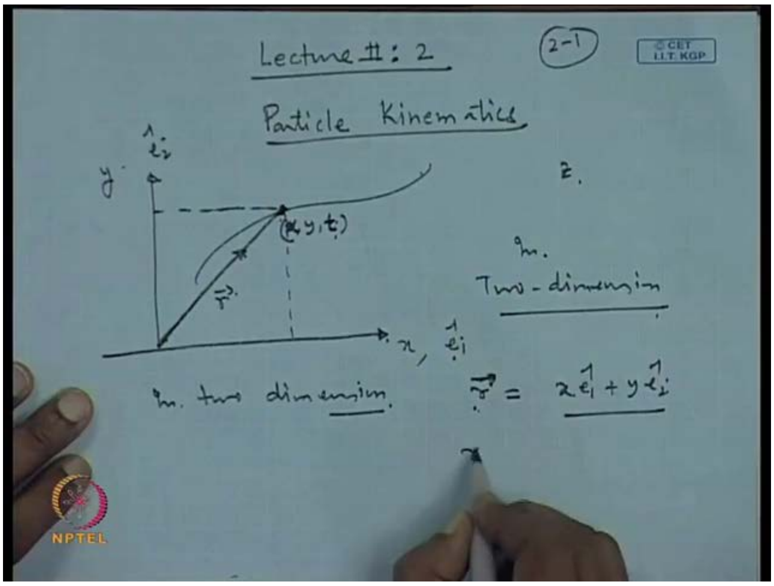

So, if this is a vector. So, for a vector the magnitude it is defined where magnitude and direction. So, this vector, once we have written in this form, automatically this includes the magnitude also. So, we can write r magnitude as x square plus y square under root. While, this angle which defines the direction, which we can write as theta, so tan theta can be written as y by x. So, vector contains both the direction and the magnitude.

Now, another important part that, we have to deal with; in the Newtonian Mechanics, also, we call this as a Non-Relativistic Mechanics, the inertial and the non inertial frame.

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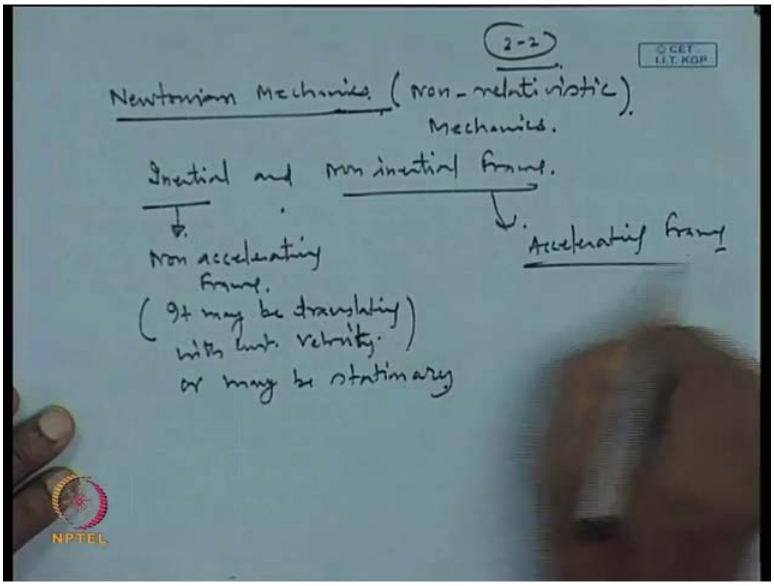

So, we will define inertial frame, as a non accelerating frame and it may be translating with constant velocity or may be a stationary, while non inertial frame, these are the accelerating frames. So, we know that, Newton's law they are applicable in inertial frames is applicable here. Now, let us start with the kinematics of the particle so, we trying to find out the velocity of a particle in 3 d. So, let us say this is a trajectory along which some particle is moving this is our particle at time t. So, the radius vector of this particle from this point, I can show as r and this is our r t and another point, we are taking. So, is radius vector can be defined as r t plus d t.

And let us say this is r 1 and to if distinguish it, we can write as r 1 equal to r t and r 2 is equal to r t plus d t. So, we cannot do a shortcut notation for this, using an shortcut notation and we can work out, which will involve just the with a summation instead of writing all the terms. So, if we follow that notation then, we can writer r as x i times e I summation.

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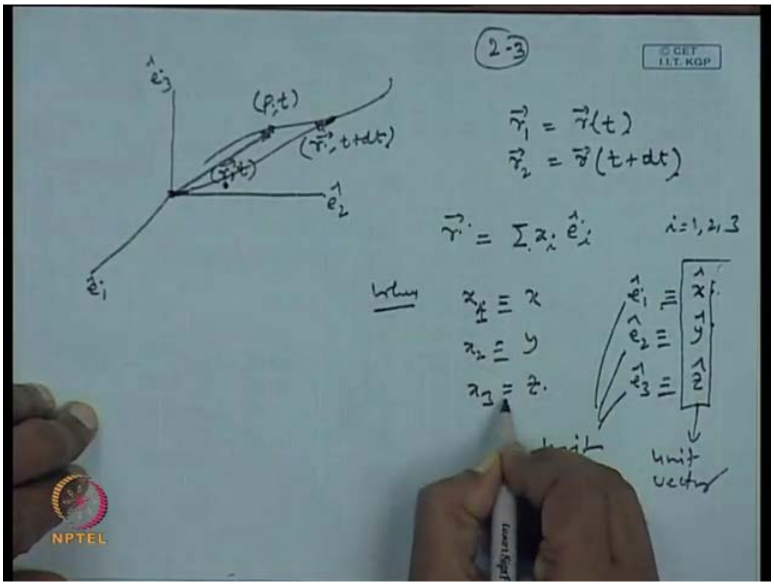

Where x i this repulse to for if i equal to 1 2 3 and we can write here x 1 equal to x, x 2 is equal to y and x 3 equal to z. And similarly, we will have the e 1 this is equivalent to here unit vector in the direction of x, e 2 is equivalent to unit vector in the direction of y and e 3 cap is equivalent to unit vector in the direction of z. So, these are unit vectors and these are also unit vectors, we can put here, this are identical.

