

Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 08

Introduction to State Feedback Controller Design

Hello everyone, in today lecture we will be starting with example how we can test the controllability of the system before we go to the controller design part. Then we will start the most important part in this course, how we can design controller for the system. If the system has noise or process noise, measurement noise, then how we can design a suitable controller so that the system noise can be mitigated, at the same time also we can track the desired value of the states. So, let us start the lecture number 8. So, in today lecture we will be discussing test of controllability of the system. So, this is very important part before you proceed to the control design whether all the states are affected by control input or not and second is controller design. So, we will start with the state feedback control concept. So, these are the topics we will cover in today lecture.

Let us start with the example. Let us assume we have a system

$$\dot{x}_1 = -2x_1 + x_2 + u$$

$$\dot{x}_2 = -x_1 - 3x_2$$

So, this is the example test for controllability of the system. If you notice in this equation, u is acting in the x_1 dynamics.

So, if you can control x_1 , also this x_1 you can control using this x_1 , you can control also x_2 , because anyways here this is the negative part, this is already in control state. So, now let us start the solution. So, as we have done in the previous lecture the controllability test method is as we know p equal to. We can change, this is the second order system.

Okay, one thing I have to write here. So, the state space formulation of the system, we can write \dot{x}_1, \dot{x}_2 . This is already in the first order ODE form. So, we can write the coefficient

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

So, basically we are getting the state space formulation of this system. So, here this is our A matrix, this is B matrix. So, using A and B we can test the controllability so we can find the p matrix

$$P = [B \quad AB] = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$$

so if you find the determinant of p it is not equal to 0. So, if this is going to satisfy, you can say system is controllable.

And another, if you want to use the MATLAB, in MATLAB code, you can write, you can put a matrix as

$$A = [-2 \ 1; -1 \ -3];$$

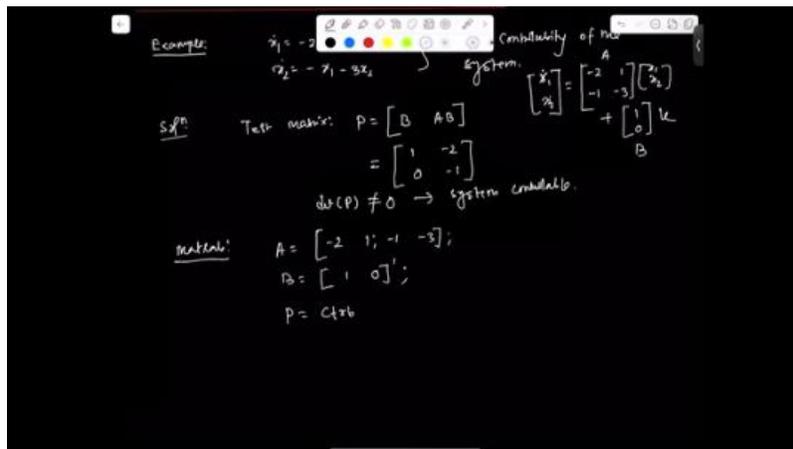
$$B = [1 \ 0]'$$

$$P = \text{ctrb}(A, B)$$

$$r = \text{rank}(P)$$

so if you Enter this command, we will get r equal to 2. So, since the system has dimension 2 cross 2, so we supposed to get the rank of the matrix 2.

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that is we can say it is basically full rank and this system is controllable. okay so this is how we can test the controllability of the system, if you have a bigger matrix you can also test, so we have taken a simple example, so now we will start the controller design process, this is very important part in this course, so we will be using pole placement design concept, so here the topic is pole placement → system design using full state feedback, from

this title it is very clear that we are going to place the desired poles, suppose the system is having some poles, for example here or here or here so let's assume these are the poles we are getting from the original system for example, this is unstable anyways, now we are going to choose some desired pole, desired poles may be in the left hand side, may be here or here or here, so how we can place these poles in such way that these poles should follow the dynamics of these poles.

