Advanced Aircraft Control Systems With MATLAB / Simulink

Prof. Dipak K. Giri

Department of Aerospace Engineering

Indian Institute of Technology Kanpur

Lecture 49

Adaptive Back-stepping Control

Hello, everyone. In today's lecture, we'll be discussing adaptive backstepping control. In the last lecture, we covered adaptive sliding mode control. In this lecture, we will apply backstepping control to the same example problem we discussed in the last lecture. So here, first, let us start with the example problem on which we'll design the adaptive backstepping control. The plant we have

$$\dot{\theta} = q$$

$$\dot{q} = a_\theta \theta + a_q q + a_u u \quad \ldots Eq(1)$$

So here, we will also assume that a_θ and a_q are unknown parameters, and we can find or estimate them using adaptive control. So here, first, we'll implement the backstepping assuming all the plant parameters are known to us. So we first implement backstepping control, assuming all the plant parameters are known. If you do not remember the concept of designing backstepping control, please revisit the lectures on backstepping control so it will be easier for you to understand. So here, we're going to begin with the Lyapunov function first. So let us consider the Lyapunov function.

$$V_1 = \frac{1}{2}\theta^2$$

$$\dot{V}_1 = \theta\dot{\theta} = \theta q \quad \ldots Eq(1)$$

And also let's assume the virtual controller as $q = \lambda$ and

$$\dot{\theta} = q = \lambda$$

so we can write equation one yields

$$\dot{V}_1 = \theta\lambda \quad \ldots Eq(2)$$

here we'll consider a virtual control here actually you should say λ is the virtual controller virtual control let us consider

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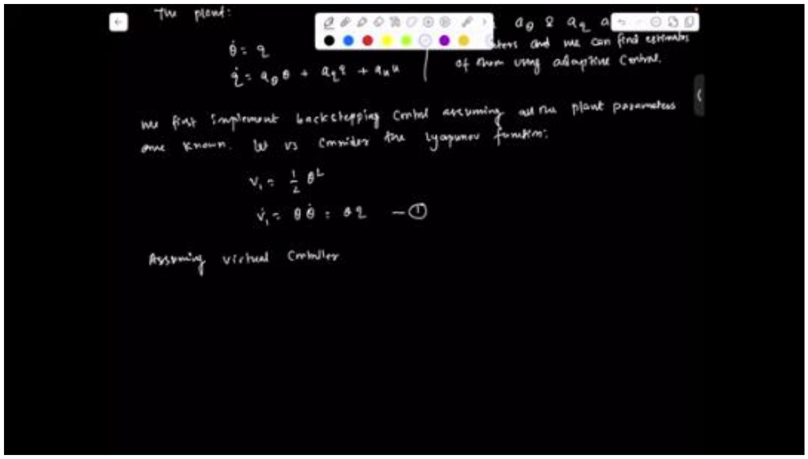

The virtual controller λ as

$$\lambda = -K_1\theta \quad \dots Eq(3)$$

Substituting equation 3 in equation 2, we have

$$\dot{V}_1 = -K_1\theta^2 \leq 0$$

Now, we can assign an error variable in between q and λ, the way we have done during the design of backstepping control. So, next, assign error as

$$e = q - \lambda = q + K_1\theta \quad \dots Eq(4)$$

$$q = e - K_1\theta$$

Hence

$$\dot{\theta} = q = e - K_1\theta \quad \dots Eq(5)$$

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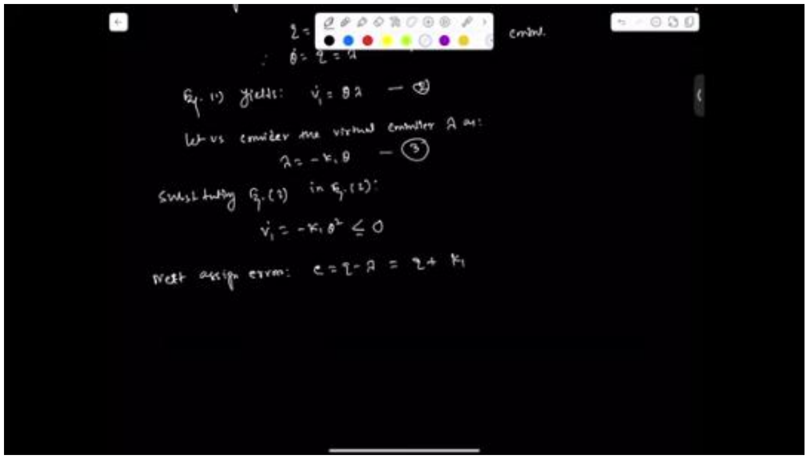