So, the velocity of as the particle is moving along the trajectory from this point to this point. So, this quantity, let us make another figure, the second axis we can show here and say the trajectory is looking like this. So, this is your delta r and here, we can write it as r t and this becomes r t plus delta t.

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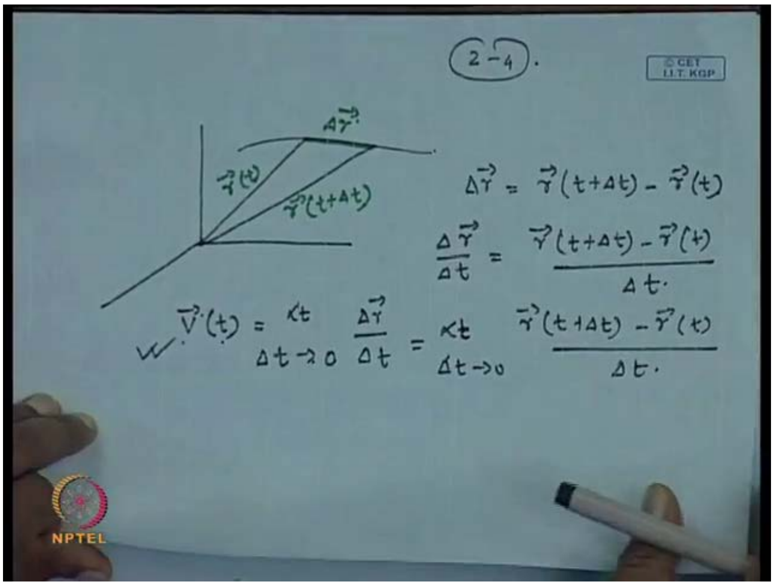

So, delta r, we can write as r t plus delta t minus r t and therefore, r t by delta t. Now, the velocity vector, this is defined as the velocity at time t, this can be defined as limit t tends to delta t tends to 0 delta r by delta t this, we can write as limit delta t tends to 0, r t plus delta t minus r t divided by delta t. So, if this, v t will get reduce to Nan limit delta t tends to 0. Now, r t we have written as summation of x i and in the bracket, we can written here, t to indicate this is a function of t and together with the unit vector in the i t s direction. So, this is the i t s component of the radius vector. So, this is the radius vector, which is r s composed of x 1 e 1 cap plus x 2 e 2 cap plus x 3 e 3 cap and here, we are put in t to indicate at this is a time t.

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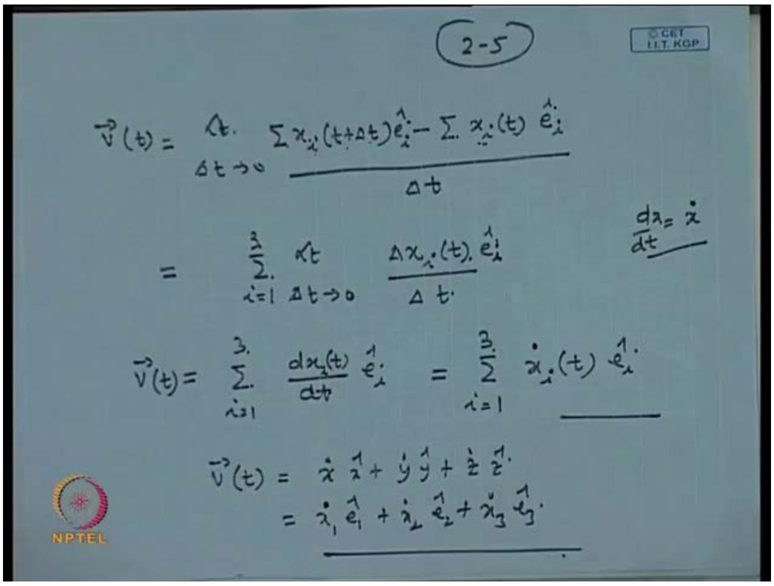

So, with x 1, x 2 and x 3 also the time t and will come which we have not shown here. And these are the components of this radius vector along the e 1, e 2 and e 3 directions. Similarly, we can write here, the r t plus delta t s x i t plus delta t and the summation and summation is obviously r is equal to 1 2 3 divided by delta t and here, we put also e i cap. Therefore, this can be written as, summation can be taken outside and we can write limit delta t tends to 0 and this will be so, if we subtract x i, if x i minus x i t plus delta t minus x i t. So, we can write this as, delta x i t times e i cap divided by delta t and then, summation is over obviously i is equal to 1 2 3.

Now, you can see from this place, that v then, becomes equal to i is equal to 1 2 3 and as the limit delta t tends to 0, we can write this as, d x t by d t and e i cap and this often, we write as, here i is their, so x i dot, this is the derivative, d by d t x this is written as x dot. So, in this simple notice and what does it say that, v t this becomes equal to x dot x cap plus y dot y cap plus z dot z cap or give aside the same thing as, x 1 dote 1 cap plus x 2 dot e 2 cap plus x 3 dot e 3. Magnitude of the velocity vector, which we write as v this will be nothing but the dot product of v dot v under root. So, we have, we contain the dot product now, e 1 cap plus x 2 dot e 2 cap x 3 dot e 3 cap. And this gets, reduce to x 1 dot square x 2 dot square plus x 3 dot square and take the under root of this and this, gives you the magnitude of the velocity and often we call this as the speed.

