**Advanced Aircraft Control Systems With MATLAB / Simulink**

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**Lecture 43**

**Backstepping Control for Lateral-Directional Dynamics of Aircraft**

Hello everyone. In today's lecture, we will be taking the example of how we can control the lateral-directional dynamics of an aircraft using backstepping-based control. We will also have the MATLAB simulation for the same. So, let's start the example. First, let me write down the equations of motion for the lateral-directional dynamics. Example. Let us consider the simplified lateral-directional motion dynamics of an aircraft. We can write it as

$$\dot{\beta} = \frac{Y}{mV} - r + \frac{g}{V}\sin\phi$$

$$\dot{p} = \frac{L}{I_{xx}} - \frac{I_{xz}}{I_{xx}}\dot{r}$$

$$\dot{r} = \frac{N}{I_{zz}} - \frac{I_{xz}}{I_{zz}}\dot{p}$$

$$\dot{\phi} = p$$

So, this is the dynamic model of the lateral-directional motion. Let us define the parameters here and the states. Here, $\beta$ is actually the sideslip angle. P is the roll rate. R is the yaw rate. And $\phi$ is the roll angle. And y is the side force. L is yawing moment. and n is the sorry L is the rolling moment this is rolling moment and N is the yawing moment and m is the mass of the aircraft total mass and this is the velocity of the aircraft so here we are assuming the expression for l y and n as so these are forces and moment so here let me define for simplicity let's consider so here

$$Y = \frac{1}{2}\rho V^2 S C_y\,, \qquad C_y = C_{y\beta}\beta + C_{y\delta r}\delta_r$$

$$L = \frac{1}{2}\rho V^2 S b C_l\,, \qquad C_L = C_{L\beta}\beta + C_{Lp}p + C_{L\delta a}\delta_a$$

$$N = \frac{1}{2}\rho V^2 S b C_n\,, \qquad C_N = C_{N\beta}\beta + C_{Nr}r + C_{N\delta r}\delta_r$$

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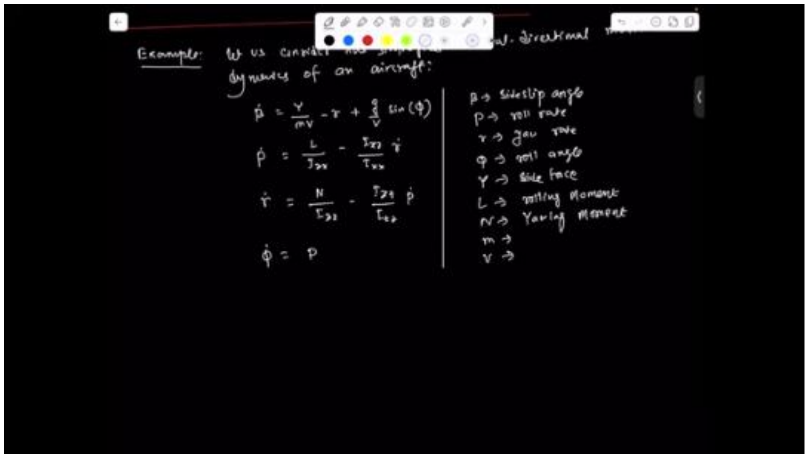


okay so here so these are basically total force and moment acting on the system and here we can define $\delta_a$ is the basically aileron reflection and to which the moment can be generated and this is basically control input control input for roll angle and we are assuming delta with the rudder deflection input rudder deflection and this deflection can be achieved through the control right so this is basically rudder deflection we can say control input for your yarn level so this is how we can define the dynamics of this system and on which we will be designing the control so our objective is control objective and design the control design the control . So, if you notice here, there is two controlling points.

