Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 57

6 DOF aircraft equations of motion

Hello friends, welcome back. In the last lecture, we simulated 3-degree-of-freedom equations of motion of the aircraft in MATLAB and Simulink using the MATLAB function block and, finally, using the MATLAB 3DOF block and Simulink 3DOF block, all right. Now, today, we will be—we are almost ready with the aircraft 6-degree-of-freedom equations. Now, we will be focusing on the six-DOF block. These blocks you can actually view in the library browser: Aerospace Blockset > Equations of Motion > 6DOF. So, we will be dealing with six-DOF equations of motion, which look something like this. So, let me go to the help model for this.

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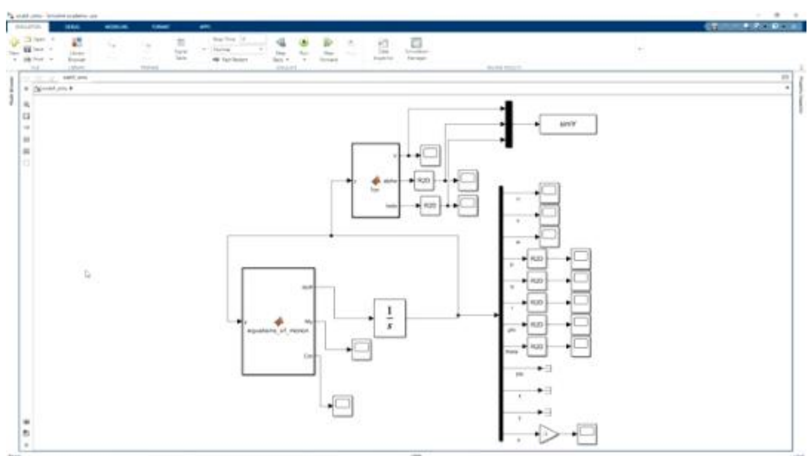

Likewise, we have done for the 3DOF block. So, this is the block for the six-DOF equations of motion. And the equations are written in this format. So here, equations will be in matrix form. For example, total velocity is in u, v, and w—body axis u, v, and w. Similarly, rates are p, q, and r. Similarly, about the moments.

And we have these equations already mentioned: $\dot{\phi}, \dot{\theta}, \dot{\psi}$. So, before we get into this directly into simulation, we need to define the parameters for six-DOF simulation. So, let us go ahead and do that. So, let me write here: six-DOF simulation. All right. So now we had already written the equations of translation dynamics: $\dot{u}, \dot{v}, \dot{w}$. Let me go back there. Yeah. So these are the equations for $\dot{u}, \dot{v}, \dot{w}$. Let me rewrite this equation there.

$$\dot{u} = \frac{1}{m}[L\sin\alpha - D\cos\alpha + T_{max}\delta_t] - g\sin\theta - qw + rv$$

$$\dot{v} = \frac{1}{m}[F_y^A] + g\sin\phi\cos\theta - ru + pw \quad \ldots Eq(23)$$

$$\dot{w} = \frac{1}{m}[-L\cos\alpha - D\sin\alpha] + g\cos\phi\cos\theta - pv + qu$$

So, here forces in the y-direction are aerodynamic forces. Let me consider this as capital Y, which is nothing but a side force. And the side force is modeled similarly to the aerodynamic forces:

$$Y = \frac{1}{2}\rho V^2 S C_y$$

$$C_y = C_{y0} + C_{y\beta}\beta + C_{yp}\frac{pb}{2V} + C_{yr}\frac{rb}{2V} + C_{y\delta a}\delta_a + C_{y\delta r}\delta_r$$

This $\frac{pb}{2V}$ is done to non-dimensionalize this parameter. so here V is the total velocity, not the side velocity.

