Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 47

Quasi-Sliding Mode control and Backstepping-Sliding Mode Control for Longitudinal Dynamics of Aircraft

Hello everyone. In today's lecture, we will be discussing how to come up with the quasi-sliding mode control. Because of this technique, we can reduce the chattering in the control. And also, we will combine backstepping with sliding mode control. The backstepping part we have already done, and now we will combine it with the sliding mode control. We will discuss in detail the whole concept of backstepping sliding mode control and how it can be applied to the aircraft system. Let's start by setting up the problems on which we'll be designing these controls. Let us consider the linearized model of an aircraft. Linearized. It is written for simplicity. The linearized model of the longitudinal motion of an aircraft. We can write

$$\dot{\theta} = q$$

$$\dot{q} = a_{\theta}\theta + a_q q + a_u u \quad \ldots Eq(1)$$

Here, a_{θ} , a_q, and a_u are the stability derivatives. Let me define these parameters: θ is the pitch angle, and we will discuss in detail how you can design this quasi-sliding mode control for this system. q is the pitch rate, and u is the elevator deflection. so let me go through the details of these derivative parameters because they are very important for understanding the dynamics here. In the context of aircraft-pitch dynamics, θ basically represents a term associated with how angles affect the moment equation. It is basically, we can also write, which is often linked to the static stability derivatives, static $C_{m\alpha}$. Here, $C_{m\alpha}$ represents how the angle of attack affects the pitching moment, and here the expression we can write

$$a_{\theta} = \frac{q_{\infty} S \bar{c} C_{m\alpha}}{I_y}$$

So the terminology is already explained multiple times in different examples. So in this term, this θ helps maintain stability when disturbed from the steady flight path. Yeah.

Similarly, we will go to the next term, a_q. So, I hope it is clear how θ is significant in this longitudinal motion. Now, a_q is the pitch damping derivative, representing how pitch rate affects the pitching moment.

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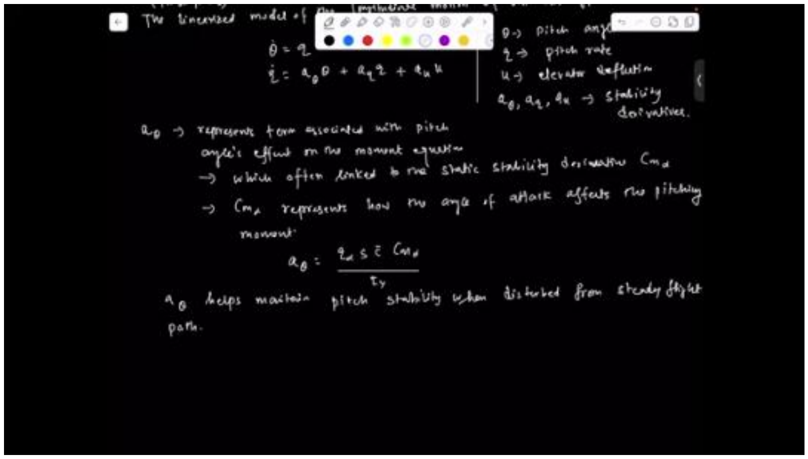

So, we can write

$$a_q = \frac{q_\infty S \bar{c} C_{mq}}{I_y}$$

$$a_u = \frac{q_\infty S \bar{c} C_{m\delta e}}{I_y}$$

Similarly, $C_{m\delta e}$, that is, which function? It is the control effectiveness derivative. We can take it as

$$C_{m\delta e} = \frac{\partial C_m}{\partial \delta_e}$$

So here, the value of $C_{m\delta e}$ means that small changes in δ_e produce larger changes in the pitching moment, and we can write: small change in δ_e produces larger changes in pitching moment, giving the elevator greater authority. over pitch control. Okay, this is desirable for effective pitch response and stability control. Now, for this example, what you are taking here, we consider for the above. Let me define the equation. Equation 1. For equation 1, we consider the values as

$$a_\theta = -0.8 \qquad a_q = -0.3 \qquad a_u = 0.5$$

Here, because as you have mentioned, the sliding mode control is sometimes called robust control, so here we are going to add some disturbance to equation one, to the dynamics. So here, let us. add a disturbance term dt to the pitch dynamics.