One is $\delta_r$ and one is $\delta_a$. But, if you want to design both the controls, so we have to go a little long mathematics. But, in this lecture, we will be considering only $\delta_a$, which is going to control the thigh angle. But also, you can go the same step to design the $\delta_r$ And to control your β angle also. Right. So the same process you can follow what you are going to follow for $\delta_a$. So our control object is we will design the control $\delta_a$ such that ϕ tracks the desired ϕ. This is a very interesting problem because we are going to track a particular value of the desired trajectory. So let's start solving the problem. First, let us define the tracking error because we have to track the particular trajectory. So we need to have the tracking error. Let us define the tracking error for the roll angle. We can write

$$z_1 = \phi - \phi_d$$

$$\dot{z}_1 = p - \dot{\phi}_d \quad \ldots Eq(1)$$

So here, if you follow the way you have done in the previous lecture, p can be assumed as a virtual control.

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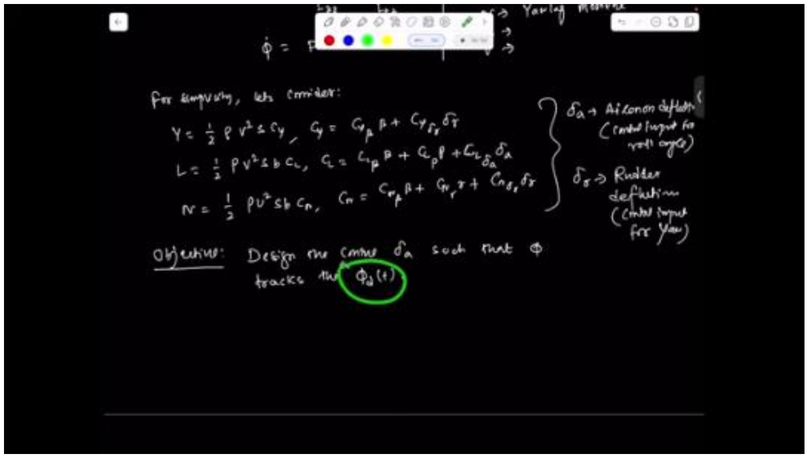


For $z_1$, which can be denoted as $\alpha_1$ or $\lambda$, whatever parameter it is, it is up to us. So let us relate p as a virtual control input for $z_1$. And this virtual control can be defined as $\alpha_1$. So here,

$$\alpha_1 = -k_1 z_1 + \dot{\phi}_d \quad \ldots Eq(2)$$

Where $k_1 > 0$, the same steps we are following what you have followed in the earlier lecture. Now, combining equation 1 and 2.

$$\dot{z}_1 = -k_1 z_1 + p - \alpha_1 \quad \ldots Eq(3)$$

Now, let us consider the Lyapunov function. Function we choose to analyze the stability of the system. So let us choose

$$V_1(z_1) = \frac{1}{2} z_1^2$$

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1\big(-k_1 z_1 + (p - \alpha_1)\big) \ldots Eq(4)$$

Now, if $p = \alpha_1$ we can say

$$\dot{V}_1 = -k_1 z_1^2$$

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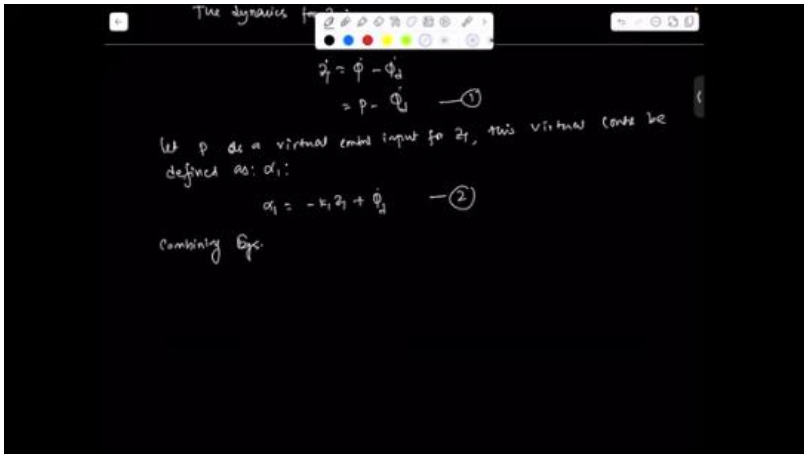