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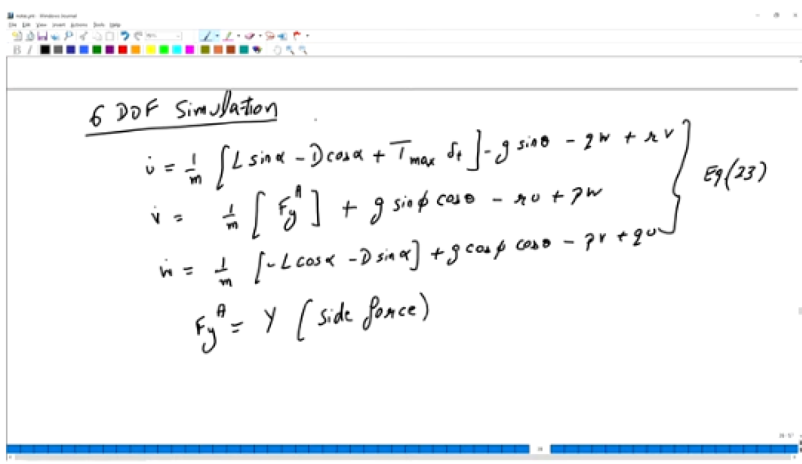

Here C_y is nothing but side force coefficient which is obviously dimensionless, so what is the unit for p for example radians per second b unit is meters here velocity is meters per second so we get radian this again unitless all right next we have from rotational dynamics we have written equation 16 let us rewrite equation 16 as

$$l = \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - I_{xz}pq \quad \ldots Eq(24)$$

$$m_p = \dot{q}I_y + rp(I_x - I_z) + I_{xz}(p^2 - r^2) \quad \ldots Eq(25)$$

$$n = -\dot{p}I_{xz} + \dot{r}I_z + pq(I_y - I_x) + I_{xz}qr \quad \ldots Eq(26)$$

So for the three-dof block, here p and r was actually zero. Then we obtained moment about y-axis as

$$\dot{q} = \frac{m_y}{I_y}$$

But if you look at equation 24 and 26, here we have rolling moment and then the yawing moment. Here we have $\dot{p}$ term. Here we have $\dot{r}$ term. here we have $\dot{p}$ term here we have we have $\dot{r}$ term so we note that lateral directional dynamics are completely coupled with each other so we do not have an ode equation in terms of $\dot{p}$ and $\dot{r}$ individually all right so hence i have written this in this way we have to convert this equation to ode form $\dot{p}$ and $\dot{r}$ equation one way to do that is find out $\dot{r}$ from here and substitute in equation number 24 or find out $\dot{p}$ from here and substitute in this equation. So, what we will do is we will find out what is $\dot{p}$ and what is $\dot{r}$. So, we have to find $\dot{p}$ and $\dot{r}$ equations separately.

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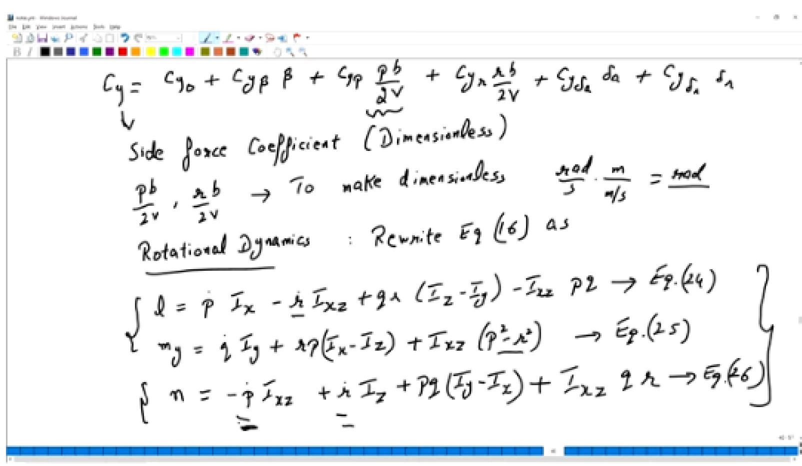

