Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 06

Example of Linear transformation

Hello everyone, in today lecture we will be solving a problem how we can find the system into diagonalized form and also we will have the MATLAB code to check the response of the system which will be in the diagonalized form and also we will find the value of the state matrix at a particular time. So, let us start the problem. Here the system is, the system is given to us

$$\dot{x}_1 = -x_1 + 2x_2$$

$$\dot{x}_2 = -x_1 - 3x_2$$

If you notice this system there is no external inputs in the system. The system we can say, this is the natural system and also it is in coupled form. So, here we can say U is basically 0 here, that is 1 plus 1 is 0. So, here the question is to calculate the state of transition matrix and then solve for the state vector, if initial condition is x naught this is given plus 1 If you notice here, the system can be determined, we can solve the problem in two ways.

We can take the system as it is and we can find the solution or we can use the previous method what we have done in the previous lecture. We can get the system into diagonalized form and we can get back to the original system solution. So, here we will solve this problem using the method of diagonalization and for that, we need to find the transformation matrix, we need to find diagonal values, diagonal vectors and we will come up the system into diagonal form. So, solution, since the system is written in the ODE form, so we need to find state space form, So, in state space form, we can write

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

This is the system given to us, there is no control. So, if you notice the system is highly, it is basically coupled system. So, if you want to convert the system into diagonal form, then we need diagonalized form, we need to find the eigen values first eigen values. So, the process is
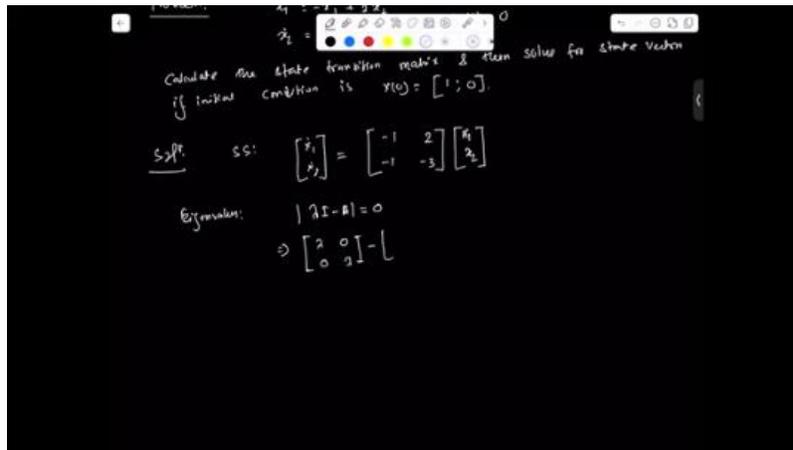
$$|\lambda I - A| = 0$$

So, this will give us the characteristic equation.So, from this the characteristic equation we get,

$$\lambda^2 + 4\lambda + 5 = 0$$

$$\lambda_1, \lambda_2 = -2 \pm i$$

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So, these are the eigenvalues and if you notice the eigenvalues are of course distinct but complex. This is like a two testing problem, let us see. If you notice the system is the location of the roots are all in the left hand side or on the left hand side. So, you can say this is basically the point if it is minus 2, then somewhere here there are two complex conjugate poles.

