Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 48

Adaptive Sliding Mode Control

Hello everyone. In today's lecture, we will be starting adaptive control. So here, we are going to discuss how we can make the sliding mode adaptive to handle uncertainties in the system or parameter variations in the control. So before we proceed to the main design concept, First, let me introduce some notes on how it is useful for practical applications. So let me write some notes. Adaptive control is used to handle systems systems with uncertainties or parameters that can vary over time. In our previous example, in the aircraft problem, adaptive control can be helpful because the aircraft's dynamics change due to varying aerodynamic

aerodynamic conditions, payload weight, or fuel consumption. The adaptive control adaptive control approach adjusts the control parameters in real time to ensure to ensure stability and preserve performance across the entire flight envelope. So this is a very important concept. to handle uncertainties and parameter variations in the system. These variations can be included in the control, and that control can handle these unwanted aspects or uncertainties in the system. We'll take the same example as before.

And we'll see how we can design the adaptive control for that system. So here, we'll be using sliding mode. The sliding mode control, which we have done for the aircraft, we'll take that example and proceed. So here, the system dynamics, we are going to consider the system model dynamics. The same dynamics, the longitudinal motion of the linearized model of the longitudinal motion dynamics, which is

$$\dot{\theta} = q$$

$$\dot{q} = a_\theta \theta + a_q q + a_u u \quad \dots Eq(1)$$

So here, first, let us design the control U using sliding mode control. So let us find. The error

$$e = \theta - \theta_d$$

$$\dot{e} = \dot{\theta} - \dot{\theta}_d = q \qquad [\dot{\theta}_d = 0]$$

And based on this error variable, let us define the sliding surface. Defining the sliding surface as

$$S = Ke + \dot{e} \dots Eq(2)$$

Now, we will take the time derivative of this surface differentiating equation 2. We have

$$\dot{S} = a_\theta \theta + a_q q + a_u u + Kq \quad \dots Eq(3)$$

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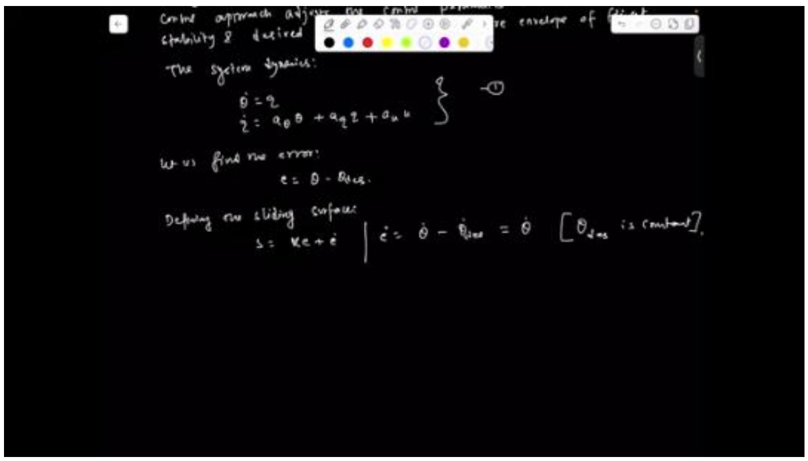

So here we can define the Lyapunov function using variable S. So defining the Lyapunov function. function as

$$V = \frac{1}{2}S^2$$

$$\dot{V} = S[a_\theta \theta + a_q q + a_u u + Kq] \quad \dots Eq(4)$$

choose u in equation 4 such that such that $\dot{V}$ is negative semi-definite. So from this, we can write

