Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 46

Sliding Mode Control for Longitudinal Dynamics of Aircraft

Finite-time sliding mode control for the longitudinal aircraft motion. In today's lecture, we'll be covering the mathematical formulation of the control, and also we'll have the MATLAB code to simulate the control algorithm we'll be designing for the dynamics. So first, let me define the dynamic model for which we'll be designing the control. The longitudinal motion of an aircraft can be described as

$$\ddot{\theta}(t) = f\left(\theta, \dot{\theta}, t\right) + g(\theta, t)u(t) + d(t) \;\; \dots Eq(1)$$

So we can write this equation in complete form for the longitudinal motion. Here, basically, we are controlling only the pitch angle(t)he aircraft, here $\theta(t)$ is the pitch angle to be controlled, and $\dot{\theta}$ (t) is the pitch rate. Here, we are applying only one control input, which is the elevator deflection control input. This is basically the command to the elevator deflection. $f(\theta, \dot{\theta}, t)$ is the nonlinear function, or you can say known nonlinear dynamics, and $g(\theta, t)$ is the control coefficient. Gain, we can say. For this particular case, we can assume $g(\theta, t)$ to be 1 for simplicity in control design. But you can take the complete full model of $g(\theta, t)$, but in this lecture, we are taking this to be one to simplify the control synthesis. Also, the disturbance d(t) is assumed to be an unknown bounded disturbance $|d(t)| \leq D$. So capital D is some maximum value. And our control objective is to design the control u of t such that $\theta(t)$ tracks $\theta_d(t)$. So this is our control objective. Let us solve this problem using finite-time sliding mode control. So here, θ is the actual, we can say, the actual state.

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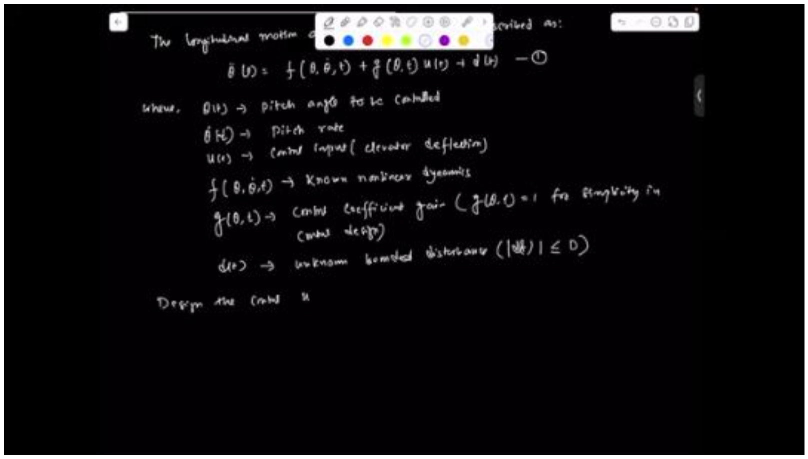

And θ desired t is the desired state. So our main aim is to design the control E(t), which helps θ to track the desired θ. So from these states, we can find the error. So let us define the tracking error as

$$e(t) = \theta(t) - \theta_d(t)$$

So our goal is $e(t)$ goes to zero as t tends to infinity. So this is our control objective. Now we can define the sliding surface based on the error function because we are trying to define the dynamics in error form. And here E(t) can be the state variable. So we are defining the sliding surface in terms of E(t). Okay. So the sliding surface, the sliding surface is chosen as

$$S(t) = \dot{e}(t) + \lambda e(t), \quad \lambda > 0 \quad \ldots Eq(2)$$

so if you notice this sliding surface it is very stable function so here if you consider e of zero is a some constant k positive value and if S(t) is 0. We can write u dot t plus lambda e(t) equal to 0. So, $e(t) = Ke^{-\lambda t}$. So, this is basically a stable function.

This is how we define the sliding surface what we have discussed in the earlier lecture. and here and lambda is is greater than zero which is basically a design parameter and which determines the convergence rate(t) of the trajectory so if you notice here in this case in this case so if you increase the value of lambda we can reduce to the equilibrium very fast right so now we'll define the dynamics in term S(t) the sliding variable And defining, taking derivative, taking time derivative of equation 2, we can write

$$\dot{S}(t) = \ddot{e}(t) + \lambda \dot{e}(t)$$

$$= f(\theta, \dot{\theta}, t) + u(t) + d(t) - \ddot{\theta}_d(t) + \lambda \dot{e}(t) \quad \ldots . Eq(3)$$