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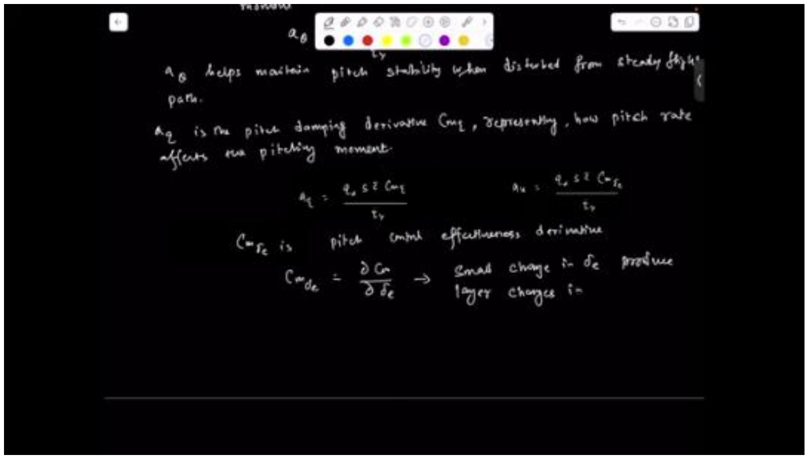

$$d(t) = 0.1 \sin(0.5t)$$

So, 0.1 is the magnitude of the oscillation, and 0.5 is the frequency. And also, we are considering the initial condition for the state. Consider the initial conditions of the states. So here, we are having two states, θ and q. So $\theta_0 = 0.3\, rad$ and $q_0 = 0$. So we have to design the control U such that θ and q are controlled or stabilized. So for this purpose, we need to choose the sliding surface. So let us choose the sliding surface.

$$S = q + K\theta \quad \dots Eq(2)$$

Where K>0.

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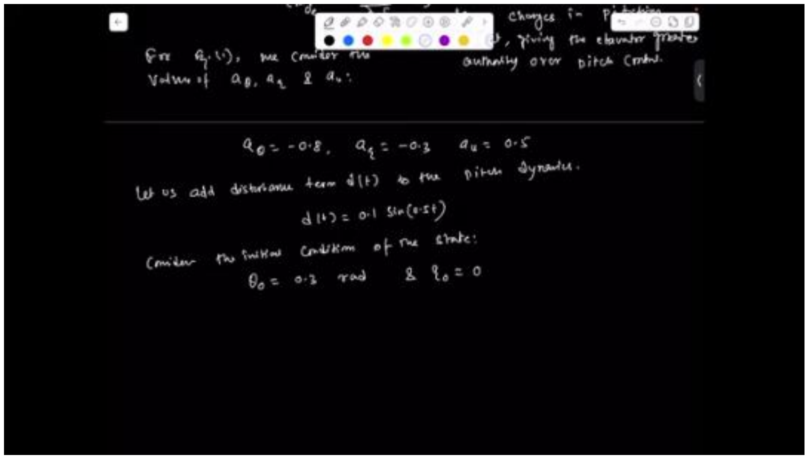

Okay, so now I'm not going to detail how we can design the control because what we're going to do is take the time derivative of this equation two. And from this, we will substitute in $\dot{q}$. We can substitute this equation. And in $\dot{\theta}$, we can substitute this equation and we can find u. Right. From that equation, in place of $\dot{S}$, we can substitute the sigmoid function to make a chattering-free control. So the same steps you followed last lecture. So, after having all these mathematical steps and analysis, the final control is found to be

$$u = \frac{1}{a_u}\left[-Kq - \rho\frac{S}{|S| + \epsilon} - a_\theta\theta - a_q q\right]$$

so this is the overall control that can be applied to the dynamics defined by equation one, which can stabilize the system. After some iterations, the values of the control gain parameters stabilize after a few iterations. The values are

$$K = 1.5 \qquad \rho = 2 \qquad \epsilon = 0.01$$

So, if you remember in the earlier lecture on sliding mode, this is basically the sigmoid function we have used, right? Why? Because it reduces the chatter in control. In the last lecture, we had the signum function in control because we have the switching control and equivalent control. We have the signum function. But that control generally causes chattering in the system. So, to reduce the chattering, we introduced the sigmoid function. So, I request the learners to please do the mathematical analysis on how you can find this U in this particular structure. Okay.