So, this is basically stable. So, the total energy is taken over time, so it is basically negative definite. So, over time, $V_1$ is decaying, so here it's stable. So now, also, you can find for this condition, for this particular condition, we can come up with the expression for $\dot{z}_1$. So, from equation 3, for $p = \alpha_1$, from equation 3, we can write,

$$\dot{z}_1 = -k_1 z_1$$

And we can say this is asymptotically stable. So now, you know, we have come up with the conclusion on $\dot{z}_1$, and it is found to be stable. Now, we will work on $z_2$.

$$z_2 = p - \alpha_1$$

$$\dot{z}_2 = \frac{L}{I_{xx}} - \frac{I_{xz}}{I_{xx}}\dot{r} - \dot{\alpha}_1 \quad ... Eq(5)$$

So, one thing: this $I_{xz}$ is actually the product terms of the inertia matrix of diagonal terms, as you can see. And these are the diagonal elements. So, we can write the L expression.

$$L = \frac{1}{2}\rho V^2 Sb\left(C_{L\beta}\beta + C_{Lp}p + C_{L\delta a}\delta_a\right)$$

This is already there. So now, we have to choose $\delta_a$ in this expression in such a way that $\dot{z}_2$ is stable. So, what you're going to do is So, first, let us substitute this expression L in this expression. So now, from equation 5, we can write

$$\dot{z}_2 = \frac{\frac{1}{2}\rho V^2 Sb\left(C_{L\beta}\beta + C_{Lp}p + C_{L\delta a}\delta_a\right)}{I_{xx}} - \frac{I_{xz}}{I_{xx}}\dot{r} - \dot{\alpha}_1 \quad ... Eq(6)$$

Now here we have to choose this $\delta_a$ in such a way that $\dot{z}_2$ is stable.

$$\delta_a = \frac{1}{C_{L\delta a}}\left[-C_{L\beta}\beta + C_{Lp}p + \frac{2I_{xx}}{\rho V^2 Sb}\left(-k_2 z_2 + \frac{I_{xz}}{I_{xx}}\dot{r} + \dot{\alpha}_1\right)\right] \ \ldots Eq(7)$$

Where $k_2 > 0$ and substituting 7 in equation 6, we have

$$\dot{z}_2 = -k_2 z_2$$

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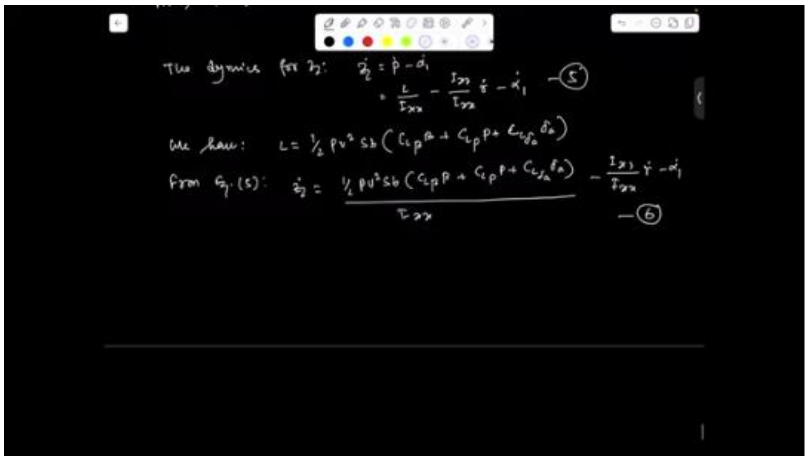