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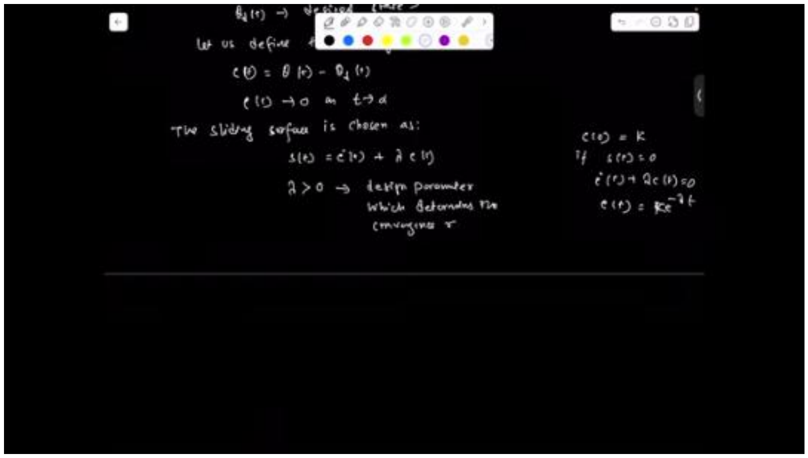

So now we'll design the finite-time control for this system. So now, as we have discussed in the earlier lecture, we have to design this control, u of t, and if you remember, we had two components of u of t: one is the sliding control, another is the uh—sorry—switching control, another is the equivalent control. So, switching control we define by the signal function, and equivalent control we can define by the functions involved in the dynamics. So, if you remember from the last lecture, we can write the control. u of t can be defined as

$$u(t) = u_{eq}(t) + u_{sw}(t)$$

So, set $\dot{S}(t) = 0$ why you are taking s dot equal to 0, that is also we have defined. In the absence of disturbance, this is basically a simply stable function, right? This is already defined before. So we can write absence of $d(t)$. We can write

$$u_{eq}(t) = -f\left(\theta, \dot{\theta}, t\right) + \ddot{\theta}_d(t) - \lambda \dot{e}(t) \quad \dots Eq(4)$$

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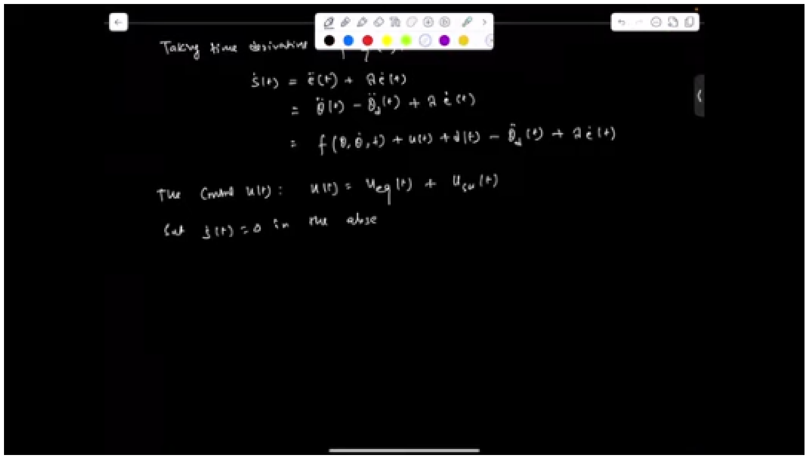

Also, the discontinuous term. So, $u_{sw}(t)$ can be expressed in terms of discontinuous term to achieve the finite time convergence. So, while you're adding this term, that is also we have defined in the earlier lecture. So, we can write

$$u_{sw}(t) = -K\, sgn\big(S(t)\big) \quad \dots Eq(5)$$

There is a simple way if you find difficulty, what you can do is you simply in place of S$\dot{S}(t)$, you can substitute this term, which gives the switching control. And this term, this term, this term and this term will provide the equivalent control. So this is the simple way you can design the equivalent control and sliding control. And disturbance anyways you are making it to zero when $\dot{S}(t) = 0$. So now the total control we can define as

$$u(t) = u_{eq}(t) + u_{sw}(t)$$

$$= -f\big(\theta, \dot{\theta}, t\big) + \ddot{\theta}_d(t) - \lambda\dot{e}(t) - K\, sgn\big(S(t)\big) \quad \dots Eq(6)$$

so for stability analysis we have to define the Lyapunov function in terms of S because our we have the system it's defined in terms of S that's the variable so here is so we can define the Lyapunov function as