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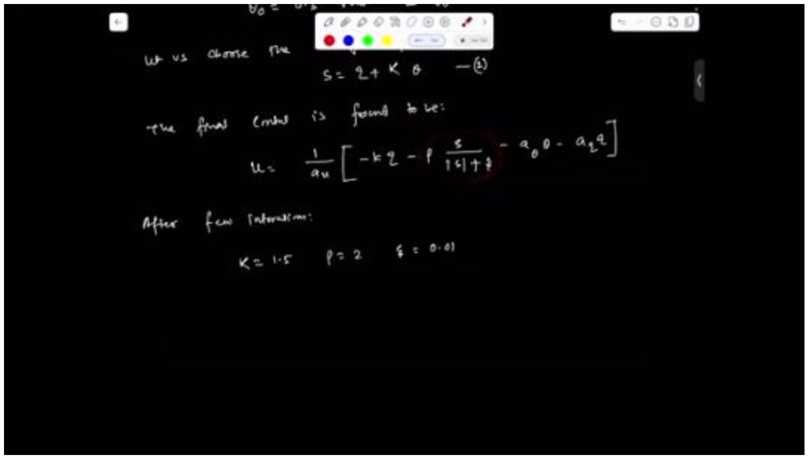

So now we'll move to the MATLAB code on how we can stabilize the system and make it chattering-free. This is the MATLAB code for this particular example. So, if you notice here, these are the initial values. This is the time period for the simulation. This is the ODE45 solver.

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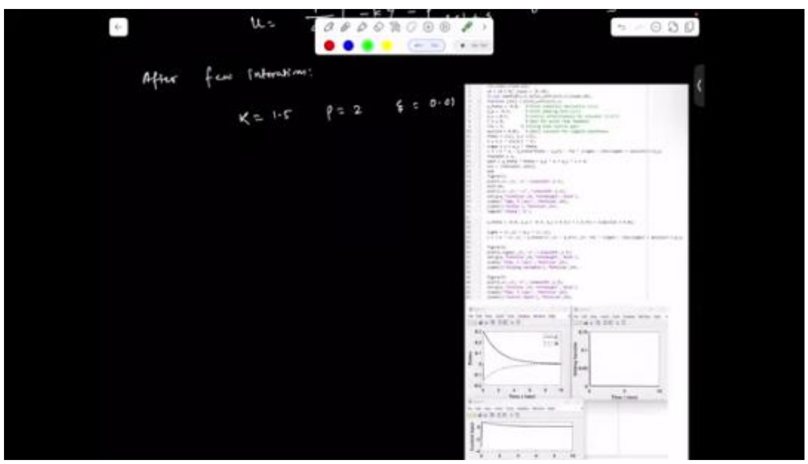

And we have the function here. And we have the figures here. We have this θ. These values—whatever values we have considered in the analysis—we have taken here. This is basically your sliding surface and your control. This is the control you have defined here, okay? And if you notice here, the control input you see is chattering-free. There are no oscillations or sustained oscillations in the control. And also the response of the states. This is the response for q. This is for θ. And you can see that over time, it is finally stabilized with the system. And this is the sliding variable. What we have chosen here. So this is how we can design the control algorithm for these longitudinal motions. And if you have nonlinear longitudinal dynamics, you can push through the same steps. You have to find the error dynamics. Using the error, you can find the sliding surface, take the time derivative of the sliding surface, substitute the dynamics, and find the control. From that. OK, so now we are going to a new topic, which is backstepping sliding mode control. OK, so I can use this. Backstepping. Sliding mode. This is a very important concept in nonlinear control synthesis. Here we will define the surface as well and also use that surface for the backstepping control concepts. As of now, we have covered the backstepping approach and sliding mode approach. Now we are going to combine them together to come up with robust control. So here we are going to consider the same dynamics we have taken before in this code lecture. So let us consider the dynamics. Linearized model of the longitudinal motions.