And this is basically you can say asymptotically stable dynamics. So now we will come up with the Lyapunov function to prove the overall stability of the system. So let us choose the Lyapunov function as now let us choose the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}z_2^2$$

$$\dot{V}_2 = z_1\dot{z}_1 + z_2\dot{z}_2$$

$$= -k_1 z_1^2 + z_1 z_2 - k_2 z_2^2$$

So now, using the fact, we have

$$z_1 z_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$$

The same process what you have learned in the earlier lecture for the longitudinal motion dynamics.

$$\dot{V}_2 \leq -\left(k_1 - \frac{1}{2}\right)z_1^2 - \left(k_2 - \frac{1}{2}\right)z_1^2$$

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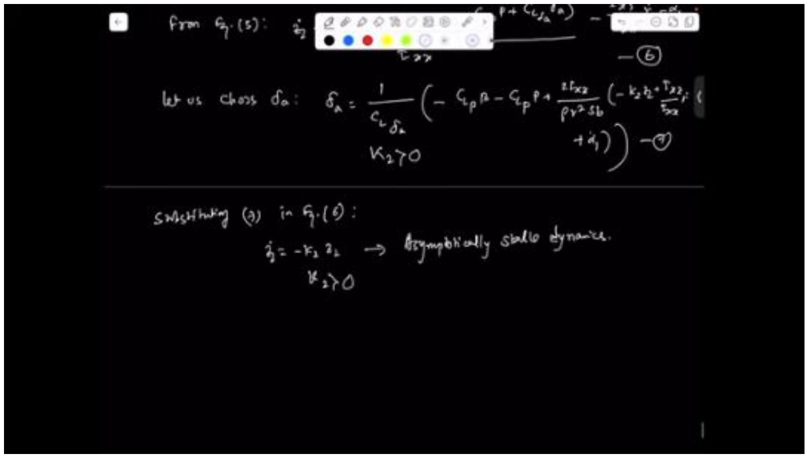

and if you choose $k_1$ is greater than half and $k_2$ greatar than half, so it will give us yields the overall asymptotic stability of the system. We have done the control design of the entire system, and since our only objective is to control the ϕ angle. Similarly, we can use the same steps to design $\delta_r$ if you want to control the β angle. Now, we can move to the MATLAB part. I will go step by step. We have initialized the total mass as 1000 kg, and these are the inertia parameters. This is the row in the dynamics. Basically, the density r and density v are assumed to be 50 meters per second. The wing area is 20, and the wingspan is 5 meters. These are the coefficient forces, roll moment, and force coefficients we have defined here. These are the other parameters used in the dynamics, as defined here. We have to choose the control gains: $k_1$ to be 2 and $k_2$ equal to 3. This satisfies the condition. These are the initial conditions for the states to solve the dynamics. The simulation time is a total of 20 seconds. We have used the ODE solver to solve the dynamics. Here are the commands for the plots and figures. This is the dynamic equation we have written here. The aircraft dynamics are defined by the states $x_1, x_2, x_3, x_4$, and so on. This is the desired roll angle and its derivative.

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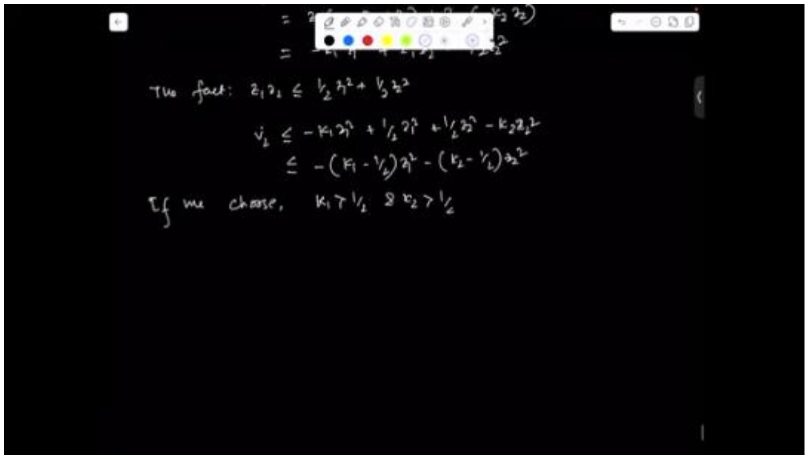