$$u = \frac{1}{a_u}[-K_1 S - Kq - a_\theta \theta - a_q q] \quad \dots Eq(5)$$

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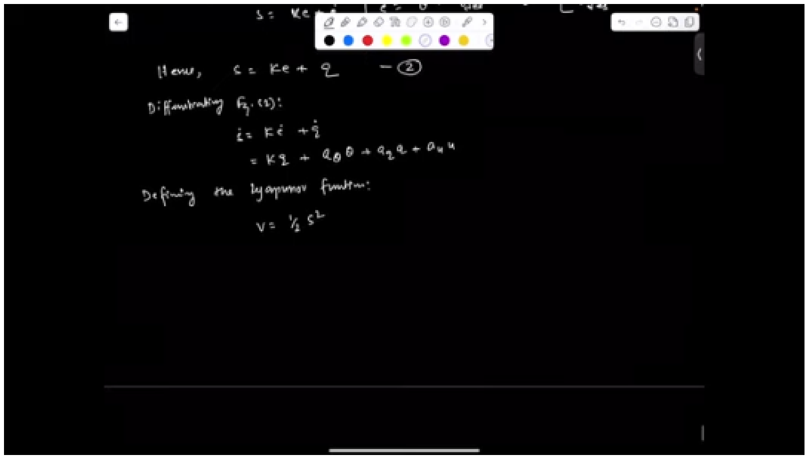

Okay. So this is, I hope this is clear. This part, this part is actually making the system—sorry, the V dot is negative semi-definite, right? This part we have done during the study of the Lyapunov function. Now, the most important part is here: how the adaptive control comes into the picture. So here, this a_θ a_q, a_u—these are basically stability derivatives.

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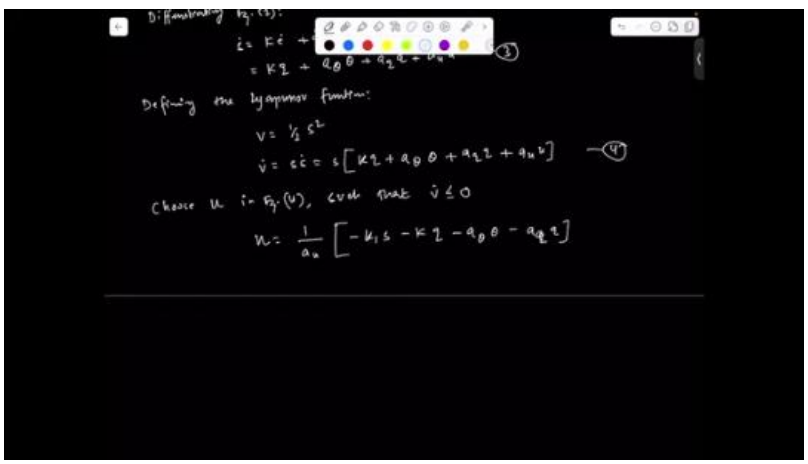

But sometimes it is difficult to find the exact values of these. So, for this, that motivation—let me write some notes here—why these are a very important part. Note: assume that the plant parameters, So, here, plant parameters—basically a_θ a_q,—are unknown. We define estimates: $\hat{a}_\theta, \hat{a}_q$—these estimates—and update them using an adaptive law based on the Lyapunov function. Lyapunov stability. Okay, for this, we first define the error between them. So, we can say these are the actual, and these are the estimates. Okay, so we will find the error between these and take them in the Lyapunov function. Through the Lyapunov stability, we can find the adaptation laws, which can be used to design the robust control.

So here, we first define the errors. Okay. So you can write,

$$e_\theta = a_\theta - \hat{a}_\theta$$

$$e_q = a_q - \hat{a}_q$$

Okay, so you're taking the time derivative of these errors.

$$\dot{e}_\theta = -\dot{\hat{a}}_\theta$$

$$\dot{e}_q = -\dot{\hat{a}}_q$$

Here, a_θ and a_q are constants. So based on this, we have this minus sign. And so here we can write. So now what you're going to do is, okay, e_θ. Okay, what you're going to do is, so this, basically parameter estimation errors, okay, which is basically e_θ and e_q, okay. So now we will modify the Lyapunov function. Now modifying the Lyapunov function and try to analyze these parameter estimation errors. So, what we are going to do is we will modify our Lyapunov function,

$$V = \frac{1}{2}S^2 + \frac{1}{2\gamma_1}e_\theta^2 + \frac{1}{2\gamma_2}e_q^2 \quad \ldots Eq(6)$$

So, here γ_1 and $\gamma_1$are actually positive adaptation gains. Differentiating equation six, we have

$$\dot{V} = S\dot{S} + \frac{1}{\gamma_1}e_\theta\dot{e}_\theta + \frac{1}{\gamma_2}e_q\dot{e}_q \quad \ldots Eq(7)$$

So from this, already we have

$$\dot{S} = a_\theta\theta + a_q q + a_u u \ + Kq \quad \ldots Eq(8)$$

And here now what you're gonna do is in the control we will replace a_θ and a_q by the estimates estimated parameter $\hat{a}_\theta$ and $\hat{a}_q$. So in equation five, to make the system adaptive because in control we will be using adaptive law u and we will find the adapting adaptation laws.