taking time derivative of equation 4 we have

$$\dot{e} = a_\theta \theta + a_q q + a_u u + K_1 e - K_1^2 \theta$$

From the above analysis, we have the equation in different coordinates,

$$\dot{\theta} = e - K_1 \theta$$

$$\dot{e} = a_\theta \theta + a_q q + a_u u + K_1 e - K_1^2 \theta \quad \dots Eq(6)$$

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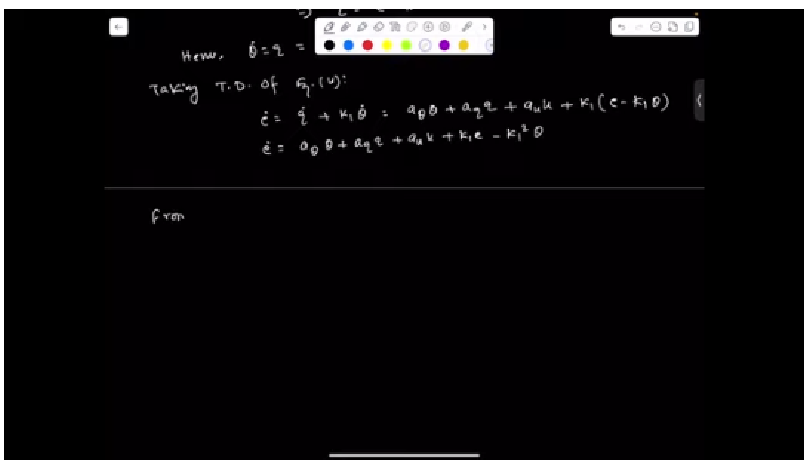

We need to redefine the Lyapunov function. So here, we need to take the e variable in the Lyapunov function. So for that, modify the Lyapunov function as

$$V_2 = \frac{1}{2}\theta^2 + \frac{1}{2}e^2$$

$$\dot{V}_2 = -K_1\theta^2 + e\left[\theta + a_\theta \theta + a_q q + a_u u + K_1 e - K_1^2 \theta\right] \quad \dots Eq(7)$$

So now we have to choose this u in this expression, in such a way that $\dot{V}_2$ is negative definite. So for that,

$$u = \frac{1}{a_u}\left[-\theta - a_\theta\theta - a_q q - K_1 e - K_2 e + K_1^2\theta\right] \quad \ldots Eq(8)$$

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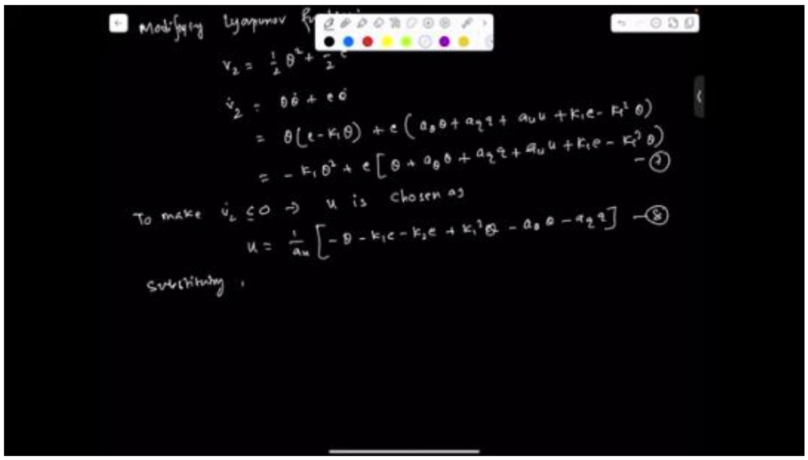