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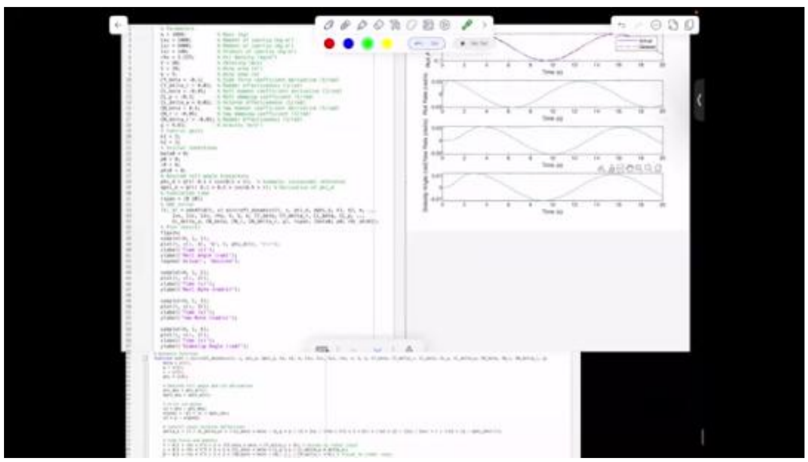

We have defined it like this. These are the tuning parameters where we store the values. This is the error variable obtained through the analysis. This is the control signal obtained through the analysis, equation 7, as written here. This is the total force and moment using the different force and moment coefficients, which we have defined there. This is the overall dynamics of the system. Now, we have the MATLAB results, and we can see that our objective is fulfilled. The roll angle tracks the desired trajectory. Over time, it is achieved within two seconds, though the simulation time is 20 seconds. These are the results for the other states, and they are also controlled. This is how we can design the backstepping control for this system. This is the continuous code. Don't worry, since it is difficult to capture in one window, I have taken it differently. But this is a continuous code. If you want to write this code in your MATLAB window, you can use it continuously. Now, we are going to start a new topic from now on. This is the last lecture on backstepping control. I hope you can design the same thing for your system, whatever system you are handling. If you have different dynamics, you can follow the same steps to design the overall control for the system.

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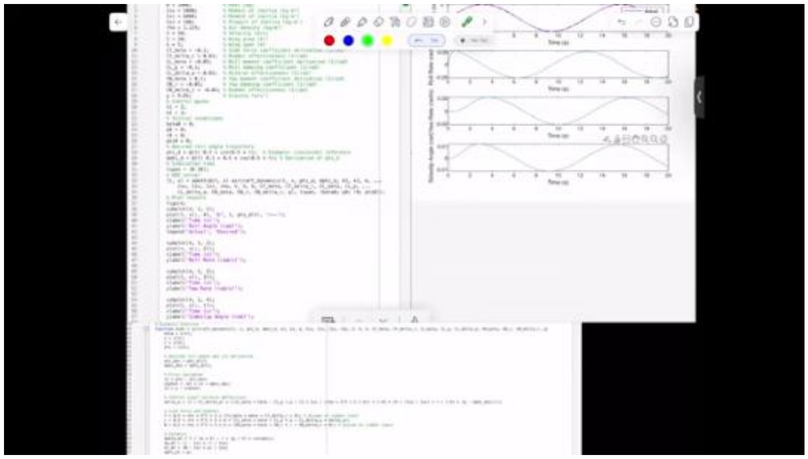

Even if it is a spacecraft or another domain in flight mechanics or robotics, you can pursue the same steps to design the controls. So, let’s stop it here. We’ll come up with a new topic, sliding mode control, from the next lecture onwards. And we’ll see how to design it for aircraft applications. Thank you.