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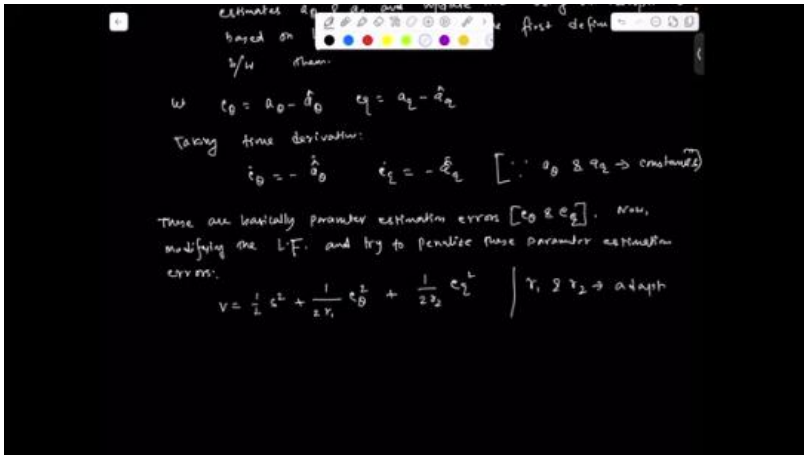

$$a_u u = -K_1 S - Kq - \hat{a}_\theta \theta - \hat{a}_q q \quad \ldots Eq(9)$$

now substituting Equation 9 in equation 8, we are having

$$\dot{S} = -K_1 S + e_\theta \theta + e_q q \quad \ldots Eq(10)$$

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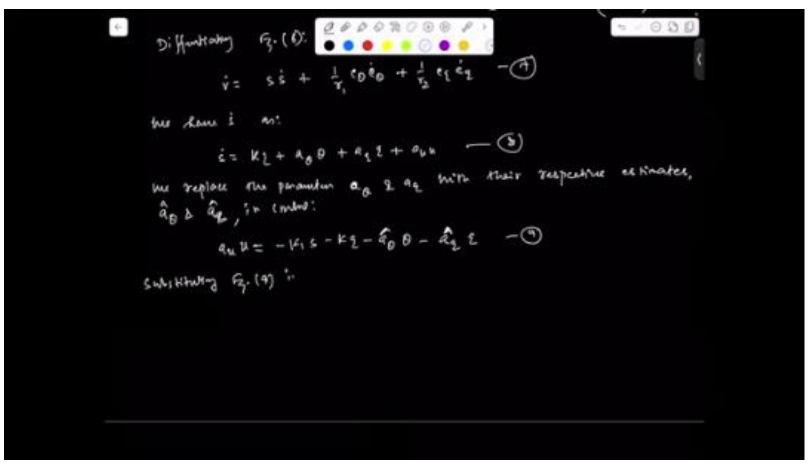

Now. So, this is $\dot{S}$ we found here. So, that we will be using while we take the final expression for $\dot{V}$. So, in equation 7, this part we have done.

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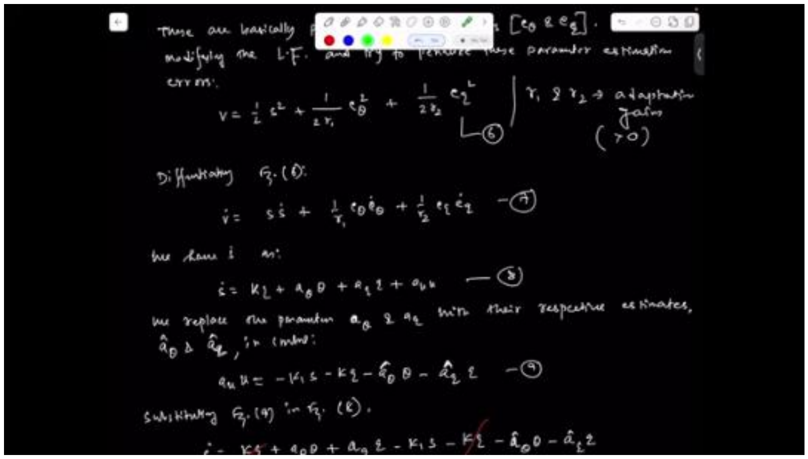