$$\dot{\theta} = q$$

$$\dot{q} = a_\theta \theta + a_q q + a_u u + d$$

So here d is the disturbance. Also, we have taken the previous example with the disturbance. So if you notice here, this is the disturbance in the MATLAB code. This is the disturbance. Right. So now we'll go step by step on how we can design the control here. So step one. Let us find the error. The error defined as. So if you notice here carefully, we have two equations. So and if you notice the system is in recursive form.

q is going to control θ and Q is controlled by u, right? First, we'll take the first dynamics, $\dot{\theta} = q$ .As you have done in the backstepping control, so here error can be defined as

$$e = \theta - \theta_d \quad \dots Eq(2)$$

So here θ_d is the desired trajectory for pitch angle. It is for the time being, let us assume this desired trajectory to be the constant value, some-constant. So we can better, if the desired value is constant, we can sometime we can call it tracking control, right? And differentiating equation two we get

$$\dot{e} = \dot{\theta} - \dot{\theta}_d = q \qquad \left[\dot{\theta}_d = 0\right]$$

now let's select the function So we are trying to find the control for θ dot equal to q, some virtual control you have to design for here. So as you have done in the backstepping control approach, so here let us consider the Lyapunov function. function as

$$V_1 = \frac{1}{2} e^2$$

$$\dot{V}_1 = e\dot{\theta} = eq \quad \dots Eq(3)$$

okay now let us design the sliding surface in terms of u because we have to change the state variable from actual to error form.

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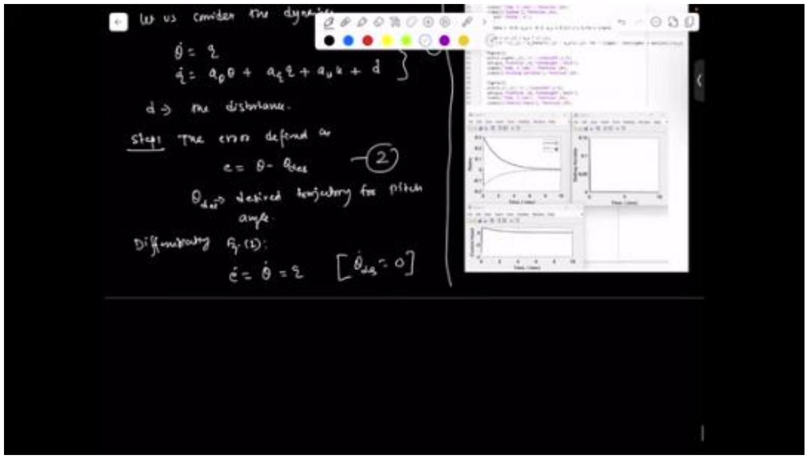

So, actual and desired to error variable. So, we can define the sliding surface in terms of error. And here, let us now design sliding surfaces as

$$S = \dot{e} + K_1 e \quad \dots Eq(*)$$

$$\dot{e} = S - K_1 e \quad \dots Eq(4)$$

Where $K_1 > 0$, So if you notice at S equal to zero, this sliding surface is stable surface. It is asymptotically stable. So it is going to equilibrium point zero because the equilibrium condition for this dynamics is zero. so substituting Eq.(4) in Eq.(3)

$$\dot{V}_1 = -K_1 e^2 + es$$

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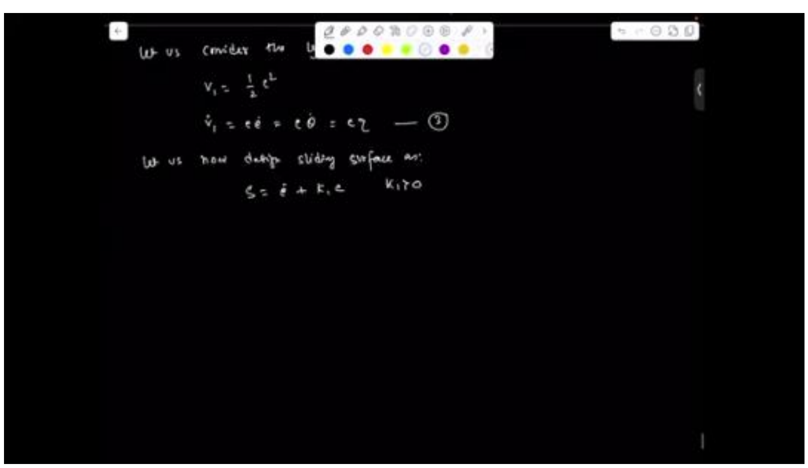