So, system is asymptotically stable because of all the poles of the system are in the left hand side. One thing in the modern control is, we use diagonal term, but in the classical control we use the poles. So, these are nothing but the poles, that are roots of the characteristic equation. what we have done in our previous course in classical control. So, since the dimension of the system matrix is 2 plus 2, so that should be 2, that we supposed to get to a L vector. So, let us find them. So, we can form the transformation matrix
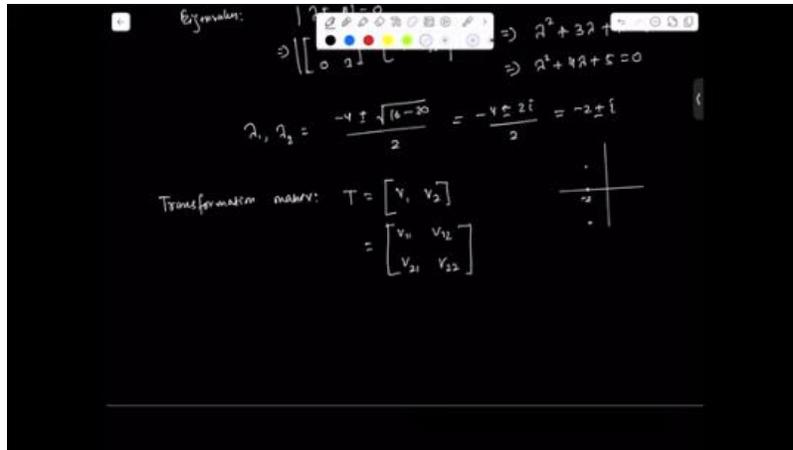
$$T = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$$

$$\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

And since we do not know the values of v 1, v 2, let us assume

$$\lambda_1 v_1 = A v_1$$

So, these are the terms of the eigenvectors. So, now, here basically these are the eigenvectors right.

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So, now we will use first eigenvalue to find v_1. So, let us start with the equation first needs to be satisfied. So, for this is the equation we know,

$$\lambda_1 v_{11} = -v_{11} + 2v_{21}$$

$$\lambda_1 v_{21} = -v_{11} - 3v_{21} \dots Eq(1)$$

If you notice from these two equations, equation 1 that are linearly dependent, that is we cannot get two unknowns, v_{11} and v_{21} by solving these two equations. The best way you can do is we can specify arbitrary value of one of the parameters and we can proceed. So, let us consider let $v_{11} = 1$ and for this case, we can get

$$v_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{-1+i}{2} \end{bmatrix}$$

And the same procedure if you follow for v_2 vector.

$$v_2 = \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{-1-i}{2} \end{bmatrix}$$

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okay so now we have the eigenvectors, so we can form the transformation matrix, t equal we can write,

$$T = \begin{bmatrix} 1 & 1 \\ \dfrac{-1+i}{2} & \dfrac{-1-i}{2} \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} \dfrac{1+i}{2} & \dfrac{1}{i} \\ \dfrac{-1+i}{2i} & \dfrac{-1}{i} \end{bmatrix}$$

So, now as you know how to find the diagonal matrix, the expression what we have done in the last lecture. So, we can find the diagonal matrix. we can find

$$A' = T^{-1}AT$$

$$= \begin{bmatrix} -2+i & 0 \\ 0 & -2-i \end{bmatrix}$$

It is great that we have the diagonal matrix. So, this is diagonal what we supposed to get, what we supposed to get for the the concept, what we have done in the previous lecture. So, this is diagonal matrix.

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Now, if you would like to find state transition matrix of the transformed system, we can write,

$$e^{A't} = \begin{bmatrix} e^{(-2+i)t} & 0 \\ 0 & e^{(-2-i)t} \end{bmatrix}$$

so this is basically the state transition matrix of the transformed system, so from this if you want to get back to the original state transition matrix, we have the formula also in the last lecture that we have derived, so the state transition matrix for the original system we can write

$$e^{At} = Te^{A't}T^{-1}$$

so if you use

$$e^{a+ib} = e^a[cosb + i\,sinb]$$

so you can use this formula for this particular matrix and simplifying the above expression we get,

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$$e^{At} = \begin{bmatrix} e^{-2t(cost+sint)} & 2e^{-2t} \\ -e^{-2t}sint & e^{-2t(cost-sint)} \end{bmatrix}$$