Now, we will go to the second and third term. So, for this, we have

$$\frac{1}{\gamma_1} e_\theta \dot{e}_\theta = \frac{1}{\gamma_1} e_\theta \left(-\dot{\hat{a}}_\theta\right) \quad \ldots Eq(11)$$

$$\frac{1}{\gamma_2} e_q \dot{e}_q = \frac{1}{\gamma_2} e_q \left(-\dot{\hat{a}}_q\right) \quad \ldots Eq(12)$$

Substituting equation 10, 11, and 12 in equation 7, we have

$$\dot{V} = -K_1 S^2 + e_\theta \left[\theta S - \frac{1}{\gamma_1} \dot{\hat{a}}_\theta\right] + e_q \left[q S - \frac{1}{\gamma_2} \dot{\hat{a}}_q\right]$$

so now we have I have some some simplified form now anyways this term always positive definite I mean the whole term is like a definite so now we'll manipulate here in such a way that $\dot{V}$ is negative definite semi-definite so here to make second and third term then is we design we have the adaptation laws as

$$\dot{\hat{a}}_\theta = \gamma_1 \theta S \quad \ldots Eq(13)$$

$$\dot{\hat{a}}_q = \gamma_2 q S \quad \ldots Eq(14)$$

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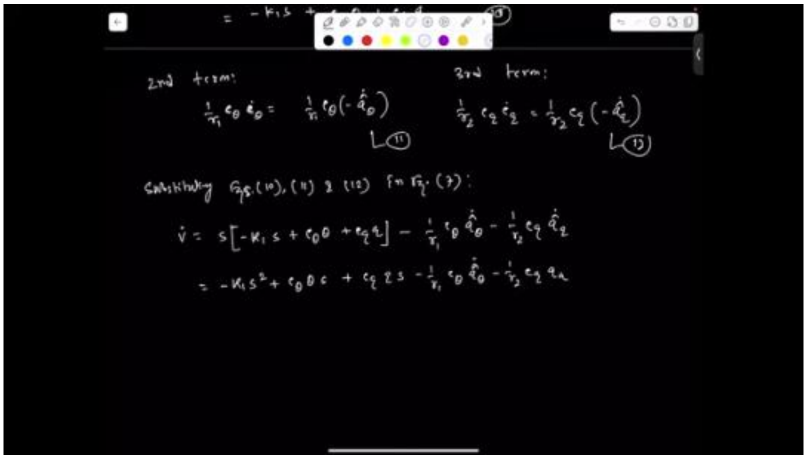

Substituting equations 14 and 15 in equation 13, we have

$$\dot{V} \leq -K_1 S^2$$

So we have done this derivative proof. And in this adaptive loss, this is the adaptive loss for this proof, for this controller, what we have done before. And now we'll have the MATLAB code, and before that, we need to have some analysis. So here, by the proper choice of $K, K_1, \gamma_1, \gamma_2$, the plant can be made stable. So this is how we can stabilize the system or we can have the desired tracking performance using the adaptive controls what you have got here. So now we'll go for the MATLAB simulation for this controller what you have done here. The values are taken as the given data. Given data are

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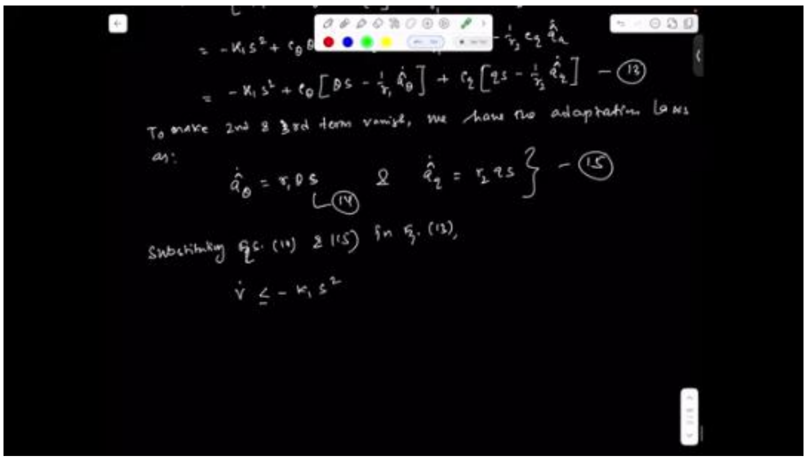