So, from equation number 26, we can write

$$\dot{r} = \frac{n}{I_z} + \frac{I_{xz}}{I_z}\dot{p} - \left(\frac{I_y - I_x}{I_z}\right)pq - \frac{I_{xz}}{I_z}qr$$

Now, substituting this $\dot{r}$ in equation 24 and writing in terms of $\dot{p}$, substitute in equation 24. All right, so then we have

$$\dot{p} = \frac{l}{I_x} + \left\{\frac{n}{I_z} + \frac{I_{xz}}{I_z}\dot{p} - \left(\frac{I_y - I_x}{I_z}\right)pq - \frac{I_{xz}}{I_z}qr\right\}\frac{I_{xz}}{I_x} -$$
$$\left(\frac{I_z - I_y}{I_x}\right)qr + \frac{I_{xz}}{I_x}pq$$

So, we have two $\dot{p}$ terms now in this equation. I am bringing that $\dot{p}$ equation to the left-hand side and taking common and further solving

$$\dot{p} = \frac{I_z}{I_xI_z - I_{xz}^2}l + \frac{I_{xz}}{I_xI_z - I_{xz}^2}n + \frac{I_{xz}}{I_xI_z - I_{xz}^2}[I_x - I_y + I_z]pq -$$
$$\frac{I_z(I_z - I_y) + I_{xz}^2}{I_xI_z - I_{xz}^2}qr$$

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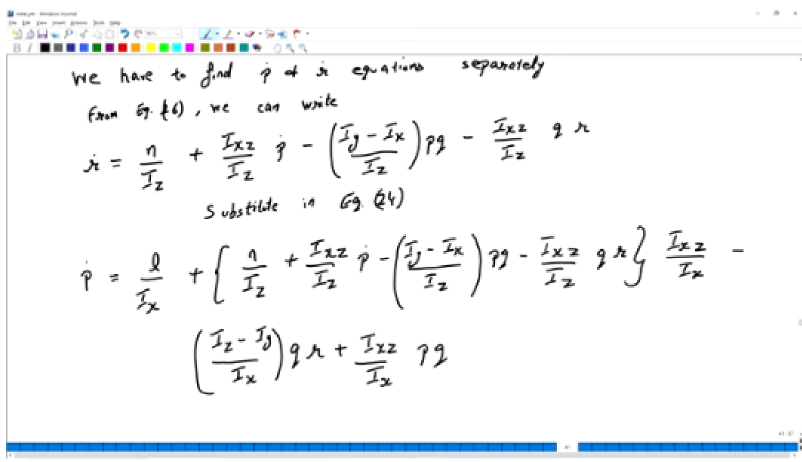

Now, for simplification, I am considering

$$\Gamma = I_xI_z - I_{xz}^2$$

$$\Gamma_1 = \frac{I_{xz}(I_x - I_y + I_z)}{\Gamma}$$

$$\Gamma_2 = \frac{I_z(I_z - I_y) + I_{xz}^2}{\Gamma}$$

$$\Gamma_3 = \frac{I_z}{\Gamma}$$

$$\Gamma_4 = \frac{I_{xz}}{\Gamma}$$

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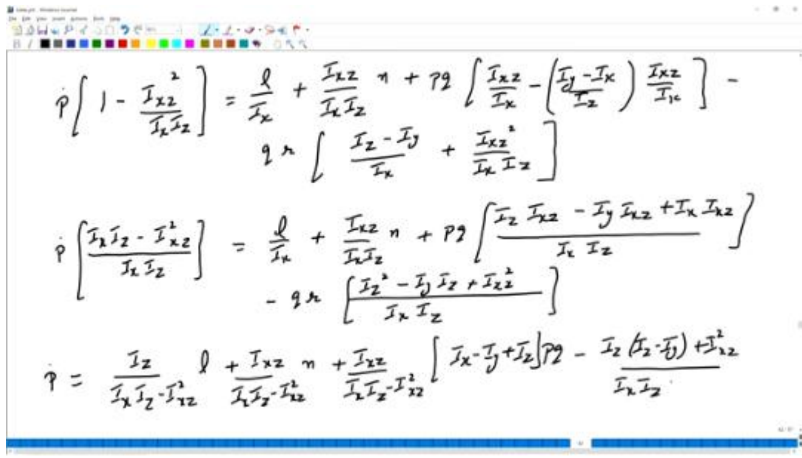