So okay, substituting Equation 8 In equation 7, we have

$$\dot{V}_2 = -K_1\theta^2 - K_2 e^2 \le 0$$

So here, the most important part is our control, what you have designed here in this control u in equation eight. Here, we have assumed that a_θ and a_q are exactly known. But in real time, it is a problem to find the exact value of these stability derivatives. So now we'll use the adaptive concept to come up with the adaptive laws for the a theta and aq. For that, as you have done in the sliding adaptive sliding mode control, assuming errors as

$$e_\theta = a_\theta - \hat{a}_\theta$$

$$e_q = a_q - \hat{a}_q$$

Okay, so you're taking the time derivative of these errors.

$$\dot{e}_\theta = -\dot{\hat{a}}_\theta$$

$$\dot{e}_q = -\dot{\hat{a}}_q$$

So we can follow the same steps. We have to modify the Lyapunov function based on these error variables, and we can come up with the adaptive laws, as we have done in the sliding mode control adaptive sliding mode control. So now we modify the function and penalize this parameter estimation errors. So for this

$$V_3 = \frac{1}{2}\theta^2 + \frac{1}{2}e^2 + \frac{1}{2\gamma_1}e_\theta^2 + \frac{1}{2\gamma_2}e_q^2$$

Now, if we take the time derivative,

$$\dot{V}_3 = -K_1\theta^2 + e\left[\theta + a_\theta \theta + a_q q + a_u u \; + K_1 e - K_1^2\theta\right] - \frac{1}{\gamma_1}e_\theta \dot{\hat{a}}_\theta + \frac{1}{\gamma_2}e_q \dot{\hat{a}}_q \quad \dots Eq(9)$$

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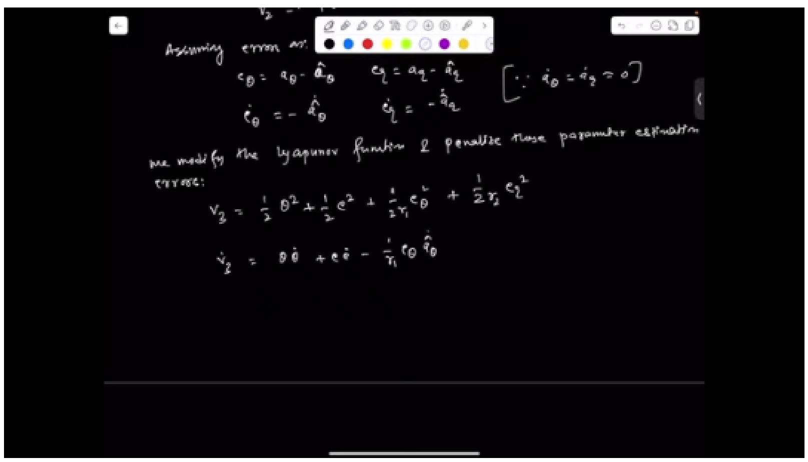

Now, substituting equation 8 in equation 9 in place of control u. So, we have

$$\dot{V}_3 = -K_1\theta^2 - K_2 e^2 + e_\theta\left[e\theta - \frac{1}{\gamma_1}\dot{\hat{a}}_\theta\right] + e_q\left[eq - \frac{1}{\gamma_2}\dot{\hat{a}}_q\right] \quad \dots Eq(10)$$

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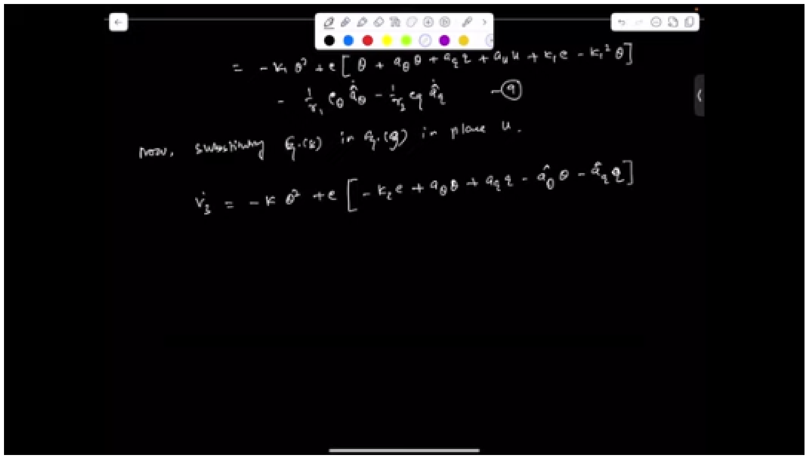