Now similarly, we can define acceleration.

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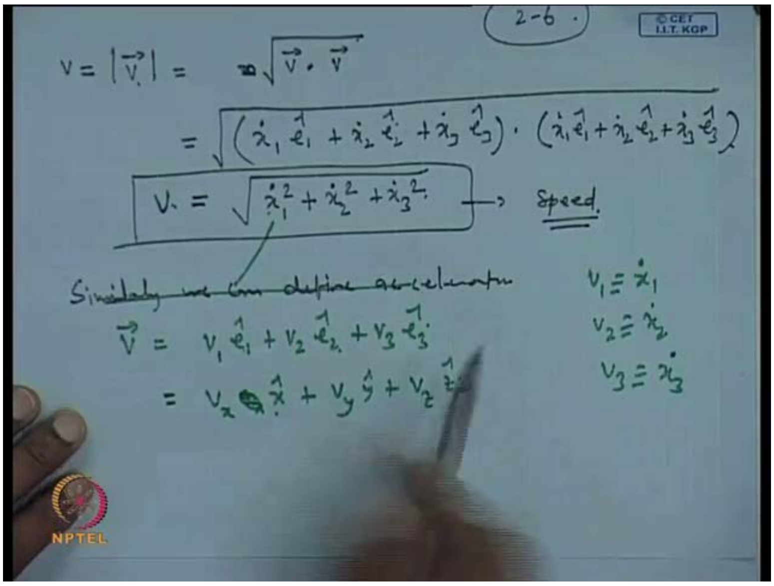

So, conclude this, this we can write this as, v as instead of writing in terms of x 1 dot, we can also write in terms of v 1 e 1 cap as v 2 e 2 cap as v 3 e 3 cap. Where v 1 equal to is identical to x 1 dot, v 2 is identical to x 2 dot and v 3 as identical to x 3 dot or if you are writing in terms of x so this, you can write as v x times v x cap, this is the unit vector in x direction, v y times y cap and v z times z cap. Actually, this e 1 e 2 and e 3 cap, what I am writing here, this is a more generalized notation for presenting a vector, while x cap y cap and z cap, it is a little restricted in sense. So, if even for a curve linear coordinate, we can precede with e 1 e 2 and e 3 cap, while x 1 y and z it is a mostly restricted to the Cartesian coordinates.

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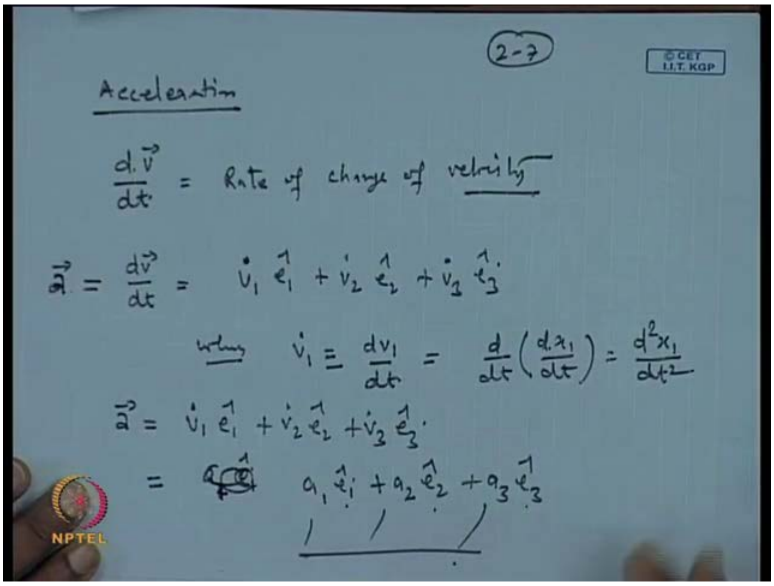

Now, acceleration is defined as. This is defined as the rate of change of velocity. So, proceeding in the same way. So, d v by d t, in the using the equation, what we have written here, we can write as v 1 dot e 1 cap plus v 2 dot e 2 cap plus v 3 dot e 3 cap. Where v 1 dot is nothing but d v 1 by d t and then, this becomes equal to, if we write v 1 equal to d x 1 by d t so, this becomes equal to d square x 1 by d t square and similarly, other follows. And the common notation for acceleration is a small a. So, we can write a small a this as; this the component wise we can write a 1 times e 1 cap a 2 times e 2 cap a 3 times e 3 cap. Where a 1 a 2 and a 3, these are the components of the acceleration along the e 1 e 2 and e 3 directions.

And the locus of the particle, as it moves through a space is called trajectory. So, quite often, they are interested in writing the velocity in terms of the trajectory variables like, what do I mean that, if I represent this trajectory here, this is the r vector. So, well we will see later on that, the velocity of the particle, this will be given by a tangent vector so, this is your velocity of the particle and this will be the tangent to the trajectory. This is the trajectory, this the particle is traversing so, at this point, the velocity is given, velocity will be along with tangent to the trajectory.

Now, this velocity can be broken in two parts. So, I can break into this part and this I can write as say the v theta and another one I can break it along this direction and this I can write as v r, so for. So, v can be written as v r times r cap, where r is the unit vector in the r direction and plus v theta times theta cap, where theta cap is the; or here, we can use the same notation as e indicating here e r and here also the e theta to indicate, this is the unit vector in this direction. So, e theta is unit vector here and e r is the unit vector in this direction. So, if we try to represent in this term so, this kind where theta is this angle, because so this kind of row presentation is called the polar notation.