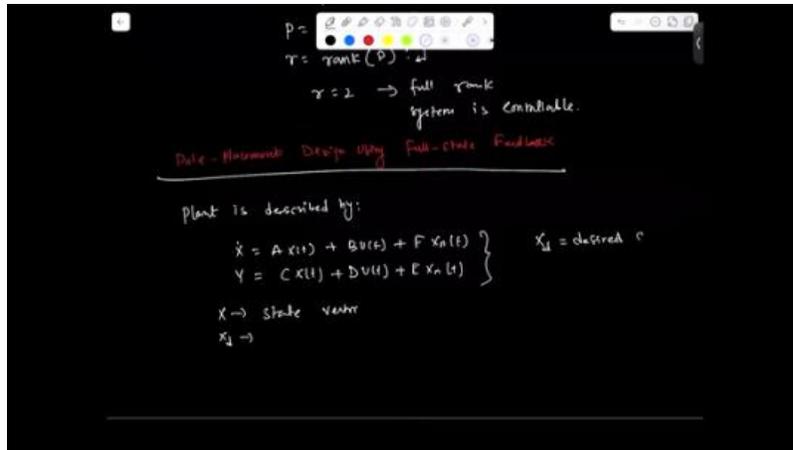
So using these poles, we will have some characteristic equation and the result of the response of this actual characteristic equation should follow the result of these pole locations, so we are going to shift this pole this side, so that overall systems response, I mean control system response will follow the desired characteristic equation, so as we proceed, you will come to know, you will understand step by step this part So, let us assume we have a system where plant is described by \dot{x} is equal to, we are taking the most critical case. taking everything into the system, so that it will not be difficult for you, even later if you want to ignore some kind of function in the system, you can. So, we are taking the most critical case

$$\dot{X} = AX(t) + BU(t) + FX_n(t)$$

$$Y = CX(t) + DU(t) + EX_n(t)$$

here x is the state vector, x desired we are choosing x desired, for example x desired to be tracked state vector that needs to be tracked, so that the system will be controlled and our x_n here noise state vector and so these are the and a and b are the system matrices, a b c and d But here the important stuff we have in the case here F and E . here F and E are the coefficient matrices of noise in the system. so this is the plant we are going to control, so now the next question is, how we can design controller so that the system will be further, so if you notice here we have desired state to be tracked and also we have noise in the system so we have to define a control in algorithm in such a way that the noise term can be tackled and also the system overall response will track the desired value, so we are defining the control algorithm, we are choosing

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$$U(t) = K[X_d(t) - X(t)] - K_d X_d(t) - K_n X_n(t)$$

so here if your desired dynamics or if you assume desired state vector. It's non-zero. While designing control algorithm, we have to take some function in the control algorithm like this. And this is the noise part that you are introducing in the control.

So, K_n should be tuned in such a way that X_n can be mitigated in the closed-loop system. this is the most critical example where we are taking all the consideration in the system but as you proceed we will come up with this complex problem, for the time being we are assuming x desired to be some zero value, so we want to design a regulator where the desired value of the state is zero, but as you proceed further i mean in later stage, we will be taking very complex case, for the time being, for making our problem simple and understandable, we are assuming the desired state to be zero, okay so assumption x desired of t we are assuming to be 0 and also we are assuming the noise of the system to be 0 for the time being but in the next lecture maybe, we will not consider these values to be 0, okay. So, in this case

$$U(t) = K[X_d(t) - X(t)] = -kX$$

Now, if you come up the closed loop diagram of the system, this is our closed loop control block diagram, it will be very helpful for you, how to understand this problem. So, this is our controller, this is the controller. So, in the controller we will be designing A and this controller output goes to the plant. This is the plant.