So, this nomenclature I have considered again from the same book, Small Unmanned Aircraft by Randal Beard. So, finally, we can write

$$\dot{p} = \Gamma_1 pq - \Gamma_2 qr + \Gamma_3 l + \Gamma_4 n \quad \ldots Eq(27)$$

And $\dot{q}$ from Eq. (16) is written as

$$\dot{q} = pr\left(\frac{I_z - I_x}{I_y}\right) - \frac{I_{xz}}{I_y}(p^2 - r^2) + \frac{m_y}{I_y}$$

Again let us consider

$$\Gamma_5 = \left(\frac{I_z - I_x}{I_y}\right)$$

$$\Gamma_6 = \frac{I_{xz}}{I_y}$$

$$\dot{q} = pr\Gamma_5 - \Gamma_6(p^2 - r^2) + \frac{m_y}{I_y} \quad \ldots Eq(28)$$

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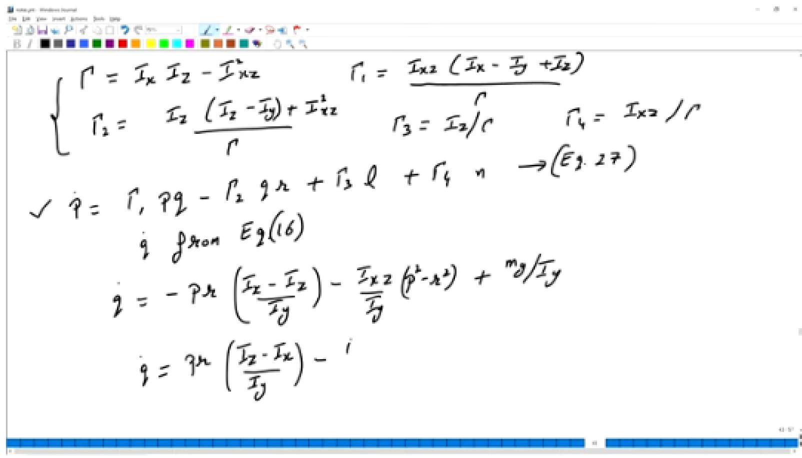

All right. So now we are only left with the $\dot{r}$ equation. So we substitute $\dot{p}$ that we have got earlier into the $\dot{r}$ equation and then solve it. Hence, equation number 26 can be written as

$$\dot{r} = \frac{n}{I_z} + \frac{I_z}{I_x I_z - I_{xz}^2} l + \frac{I_{xz}^2}{I_z(I_x I_z - I_{xz}^2)} n + \frac{I_{xz}^2}{I_z(I_x I_z - I_{xz}^2)} [I_x - I_y + I_z] pq -$$

$$\frac{I_{xz}}{I_z} \left\{ \frac{I_z(I_z - I_y) + I_{xz}^2}{I_x I_z - I_{xz}^2} \right\} qr - pq \frac{I_y - I_x}{I_z} - \frac{I_{xz}}{I_z} qr$$

$$\dot{r} = n \left\{ \frac{1}{I_z} + \frac{I_{xz}^2}{I_z(I_x I_z - I_{xz}^2)} \right\} + \frac{I_{xz}}{I_x I_z - I_{xz}^2} l + pq \left\{ \frac{I_{xz}^2}{I_z(I_x I_z - I_{xz}^2)} [I_x - I_y + I_z] - \frac{I_y - I_x}{I_z} \right\}$$