And let see easy to work out, as we can see that, if I write d r by d t say, if we start with r equal to r times e r cap. So, if I differentiate this with respect to time, this can be written as r dot times e r cap plus r times e r cap dot, so this is what I indicated here, this is your v r is nothing but r dot. And this part, we need to evaluate so, if this will be using frequently in our discussion later on.

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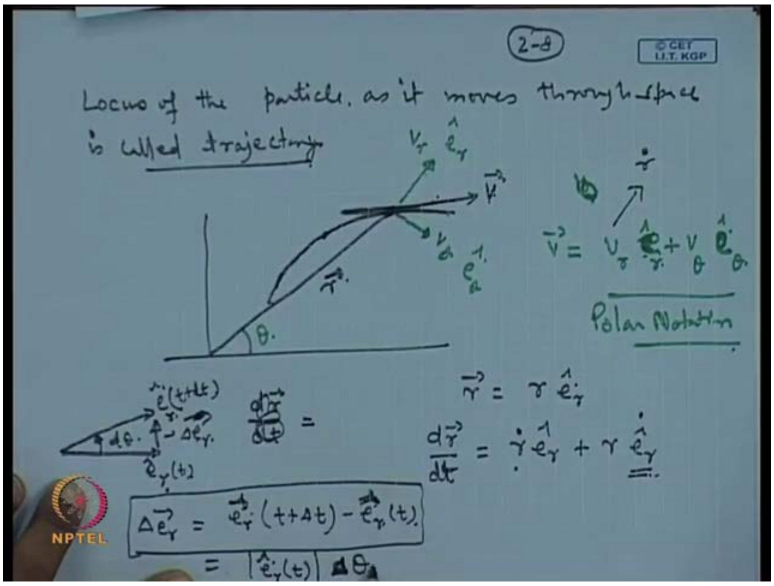

So, if this is e r vector and if we turned it by d theta and let us say, this is a time t and this is a time t plus d t, so this is the unit vector. So, the change will be delta e r and delta e r, we can write as e r t plus delta t minus e r t. And from here we can see that, this delta e r is nothing but the magnitude of this e r so, magnitude of the e r cap, we have written e r cap for unit vector, so e r cap t times, this angle d 5 or here, we write it as delta theta.

So, delta e r. Now, we are writing that, this is the magnitude of the unit vector so this becomes 1. So, delta e r magnitude wise. So, here, we will write delta e r, this is the vector and here we are writing the magnitude wise so, we are just taking the magnitude of this. So, delta here, becomes delta theta and therefore, delta e r by delta t this becomes delta theta by delta t in the limit delta t tends to 0, this becomes theta dot. So, what we get that, d e r by d t this is nothing but theta dot. But this is magnitude y, what about the direction? So, direction we can see here. So, for writing delta e r by delta t, we can follow; the example here, so as delta theta this v, if we write as the delta theta angle so, as delta theta becomes a small and a small you will that, this vector will be just perpendicular to the e r vector.

Therefore, this is directed along the e theta direction, limit delta t tends to 0, you can write as e cap theta. So, supplementing here in this place, we can write r dot e r cap plus r times theta dot e theta cap.

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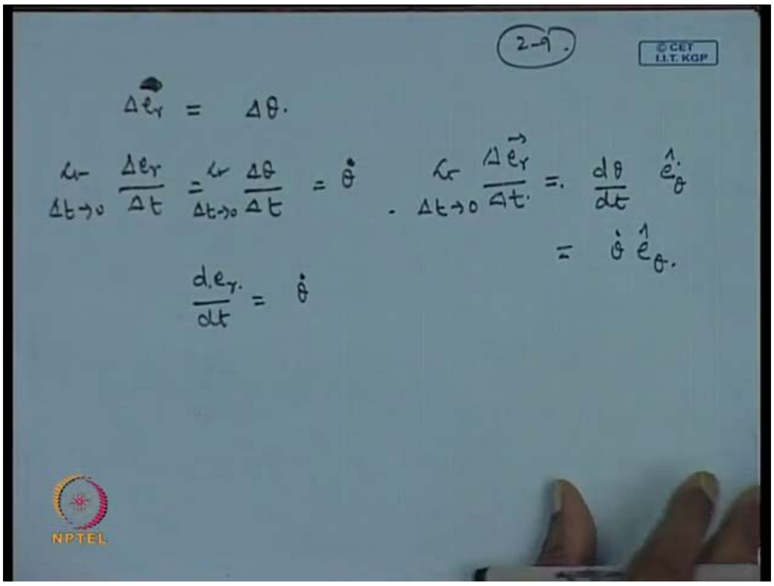

And this is your v theta, which nothing but r times theta dot. So, this is all about the polar coordinate. So, if whatever the derivation, we have done right now, this will be very useful the way we have represented the derivative of a unit vector and the derivations we do later on.

So, if these are is representing unit vectors in the directions e 1 e 2 and e 3, r is the vector at time t, this is at t plus d t or delta t. So, the vector from here to here, this is your delta r t we can write as delta r t. So, we are trying to represent this in different terms so you can see that, the black curve which is seen, this black curve we will write as d s this segment from here to here.