$$V = \frac{1}{2}S^2$$

and if you take the time derivative taking time derivative we have

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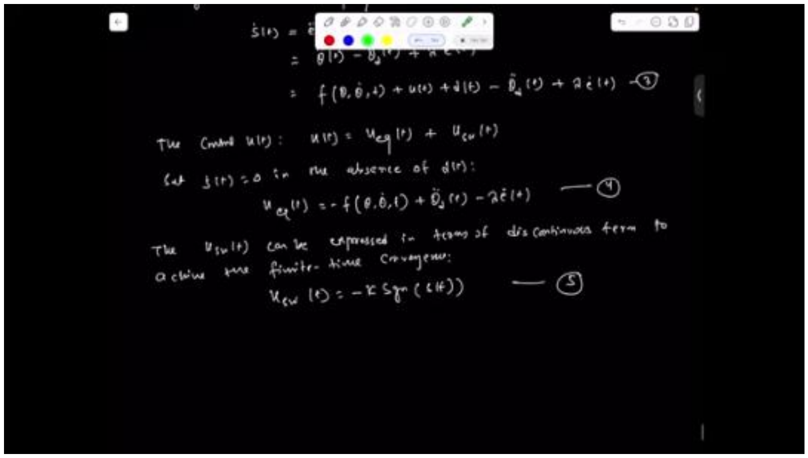

$$\dot{V}(t) = S(t)\dot{S}(t)$$

$$= S(t)\left[f(\theta, \dot{\theta}, t) + u(t) + d(t) - \ddot{\theta}_d(t) + \lambda\dot{e}(t)\right] \quad \ldots Eq(7)$$

Here, we have to consider disturbance also because we are checking the stability in the presence of disturbance, right? Substitute Eq.(6) in Eq.(7)

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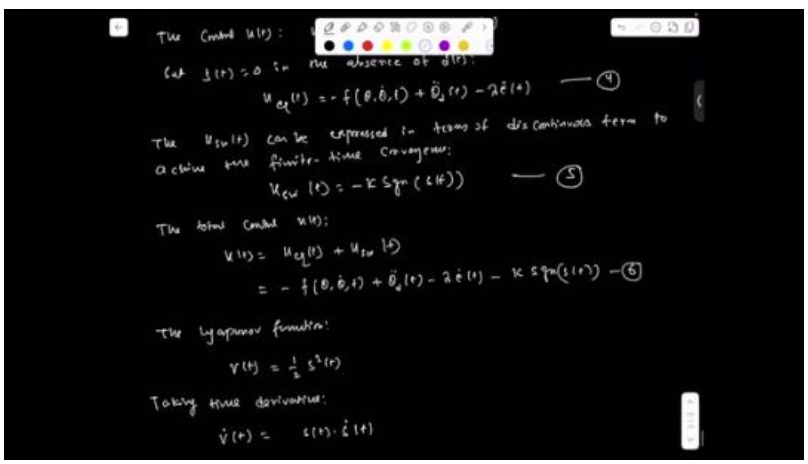

$$\dot{V}(t) = S(t)\left[d(t) - K\, sgn\left(S(t)\right)\right]$$

Here $|d(t)| \leq D$ and $K > D$ and therefore

$$\dot{V}(t) \leq -(K - D)|S(t)| \ldots Eq(8)$$

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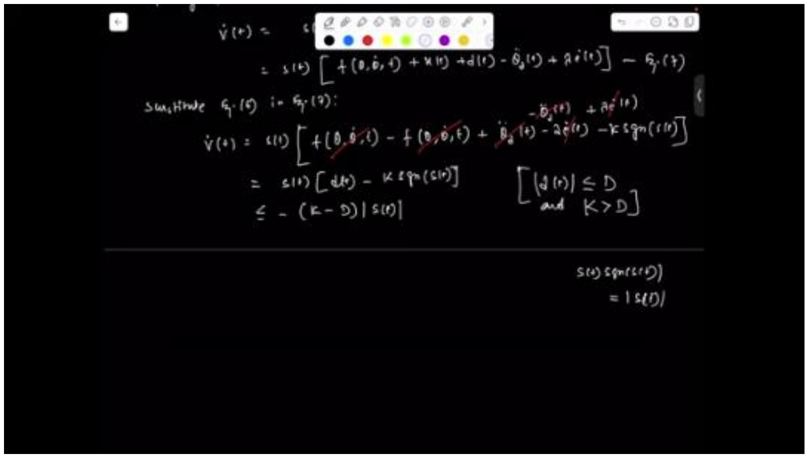

And here we can, further we can simplify this expression, this equation number, 8. So, equation 8 can be simplified as we can write

$$\dot{V}(t) \le \sqrt{2}(K - D)\sqrt{V(t)}$$

$$\le \alpha\sqrt{V(t)} \quad \ldots Eq(9)$$

Where $\alpha = \sqrt{2}(K - D) > 0$

so actually if you remember the last lecture if you have the expression $\dot{V}(t)$ in this expression then we can comment that this is finite time convergence so you can write the above inequality ensures finite time convergence. So this convergence and from equation 9, we can write

$$V(t) \le \left(\sqrt{V(0)} - \frac{\alpha}{2}t\right)^2$$

This easily can compute this expression. And as time proceeded to infinity, so we can say this goes to 0, this term. And we can comment from this expression, we can write S. reaches to zero in finite time which is defined as

$$t_f \le \frac{2\sqrt{V(t)}}{\alpha}$$

This part is going to 0 because S(t) is going to 0 and V(t) is defined in function of S(t). Right. So, as time tends to infinity, this part is going to 0 and we are left with this and from this we can find this expression t_f. So now this is basically stability analysis this part our main part is the control which is defined by equation six now we will see the