$$-qr \left\{ \frac{I_{xz}}{I_z} + \frac{I_{xz}}{I_z} \left[\frac{I_z(I_z - I_y) + I_{xz}^2}{I_x I_z - I_{xz}^2} \right] \right\}$$

$$\dot{r} = \frac{I_x}{I_x I_z - I_{xz}^2} n + \Gamma_4 l + \frac{I_{xz}^2 + I_x(I_x - I_y)}{I_x I_z - I_{xz}^2} pq - \frac{I_{xz}(I_x - I_y + I_z)}{I_x I_z - I_{xz}^2} qr$$

$$\dot{r} = \Gamma_7 pq - \Gamma_1 qr + \Gamma_4 l + \Gamma_8 n \quad \ldots Eq(29)$$

Where

$$\Gamma_7 = \frac{I_{xz}^2 + I_x(I_x - I_y)}{I_x I_z - I_{xz}^2}$$

$$\Gamma_8 = \frac{I_x}{I_x I_z - I_{xz}^2} = \frac{I_x}{\Gamma}$$

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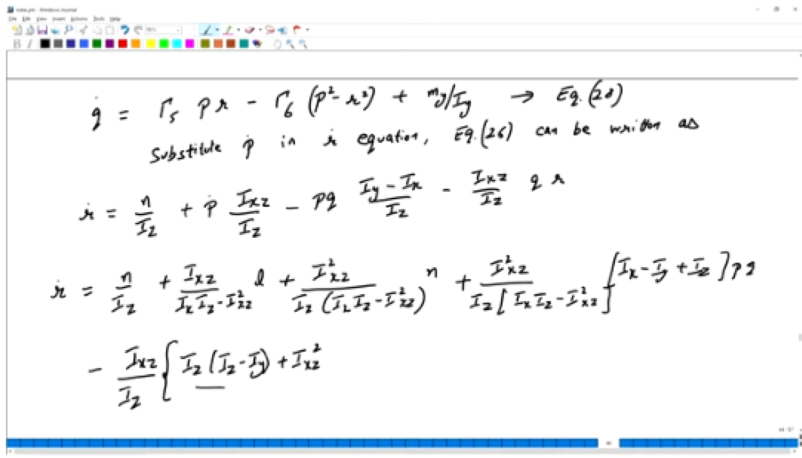

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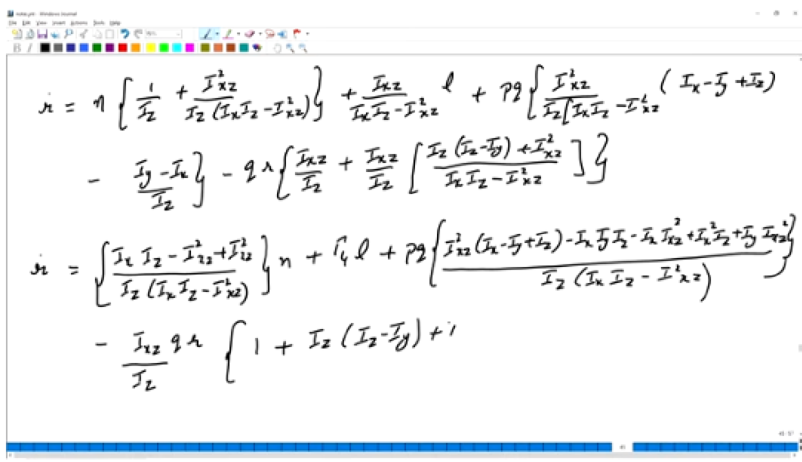

So, this was a little bit lengthy over here, but as a flight dynamics and control person, you only need to derive this once. Now, we will consider the same UAV that we took before.