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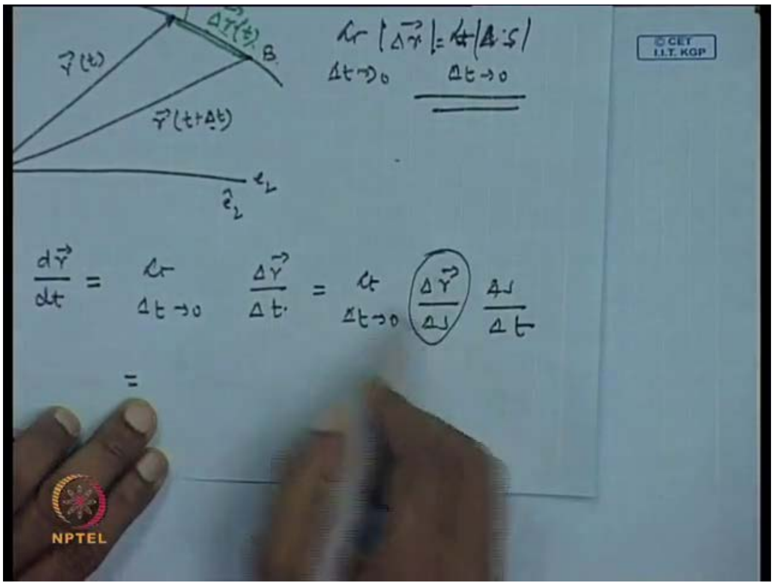

Let us say, this point is A, this point is B so, the black segment and the green segment they differ in length little way. But as, delta t becomes a smaller and a smaller that is B approach is A, if you reduce the delta t towards 0 so, it will be closing to A. So, as it closes to A so, d r magnitude it becomes equal to delta s, in the limit delta t tends to 0, these are equal.

So, if this is magnitude wise, what we are writing here. Now, d r by d t, we can write as, limit delta t tends to 0 delta r by delta t so, this can be written as, limit delta t tends to 0 delta r by delta s times delta s by delta t. So, already we have seen that, from here, that has the limit delta t tends to 0, the magnitude of delta r and delta s it is becoming same. And from here, we can also see that, as delta r becomes closer to 0 means, the B approaching A. So, this vector delta r vector this will be just tangential to the tangential at point A.

So, as delta t tends to 0, delta r becomes, tangential to the point; to the curve at point A. Therefore, delta r can be written as, delta r by delta s times delta s by delta t in the limit delta t tends to 0, this can be written as the magnitude of this obviously it becomes 1 and the unit vector tangential to the curve at that point and times this quantity will become d s by d t. So, what we are getting, we now, we can write as d s by d t times e t. Now, referring back to the representation that we give in this place. So, we can see that, we have written in this way here.

So, if e t this, such a vector, which is tangential to this point. So, if here, what it is doing is, at least once, we are showing here as e t, e t is nothing but in the direction of the p direction that, we have shown and this can be composed of so from here, also you can get, the unit vector in the direction of t. So, if you try to write from this place. So, this you can write as v divided by v so, this will give the unit vector in the direction of t so, this will be v r e r cap plus v theta e theta cap divided by v. It is, because this is the magnitude so in this direction this is the e t vector. And v r is obviously, we have written as r dot so this is r dot by v e r cap and v theta, we have written as r times theta dot. So, this is divided by v theta cap. So, this is your e t vector.

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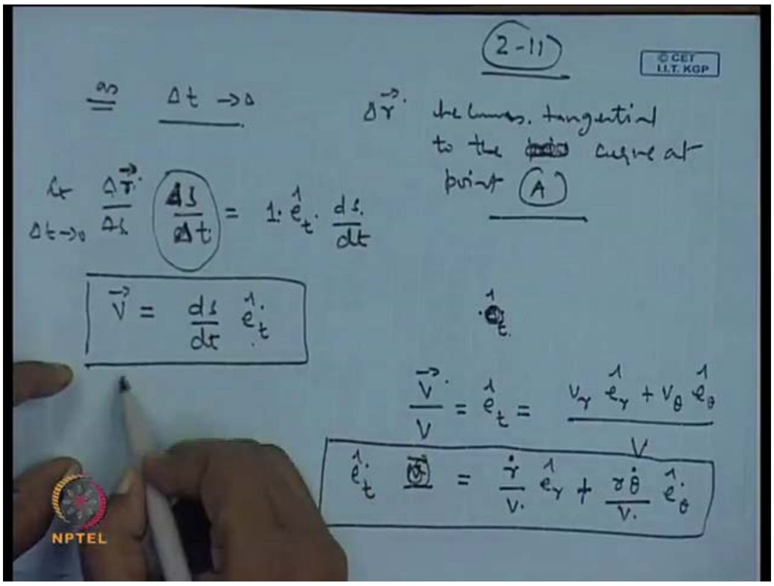

So, we can see that the same thing can be represented in various way and they have their own significance has far as this representations are concerned. Sometimes, we use the representation in the polar notation where rate is convenient and wherever the if this; just now, we have derived in terms of d s by d t, wherever this notation is convenient we will be using this. Now, the next step will be to find out the acceleration. This, we have done earlier. Now, a we can write as d by d t.

So, this is the one way of working out the acceleration also, we can do this in the polar notation. So, if we keep the derivative here, this becomes d square s by d t s square times e t cap plus d s by d t times e t cap, d by d t e t cap this is clear, this factor, we need to work out.