$$\theta_d = 5^{\circ}, \quad \theta_0 = 0.3\, rad, \quad q_0 = 0$$

$$a_\theta = -0.8, \quad a_q = -0.3, \quad a_u = 0.5$$

$$K = 2.5, \ \ K_1 = 2.5, \ \ \gamma_1 = 0.1, \quad \gamma_2 = 0.1.$$

Okay. So these are the given data we have. And now we'll proceed with designing the control. So we'll see the MATLAB simulation for this particular control. This is the MATLAB code. These are the initial values, timespan 10 seconds. This is the ODE45. These are the data, control, and error variables, as we have designed in the analysis.

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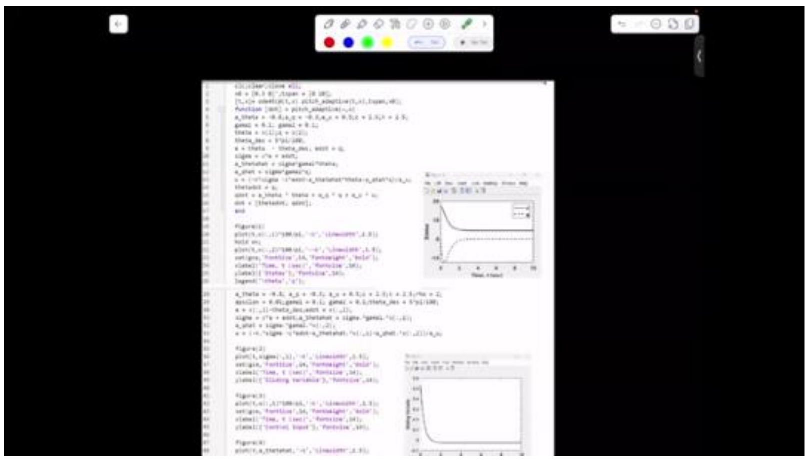

Please cross-check line by line what you are doing in the analysis and what data we are using in the code. This is the figure, and this is theta and epsilon for the control purpose. Different values of the parameters are also given here. This is the figure plot. We can see that the adaptive law control is more robust.

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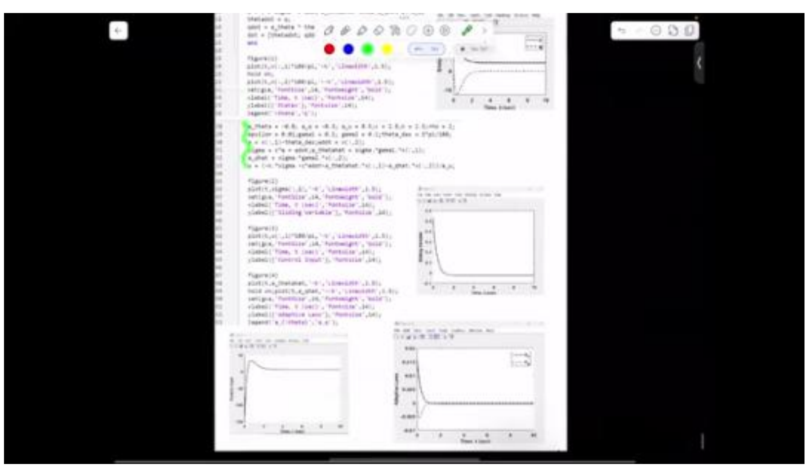

And strong compared to the control we have done. Here, if you notice, this is the control input. There is no chattering—nothing is there. And this is the state variable; it is converging to the desired value very fast. So, this is how we can design these adaptive controls in MATLAB. So, I'm not going to detail it line by line in the code. So, try to do it in your MATLAB window and simulate whether you are getting this result or not. And if you want to design the same control for other systems or the system you are currently handling, you can test it and see the result.

And you can even check the system robustness for different disturbance signals. Yeah, you can verify your control efficacy—what you are doing. So, let's stop it here. We'll continue with another slide on adaptive control techniques in the next lecture. Thank you.