Now, we can choose the adaptive laws in this expression

$$\dot{\hat{a}}_\theta = \gamma_1 e\theta$$

$$\dot{\hat{a}}_q = \gamma_2 eq$$

Now Eq. (10) yields

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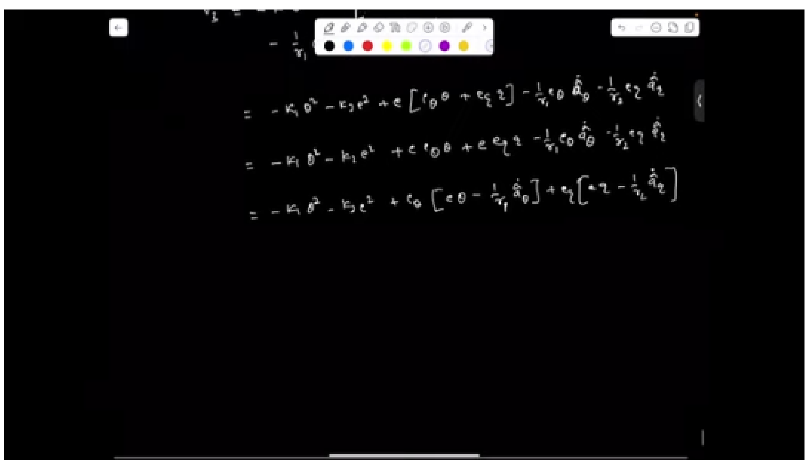

$$\dot{V}_3 = -K_1\theta^2 - K_2e^2 \leq 0$$

This is the stability proof. So, with the proper choice of K_1 , K_2 , γ_1 , γ_2, the plant can be made stable. So, also from this analysis, it can be—we can write the adaptation laws approach zero as e goes to zero as t tends to infinity. Yeah, so this is how we can design the adaptive backstepping control for the nonlinear system or linear system. If you have some very good novel system and if you want to design this kind of control, you can come up with a very good research output from the work you are doing. So, we are almost close to the end of this nonlinear control part. And so, we have covered almost this adaptive nonlinear Lyapunov-based control, feedback linearization-based control, backstepping control, sliding-mode control, adaptive control. So I hope you have a strong base now to design nonlinear control for your system, what you're doing. From next lecture onwards, we'll have the full MATLAB part. That is around seven to eight lectures. And I hope you'll enjoy that part also. So let's wind up here.

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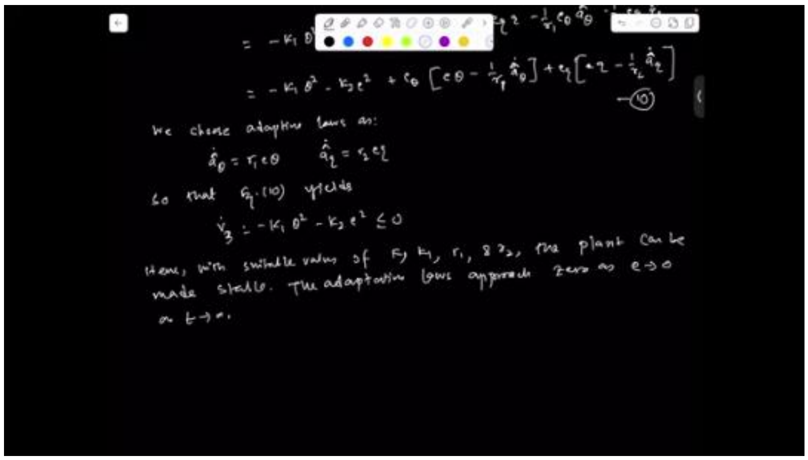

And if you want to write the code for this control, please refer the previous MATLAB code that we have done. So you can follow the same step. to come up the MATLAB code to see the results of this control. And this is I'm leaving to the learners. Please try to simulate this control in your MATLAB for this particular system.

Thank you.