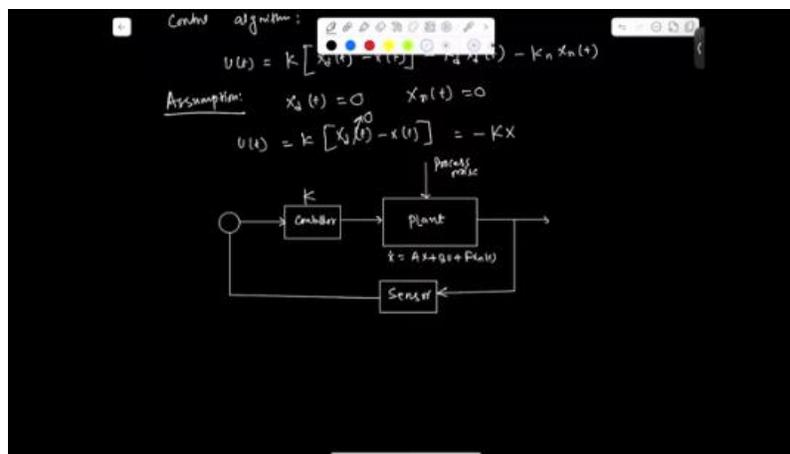
So, plant is nothing but our

$$\dot{X} = AX + BU + FX_n(t)$$

This is our plant. and also so you can assume some process noise coming into picture and this is the output from the plant and we are having the feedback here, also we can take sensor noise, maybe sensor noise is there Instead of these, we can change the diagram a little bit. So here we are having the sensor. This is basically the sensor part.

So in the sensor also you are having some measurement noise. So, this is the, and also we are having the desired state, X_d . So, in this case, X_d is 0. So, our, the state vector, Y equal to CX , should track the desired value. Instead of S , I can write simply X . So, our main image as, as time,

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tends to infinity, x to track x desired. Okay. So, now, if you notice here, so, this is the error from the system, this is negative, this is positive, right? So, here, if we assume that Since we have assumed there is no noise, so x_1 we are assuming to be 0, right.

So, what you are doing? Whatever output is available here, let us assume the same output also is available here, to get the error. So, from this we can write, error equal to x desired minus x . But for this problem, we are assuming this to be 0, right. so we can write,

$$E = X_d - X = -X$$

$$U = -KX$$

so whatever things we have done here this is validated also with this equation So, I hope it is clear. So, now, based on this assumption, our system dynamics get reduced

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

This is our system dynamics. But if you follow some standard book, most of the books, they are using this structure. Most of the books, they are not considering the noise, but we are trying to see how the controller can be designed if there is noise in the system. Okay.

Now, since we are working on control part, so we will substitute. So, our control is equal to, this is our controller. This is the controller equation. So, we will substitute this controller. So, in this case, if you substitute e equal to minus case, then you can write

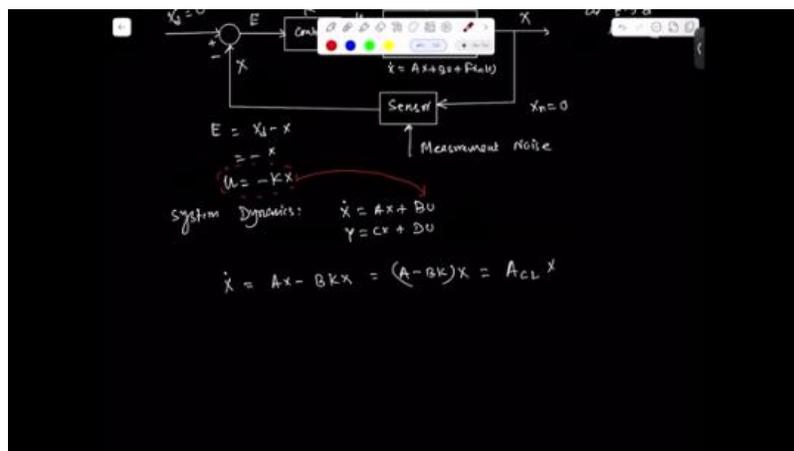
$$\dot{X} = A_{CL}X$$

So, this is if you notice carefully, this system become in some very nice form. So, now if you can find the eigenvalues You can easily find the eigenvalues of this. And these eigenvalues will be function of K here, because A and B are known for the particular system. Right.

So, we are defining this as equal to, we can define A_{CL} . CL for closed loop. Why closed loop? Because we have closed the system. Initially it was open loop.