MATLAB code how we can get the finite time convergence for this particular problem so let's go step by step this is here actually the MATLAB code for the above problem so here we have defined the parameters

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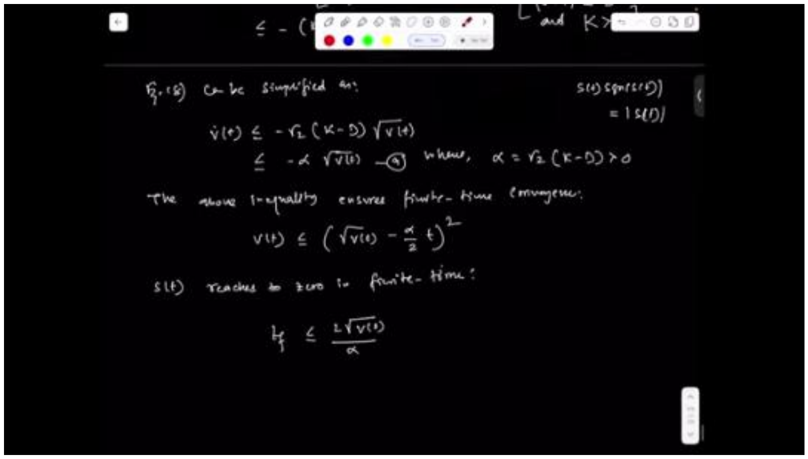

what you have considered before. So here, if you notice K is five and D is one, so K minus D always positive. And the initial conditions are here for the different states. And we are simulating this program for 10 second, which is zero to 10 second. And initial state vector based on this initial values, we can define the initial state vector like this.

And this is the ODE solver, which can find the states and basically there are four states it can get the four states and these are the states how can extract from z because z has four component we'll have later this z has component and these are the same simulation actually plotS(t)he states and this is the function on which the dynamics is defined so we have the θ θ dot e and e dot and these are our desired values we have defined and this is the known dynamics what we have defined in the beginning of the problem and these are the disturbance we have considered so we have some sinusoidal disturbance acting on the system and this is the sliding surface the same structure what we have considered in the analysis this is the equivalent control and this is the sliding control And this is the total control going to affect the dynamics.

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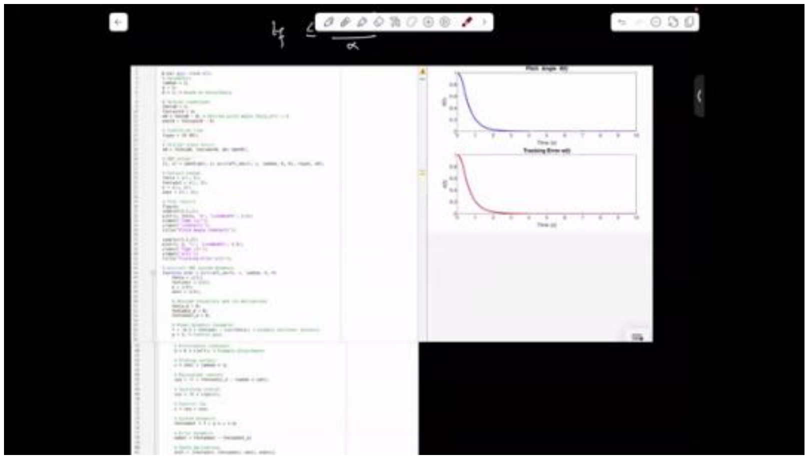

This is the $\ddot{\theta}$ equation: Here, we have also considered g equal to 1. This is the f we have defined here. Some nonlinear dynamics for f are considered. This is the error dynamics. These are the state derivative S(t)he different states. These states are going to this function. This function is simulated using the ODE45 solver. These are the results. If you notice here, we are getting very fine results for θ(t). E(t) is also going to 0, and θ(t) is also going to 0 because our desired objective is (0, 0). This is how we can design the finite-time sliding mode control for the nonlinear system. Let's stop it here. In the next lecture, we'll have a different model of the longitudinal dynamics and how we can design the sliding mode control—finite-time sliding mode control. We'll provide the full details.

Thank you.