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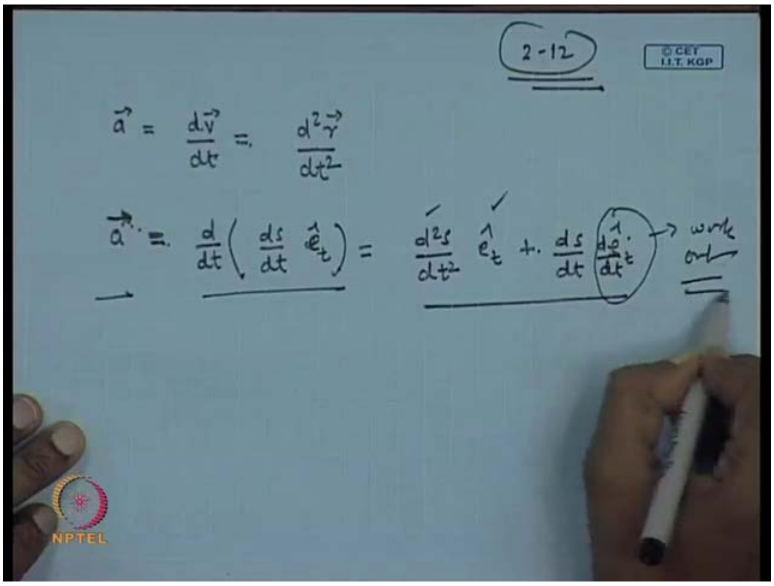

This is the r vector at time t plus delta t and the tangent to this, we are showing as, this is the tangent vector at time e t and here, this is the tangent vector at time e t plus delta t. And, if we draw, this quantity we have already written as d s and let us say, this is the radius vector r and this also called the radius of curvature, not radius vector this is the radius of curvature r. And this angle, we write as delta phi. So, by the time here, this vector here this is the center of the curvature. So, as this say if here this is the e t vector so, perpendicular to this we have drawn this vector so, this is perpendicular to this 90 degree and as it rotates from here to here.

Again, we are showing this is the angle 90 degree. So, rotation from here to here, this is the d s and this is a very small segment and we can write here delta s instead of d s. So therefore, your unit vector also, it will rotate by the angle this delta phi. So, delta S by R this is nothing but the angle delta phi and this is the angle by which your tangent vector will rotate. So, this angle is here, delta phi if we extend it back from this place to this place. So, this angle will be represented by angle delta phi.

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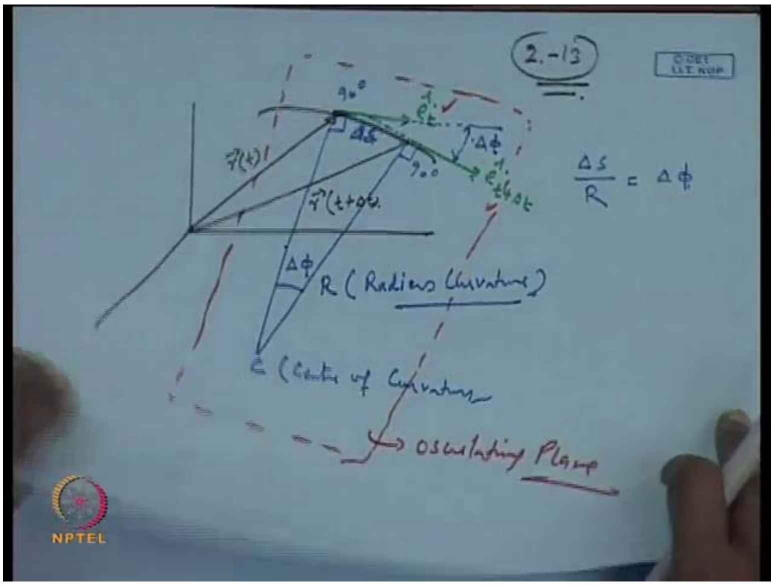

So, if the plane bracket constitutes, this a small segment delta S and this two radius of curvature lines, it is called a oscillating plane, this constitutes one plane. And this is basically constituted by your e t vector and e t plus delta t vector. So, these are the two vectors lying in this plane, which we call as the oscillating plane. And, because this is perpendicular to this and this is also perpendicular to this. So, therefore you are to the center of curvature this also lays in the oscillating plane.

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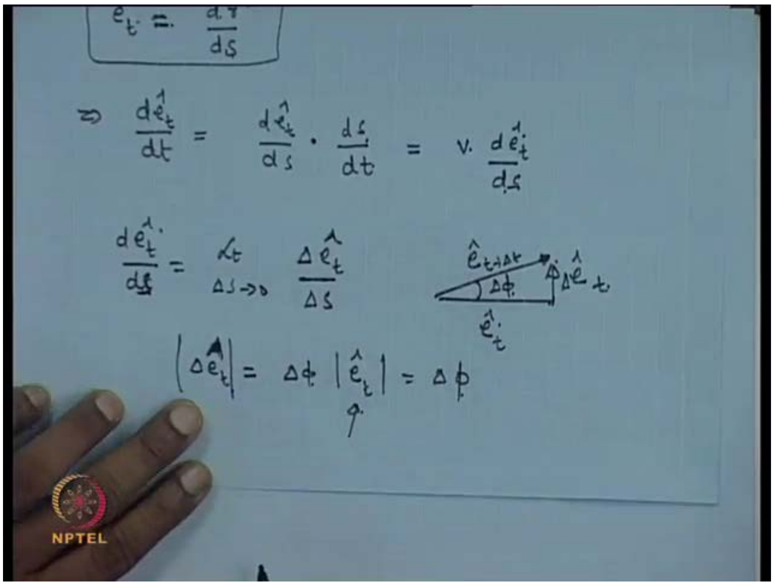

And now, from here, we can work out what will be the d t by, what we were looking for this quantity d t cap by d t. So, e t cap already, we have earlier work as this can be written as d r by d s. So, this implies d e t cap by d t this, we have worked earlier, just few pages of back. So, d e t by d t, we can write as d e t cap by d s times d s by d t. This quantity is nothing but v.

Now, this needs to be further revaluated, d e t cap by d s this, we can write as limit delta s tends to 0 delta e t cap divided by delta s. And as earlier, we have done this is your e t cap and this is your e t plus delta t cap and this is delta e t cap and this angle is your as shown here, in this Figure this is the delta phi angle.