So, this is the state translation matrix of the original system. Now, if you notice our the solution of the state equation, we know that

$$X(t) = e^{At}X(0) + \int_0^t e^{A(t-\tau)}BU(\tau)d\tau$$

This is you know the solution of the system if it is written in the state space form. It does not matter it is in diagonalized form or in coupled form. So, this is the standard expression. So, now here if you notice here our control input is 0 here, in the problem it is defined. The control input is 0 here, so this the exponential term here, it will not be there in the solution, so we can now write with u of t equal to 0, we can write
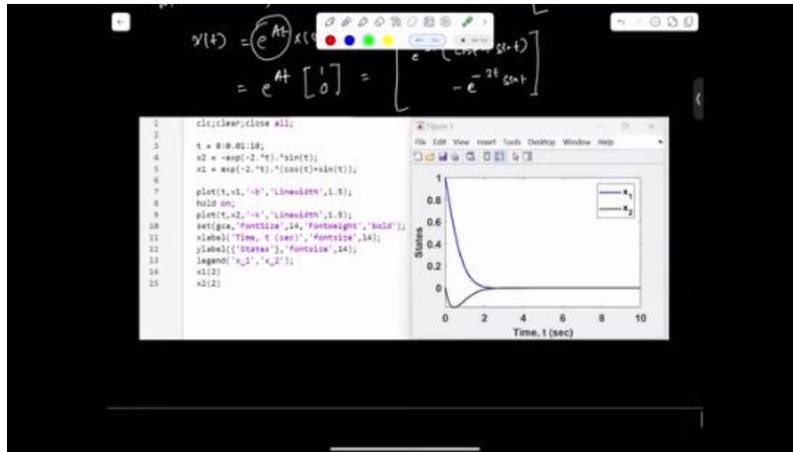
$$X(t) = e^{At}X(0) = e^{At}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-2t(cost+sint)} \\ -e^{-2t}sint \end{bmatrix}$$

This is the solution of the original system. Now, we will see the response in MATLAB for this particular case. So, it is quite obvious that there should be some overshoot, undershoot because in both the solutions, we have the sine and cosine terms. So we will go through some oscillation but over time the oscillation will go to zero because we have this term e^{-2t}. So, as t tends to infinity, this part is going to 0 and this is going to decay the oscillation in the system. So, let us check the MATLAB result. This is the MATLAB code of this particular solution.

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So, here we are having, we are solving the system for 10 second, 10 second here. and we have the expression, so it is nothing but here, this is actually x1(t) and x2(t), so x1(t) our expression is, this is the expression, so this is here we have written x1(t) and x2(t) is here and the other things are to modify the plot and here if you notice the system actually the initial condition is given to us is x1 is, it should start from 1 and x2 should start from 0, let's see x2 is starting from 0 and x1 is starting from 1 and as time goes to infinity, the system supposed to go to 0 and it is equal to 0, the zero line, so it is now whatever result we have obtained through numerical analysis, it is also satisfying in the simulation.

So, now I think it is correct, that is the solution what we have obtained, it is correct. Now, we will have another MATLAB plot. So, that MATLAB plot actually we are going to see, here the thing is Let's find, if you would like to find, because if you remember two lectures, before two lectures we had some comment that state transition matrix actually comes from any initial state to final state or any other time. And e^{At}, so this is basically state transition matrix which helps to transform from one state to another state, now our next MATLAB plot is, how to find the result of e to the p at two second

okay this is the result, so what should be the result for e to the power a t, at t second, so this is you can simply use some code but we have the code for that as well, so this is how we can find the value of state transition matrix after a few second, so we have defined the matrix and this is the a state transition matrix and we are using Zeigler Nicholas eigenvector on this term and we are finding the new state transition matrix at t equal to 2. So, this is basically the main value and this is the we are showing the result, this is the result basically at t equal to 2. So, this is how we can find the state transition matrix result at particular time.

But here the main point is this, how we can find the system response using state transition matrix obviously, but the procedure we have followed, first we had the coupled system, then we convert it into decoupled form, then again we get the solution of the original system coupled system, this is the coupled system solution, And we get the result. And in our numerical result, the analysis what we have done is validated with this simulation. So, let us stop it here. We will continue from the next lecture on some very good example of the aircraft, how we can extend all this concept for the aircraft system.

Thank you.