We have introduced some control here, which makes the system closed loop. So that's why A_{CL} is nothing but the system matrix of the closed loop system. This is sometime we call augmented matrix. Okay. Augmented system.

(Refer Slide Time: 20:05)



So now I will write a note here very important here. So, the closed loop, we know that closed loop poles are eigenvalues of A_{CL} that is obvious, so if you would like to find the closed loop eigenvalues, then I can simply find the eigenvalues of this system, this matrix right, hence by selecting K we can place closed loop poles at desired locations. What I have discussed before that how to shift the poles from some place to other place.

So, I have explained somewhere in the first page. So, note that A is also a vector. and for a plant of order n and if there are inputs size of k will be is r plus n This is simply you can find from the relation between the U and A matrix. Now, so this is how we will be designing controller for this particular simple system.

For this simple system, if the system is regulator, if the desired state should be 0, if it is 0, then we get the augmented system like this. now we will take an example and we will validate this concept okay, let's take an example, so we have system matrices given to us,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

now design full state feedback regulator once we are tracking a constant value or zero of the desired state, then it sometimes called regulator, if you are tracking a time varying function then it is called a tracking algorithm. So, here regulated such that closed loop poles $S = -0.5 \pm i$.

So, if you notice these poles are in the left hand side the S plane, so maybe somewhere here okay, so these are the poles which are in the left hand side the system, so we need to find a controller which will help to shift the original system poles to follow this trajectory okay this is our target, so we have to design a controller, so as a control engineer we can choose this pole, it does not matter because for this particular example i have chosen this but you can choose any other values which are stable poles, so based on this, you can design the controller, so first we have to try the pole location of the original system, so let's find if it is already stable, so there is no requirement of designing controller, so that's why let's place the pole location of this original system okay, so let's do it, first check the pole location of the original system So, here you can use this characteristic equation $S^2 + a$. This is how we find the eigenvalues.

So, eigenvalues are nothing but roots or poles. So, here we can find

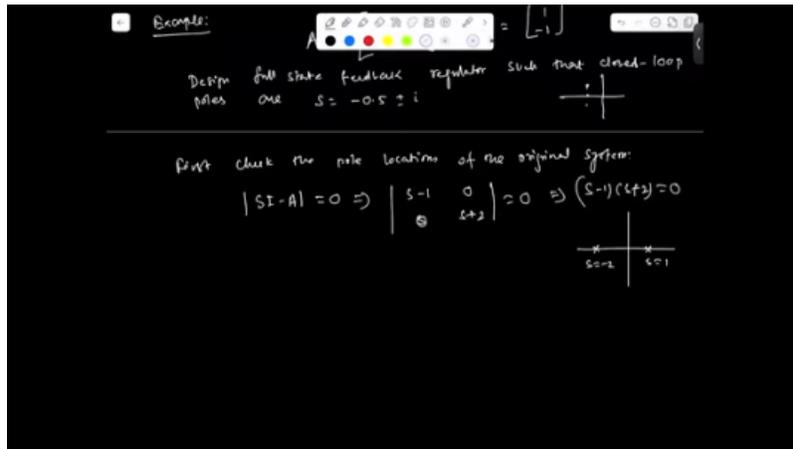
$$|SI - A| = 0$$

$$(S - 1)(S + 2) = 0$$

so here if you notice carefully there is one pole of the original system in the right hand side, you can write here the location of the pole, so one pole is maybe here Another pole is written here.

This is $S = 1$ and $S = -2$. So, system is unstable. If any of the pole lies in the right-hand side, system is unstable. This is the knowledge already we have in the classical control. So, $S = 1$ in stable pole.