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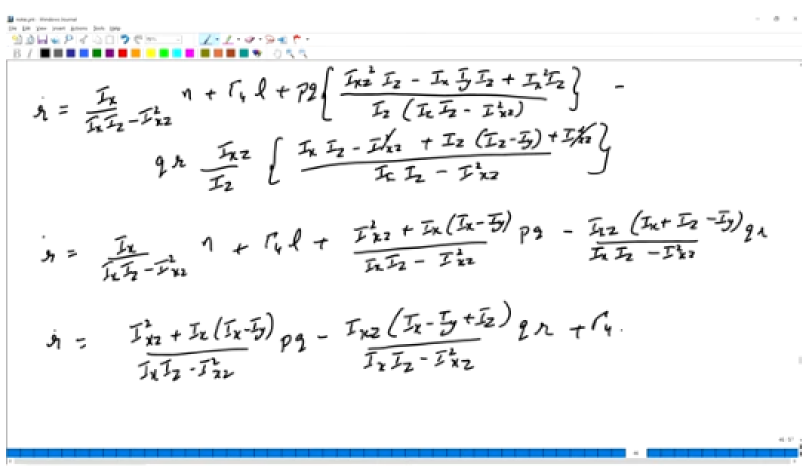

And we have a few more parameters left to define, that is, rolling and the yawing moment l and n are generally modeled as

$$l = \frac{1}{2}\rho V^2 SbC_l\,, \qquad n = \frac{1}{2}\rho V^2 SbC_n$$

$$C_l = C_{l0} + C_{l\beta}\beta + C_{lp}\frac{pb}{2V} + C_{lr}\frac{rb}{2V} + C_{l\delta a}\delta_a + C_{l\delta r}\delta_r$$

$$C_n = C_{n0} + C_{n\beta}\beta + C_{np}\frac{pb}{2V} + C_{nr}\frac{rb}{2V} + C_{n\delta a}\delta_a + C_{n\delta r}\delta_r$$

Now we know that, likewise, we modeled alpha similarly. Here, the side slip also will be changing over time. So, let me draw the free body diagram to model how to find the side slip. So, if we see the top view of any aircraft, consider this is an aircraft model. And if we have velocity in this direction, free stream velocity, the angle which it makes with the body x-axis is the side slip angle beta.

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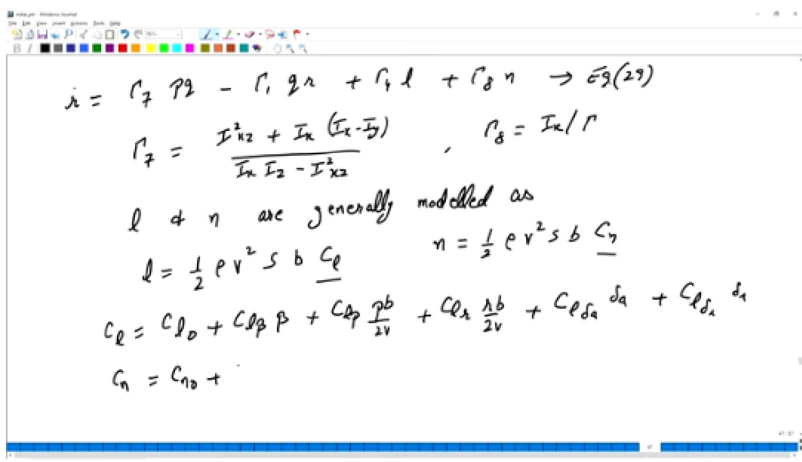

All right. And in this direction, we have small v, side slip, side force, sorry. So, to find the side slip angle, we can easily get it from

$$\sin\beta = \frac{P}{H} = \frac{v}{V_\infty}$$

Then the process for finding $\delta_{e,trim}$ and α_{trim} remains the same. So, now we are only left with the aircraft lateral parameters that we will input in the next lecture. So, let us stop here. Thank you.