So it is a very nice form that if S is equal to 0, we can say $\dot{V}_1$ is negative definite. So we can say If you forgot the step in backstepping, please revisit it again to understand this thing, because we are simply following the same steps you have done in the backstepping design process. So here, if S equals zero, we get

$$\dot{V}_1 = -K_1 e^2 \leq 0$$

which is negative semi-definite. Hence, We can write here, hence, for the system to be stable, we need to ensure S equals 0. Now, what we are going to do is take the second dynamics. Second dynamics in the picture, for that, we need to redefine the sliding surface function. So, ensure this. We can write here. We do so by adding S in the Lyapunov function. So, we can write

$$V_2 = V_1 + \frac{1}{2} S^2$$

Now, if we take the time derivative of this expression, we can write

$$\dot{V}_2 = -K_1 e^2 + S\left[e + \dot{S}\right] \quad \ldots Eq(5)$$

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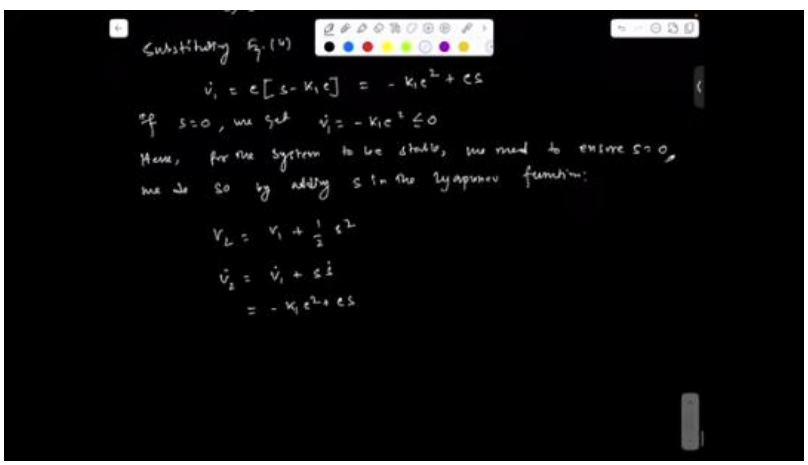

Now, we will take the sliding surface to design controls. So, the sliding surface is this. Now, taking the time derivative of Eq.(*) , we get

$$\dot{S} = a_\theta \theta + a_q q + a_u u + d + K_1 \dot{e} \quad \ldots Eq(6)$$

Substituting Eq.(6) in Eq.(5), we get

$$\dot{V}_2 = -K_1 e^2 + S\left[e + a_\theta \theta + a_q q + a_u u + d + K_1 \dot{e}\right] \quad \ldots Eq(7)$$

And if you choose, now we have to choose u in such a way that $\dot{V}_2$ is negative semi definite.

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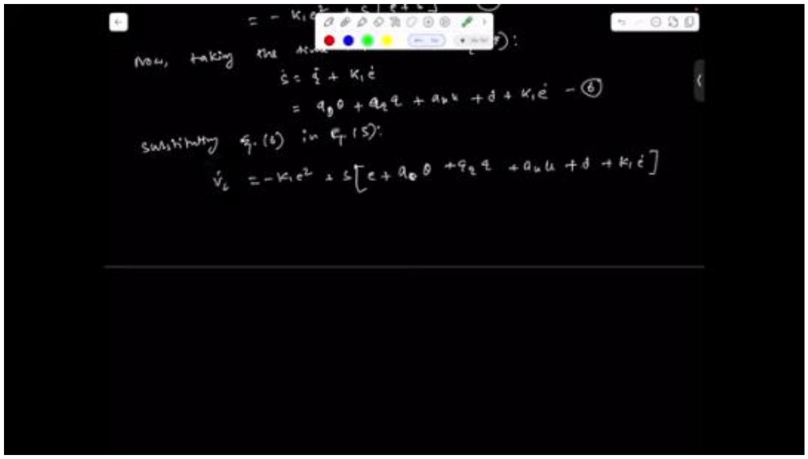

$$u = \frac{1}{a_u}\left[-K_2S - e - a_\theta\theta - a_q q - K_1\dot{e} - \rho\frac{S}{|S| + \epsilon}\right] \quad \ldots Eq(8)$$

Now, substituting equation 8 in equation 7, we have

$$\dot{V}_2 \leq -K_1e^2 - K_2S^2 \leq 0$$

This is very simple; you can do it by yourself. This is not a difficult job.