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Right. so now we will check, we will design the controller part so that the response of this current system should follow this desired dynamics, so our next part is, we have to just since the system is second order system 2 plus 2 matrix, you can find the gain matrix in this form, so control matrix you can choose

$$K = [K_1 \quad K_2]$$

this is the control matrix and

$$\begin{aligned} A_{CL} &= A - BK \\ &= \begin{bmatrix} 1 - K_1 & -K_2 \\ K_1 & -2 + K_2 \end{bmatrix} \end{aligned}$$

This is the system, this is the closed loop system matrix. And again we can find the characteristic equation of augmented matrix or augmented system, we can write. So, what do we have?

$$|sI - A_{CL}| = 0$$

$$s^2 + (K_1 - K_2 + 1)s + (2K_1 + K_2 - 2) = 0 \dots Eq(1)$$

So, this is the root of this characteristic equation. So, now you can use simply the minus b plus minus b square minus 4 ac divided by 2a. Simply you can use this formula because it is second order system you can even find. But now this is the basically roots of the augmented system, right. Now, also we have to find the characteristic equation, ok.

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The C.E. of Augmented system is

$$|sI - A_c| = 0$$

$$\Rightarrow \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{bmatrix} 1-K_1 & -K_2 \\ K_1 & -2+K_2 \end{bmatrix} = 0$$

$$\Rightarrow s_{1,2} = -\frac{1}{2} (K_1 - K_2 + 1) \pm \frac{1}{2} (K_1^2 + K_2^2 - 2K_1K_2 - 6K_1 - 6K_2 + 1)^{1/2}$$

One thing I also should write here the characteristic equation here, So, this is the characteristic equation we are getting from the matrix, ok. So, now and from this characteristic equation we are getting these are the poles. Now, that from the desired dynamics from the desired poles, we can find the characteristic equation as desired pole we have been given already, this is our desired poles given to us.

So, we can find the characteristic equation. So, we can write

$$S^2 + S + 1.25 = 0 \dots Eq(2)$$

this is our desired characteristic equation and this is the augmented characteristic equation, right, this is the augmented system characteristic equation. So, we can compare both the equations,

$$K_1 = K_2$$

$$2K_1 + K_2 - 2 = 1.25$$

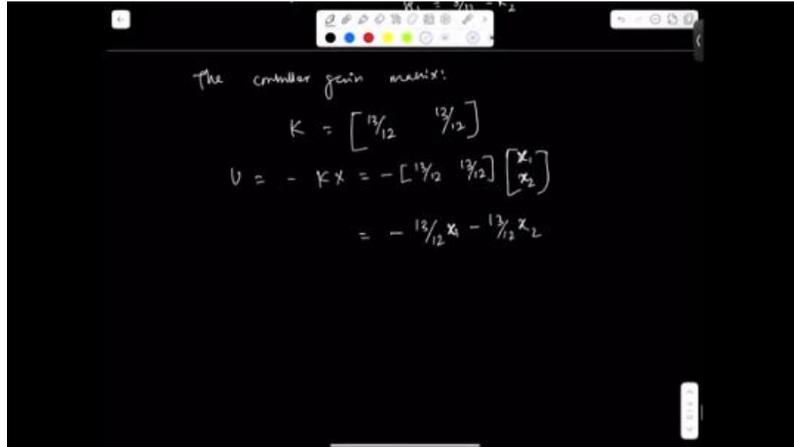
$$K_1 = K_2 = 13/12$$

this is the control matrix which will come in the controller part to design control input to the plant, so you can write, p equal to minus or we have kx, so you can write

$$K = [13/12 \quad 13/12]$$

this is So now if you can simulate this program in MATLAB, you'll get nice results. So this is the homework for you, how to write the simulink code for this space.

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A photograph of a blackboard with white chalk writing. The text reads: 'The controller gain matrix:' followed by the matrix equation $K = \begin{bmatrix} 13/12 & 13/12 \end{bmatrix}$. Below that is the control law $U = -KX = -\begin{bmatrix} 13/12 & 13/12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, which is then simplified to $= -13/12 x_1 - 13/12 x_2$.

The controller gain matrix:

$$K = \begin{bmatrix} 13/12 & 13/12 \end{bmatrix}$$
$$U = -KX = -\begin{bmatrix} 13/12 & 13/12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= -13/12 x_1 - 13/12 x_2$$

So let's stop it here. We'll continue from the next lecture, how we can take the noise in the system and how we can design controller. So we have some example on aircraft system. Thank you.