So therefore, delta e t cap this can be written as, delta phi times this delta e t cap magnitude, we will write this magnitude is equal to 1 so, this becomes delta phi. And, you can see from this place, what will gets the direction? This direction will be perpendicular to the as delta phi becomes a smaller and smaller so, this quantity will also reduce and as this point, this is B suppose and this is A. As this approaches here in this point, here, we will write as B prime and A prime to distinguish from any or earlier notation. So, as this point approaches here delta phi becomes a smaller and smaller and this delta e t vector, this will become perpendicular to the e t vector and perpendicular to this e t vector let us write this as e n, e n cap.

So, e n cap is a vector which is perpendicular to e t cap, this is a perpendicular notation. So, e n cap is perpendicular to the e t cap. So, therefore, d e t cap by d e t this can be written as v times d e t cap by d s, which is v times limit delta s tends to 0 delta e t cap by delta s and this reduces to delta phi by delta s times e n cap delta limit extends to 0. And now, delta phi, we know from this place this is nothing but delta s by R. So, we insert here v is equal to limit delta s tends to 0 and delta phi is nothing but delta s by R so, this is delta s.

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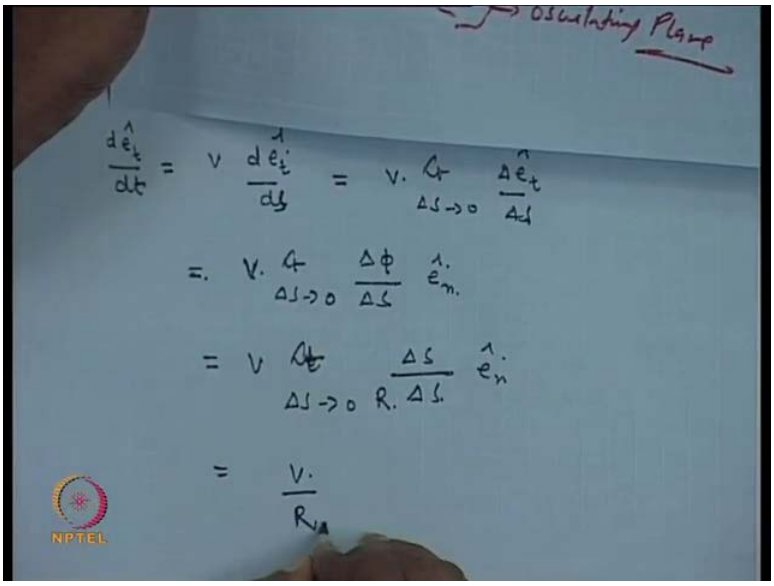

So, this becomes v by R times e n cap and now, putting back into our original equation for a. Now, here we need to insert this and where v is nothing but v equal to d s by d t in this place. Therefore, we can write a equal to d s square s by d t s square e t cap plus d s by d t time d s by d t times e n cap.

Now, e n cap as you can see here, e n cap as this point, this is point P and this is point A So, as the point B is approaching A so, the plane which it will be defining, this will be an instantaneous plane. This is an as delta t tends to 0, the oscillating plane, this is basically an instantaneous plane, at every instant this plane is changing and simultaneously the center of curvature will also be changing and the reduce of curvature also this will be changing.

So, as this point is approaching here and in vector what we are writing here. So, this is approaching here, in this place and the perpendicular to the e t vector. Just now, we have defined that this is approaching here and therefore, delta phi is becoming a smaller and smaller and the change in the vector e t plus delta t minus e t, it becomes perpendicular to this vector. So, perpendicular to this vector is a where this is a direction, we can see from this place, this is the direction it is along, this direction perpendicular to this vector.

So, perpendicular to this vector at point A is points in this direction. So, in this direction is your e n you write by e n cap, it is direction is along the center of curvature so, thus, we have completed here. Now, we can see that, this is the e t vector and this is the e n vector so, your acceleration total acceleration of the particle, it is composed of two components d s by d s square s by d t square. Which is written in terms of trajectory length and then also d s by d t this is a d s by d t, this trajectory velocity also you call as this as the speed. So, d s by d t and e t n cap which is along this direction.

So, this term also, we call as the tangential acceleration and this the normal acceleration. And the product of the vector now, you can see from here the product of the vector, if we take a product of this e n cross e t. So, it will be perpendicular, vector will be which is perpendicular to this plane this is called the Binomial vector. So, e n cap cross e t this, we can represent as e b and this will be called the Binomial vector, this is also unit vector.

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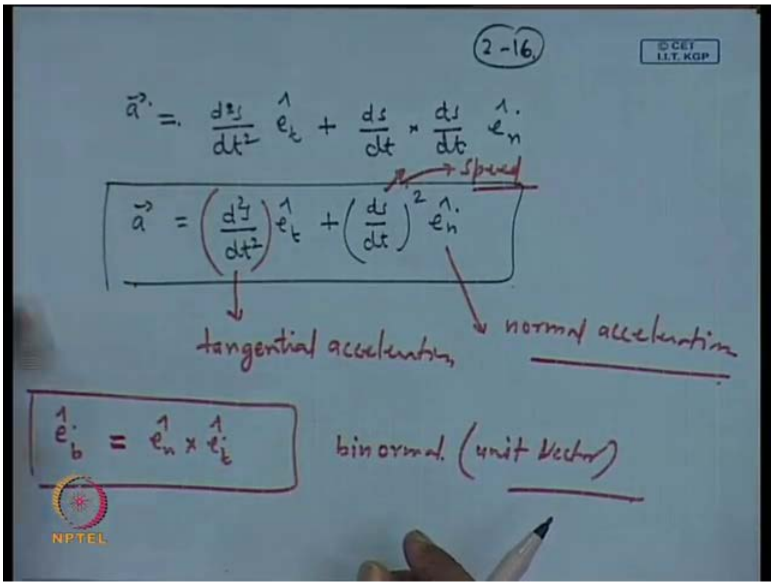

So, this is your tangential vector, tangential unit vector and this is your normal unit vector and this is also called the principal normal direction. And while the perpendicular to this will be called e n cross p, e n cross e t this will be called the bi normal direction. Now, once we have done this here, we are missing one term e n 1 by r is missing in this so, we introduce 1 by r here more over, this is d s by d t times 1 by r. So, here we will have a term 1 by r times d s by d t square. After finishing this, later on, we can utilize while solving various spacecraft problems in aircraft this is, this constitutes a very important equation, while doing the; finding out the equation of motion or the aircraft.