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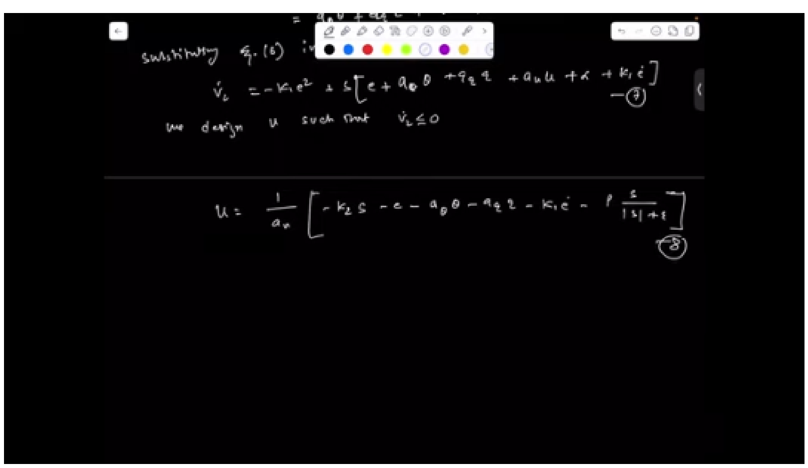

So, we can say e tends to 0 and S tends to 0 as t tends to 0. So, this is our control equation 8. Now, we'll see in simulation how the states are being controlled. So, this is the MATLAB simulation for this.

This control, so here, we can see that these are the codes. You can please follow these codes to try in your MATLAB command window and see how this control helps stabilize the system. Here, this is basically the same function you have taken before, so only the control part will change. And other than that, the things are almost the same. So from this plot, it is clear that the pitch angle—this is the pitch angle, θ—actually converges to 5 degrees. Right. So here we have the desired phi; θ is here, 5 degrees.

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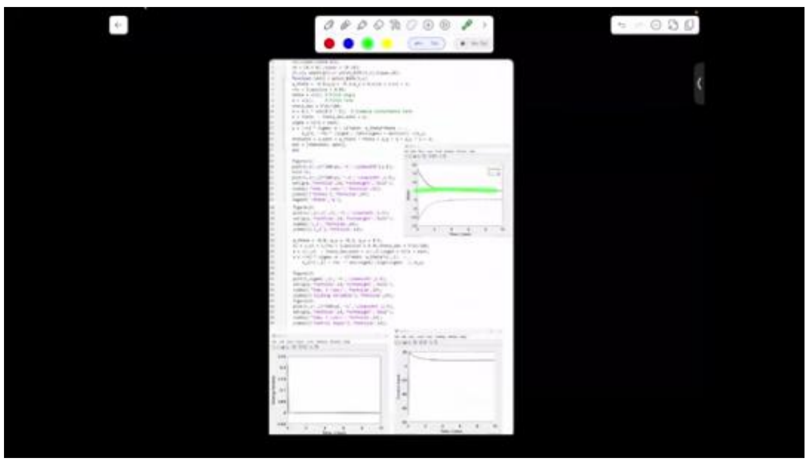

So you can see that the pitch angle converges to 5 degrees. And the states are approximately converged to the sliding surface as S goes to 0, right? We can also see that this control is highly robust because, in the presence of disturbance, the system stabilizes and reaches the desired values in finite time. This is how we can design sliding mode control for linear or nonlinear systems. If you need to design the same control techniques—sliding mode control or combined sliding mode and backstepping for your system—you can use this MATLAB code as a reference. You can come up with your own autopilot and control design algorithms for different subsystems or systems.

Whether it is a robotic system, aircraft, or missile, you can easily do it. So let's stop it here. From the next lecture onwards, we'll cover a new topic: adaptive control and how we can design adaptive control for the aircraft system. Then we'll wrap up this control part. Thank you.