Now, next what we are will be working out, it will be quite important from the point of view of our attitude dynamics.

Consider, a rigid body and in which, we have a particle here, whose position vector is row and this is the particle p. And suppose, e 1 e 2 e 3 this is the body reference frame located at center of mass, this is located at the centre of mass here and fix to the body. So this is fixed to the body and this body is rotating at an angular rate of omega. So, if what I will try to do here, instead of writing a let us in the beginning will indicate this vector as instead of indicating as a row we indicate this by A.

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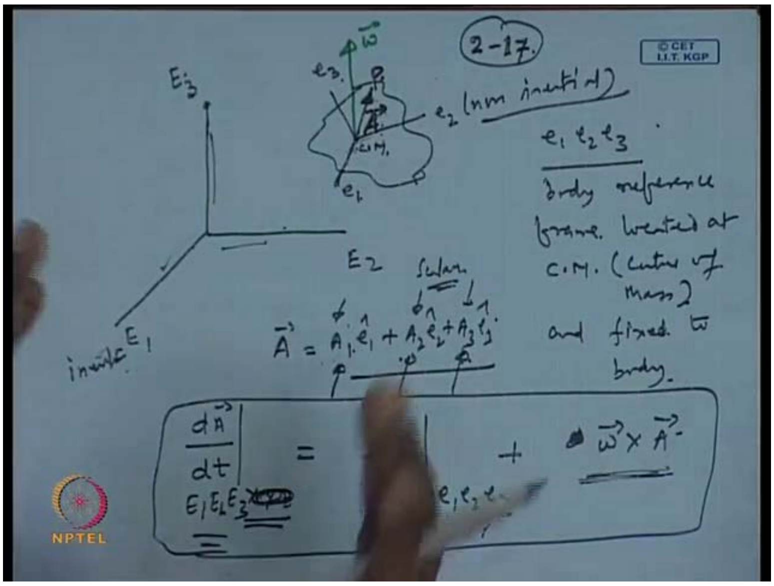

So, this A vector, this will have components in the body reference frame, where A 1 A 2 and A 3 are the components along the e 1 e 2 and e 3 direction in the body reference frame. So, if somebody is sitting in the this reference frame are attached to this body and the person is also fixed so, he will see that, these magnitudes are fixed, if this point is fixed, say rigid body means, this point obviously will not move. So, this quantity will remain fixed and these are basically scalar. An important property of this scalar are so, they are independent of the reference frame just like, take the case of the temperature. So, temperature, if you the temperature of the body in a moving train it will be same as at the temperature of a body, if you from a non moving position so that body temperature will automatically it will not change.

A train is moving in which I have kept this pen and the temperature of the pen whatever it is there, it is a scalar reality it will not change. Similarly, if you look at, the top of your ceiling fan and if you are looking from the below, you will see that the length of the blades, it remains same in this reference frame if in the of our inertial reference frame in which, we are sitting say on this table I am chair, I am sitting and looking at that fan on the top. So, this length will not change and also to a person who is sitting on the fan and the fan is moving now; so, it is just like a fan is like a rigid body and it is moving. So, for that person also the length will not change with time. So, the scalars they if do not change with time.

So, if basically, they do not; the scalars they do not depend on the reference frame rather they may vary with the time, if they are variable, but if is not that it is a it is a dependent on the reference frame. So, the length of the fan blade whatever it is a person will look from there or if the same length I will see from this place. Now, I fusing this concept, we will be developing that, if there is a vector A and this is a rotating reference frame. And I am sitting here in this place then, how the change of this vector with respect to time will appear from the inertial reference frame, which a here we write as E 1 E 2 E 3 this is an inertial reference frame and this is a non inertial reference frame.

So, how the change of this vector will appear from this, from the inertial reference frame. So, this can be correlated to the; as we will see in the next class e 1 e 2 e 3, we can write as, this can be written as in this way. So, basically, it gives a correlation this equation gives a correlation between the rate of change of this vector A as from the E 1 E 2 and E 3 reference frame.

So, if we; if it is fixed here, if this point is fixed so obviously, the rate of change of A in the body reference frame, which we are writing as e 1 e 2 e 3, because it is a fixed so this will not change here so this quantity will become 0. And therefore, the components of this vector, as you see from the inertial reference frame, if the components of this vector along this direction, this direction, this direction it will be given by omega cross.

So, p you write in terms of the is a break it up and that, will provide you the components along this direction. While in this reference frame in the body reference frame, you will see that, the components will be different, if here, it depends on how it is oriented with the body. So, depending on the orientation with the body it is a component will be defined here, but this component will also remain fixed here in this place. The irrespective of the components either in this frame or either in this frame the one reality is that the magnitude of this vector, which is the length of this vector from this place to this place. Whatever, it remains in the rotating reference frame the same it will remain from the non rotating reference frame and this is the crush of this conclusion that, we are going to work out in a; in the next lecture. So, we stop here, Thank you very much.