Advanced Aircraft Control Systems With MATLAB / Simulink

Prof. Dipak K. Giri

Department of Aerospace Engineering

Indian Institute of Technology Kanpur

Lecture 42

Backstepping Controller for Longitudinal Dynamics of Aircraft

Hello everyone. Today, we will be taking the example of an aircraft system to design the control using backstepping. Here are the topics we are going to cover: we will take the nonlinear system longitudinal motion of the aircraft, and we'll also have the MATLAB simulation for the same. Whatever control we design for the system, I will validate it in MATLAB. So here, first, let's start with the dynamics model example. Let us consider the following simplified longitudinal dynamics of an aircraft. The equation of motion we are considering is

$$\dot{\alpha} = q - \frac{L}{mV} + \frac{g}{V}\cos\theta$$

$$\dot{q} = \frac{M}{I_{yy}}$$

$$\dot{\theta} = q$$

This is the simplified model we are considering. For different aircraft, we have different equations of motion. But this is the basic one. Here, the parameters and states we can define are: α is the angle of attack in radians, θ is the pitch angle, and L is the lift force in Newtons And M is the pitching moment in newton meters, m is the mass of the aircraft, and v is the velocity in meters per second, and I_{yy} is the moment of inertia about the y-axis, and g is the gravitational constant. Assuming for simplicity, let us assume

$$L = \frac{1}{2}\rho V^2 S C_L, \qquad C_L = C_{L\alpha}$$

$$M = \frac{1}{2}\rho V^2 S\bar{c} C_m, \qquad C_m = C_{m\alpha} + C_{mq}q + C_{m\delta e}\delta_e$$

Here, δ_e is the important control parameter we are going to design through control, which is going to provide the moment. Our main motivation is how to control the angle, which is θ.

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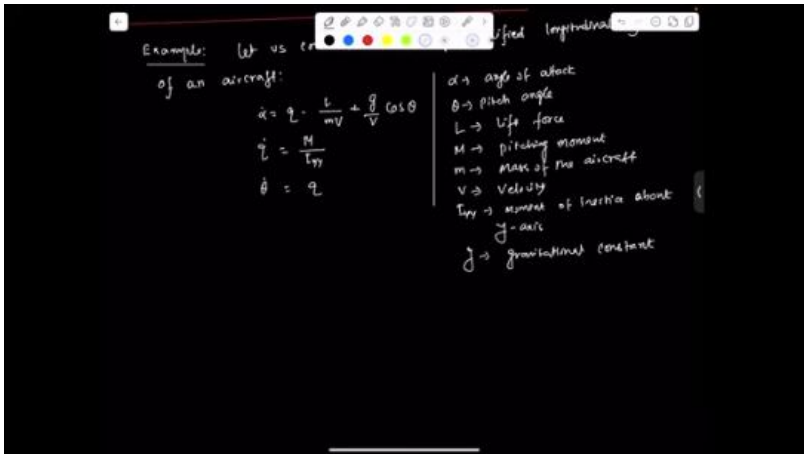

So, this is the parameter we are going to control through the backstepping design process. Here, we can write that delta is the elevator deflection, basically the control input. Now, our objective for this particular problem is... Design the control command δ_e such that the pitch angle θ tracks the desired reference trajectory. That is the desired reference trajectory $\theta_d(t)$. So, this is the problem we are going to solve—this objective. Let's start the problem: how we can tackle this to design the control for this dynamic solution. Now, let us define the tracking error since we have the desired dynamics. Here, we need to have the tracking error, and that error variable will be one of the states of the system. So, let us define the tracking error as

$$z_1 = \theta - \theta_d$$

$$\dot{z}_1 = q - \dot{\theta}_d \quad \dots Eq(1)$$

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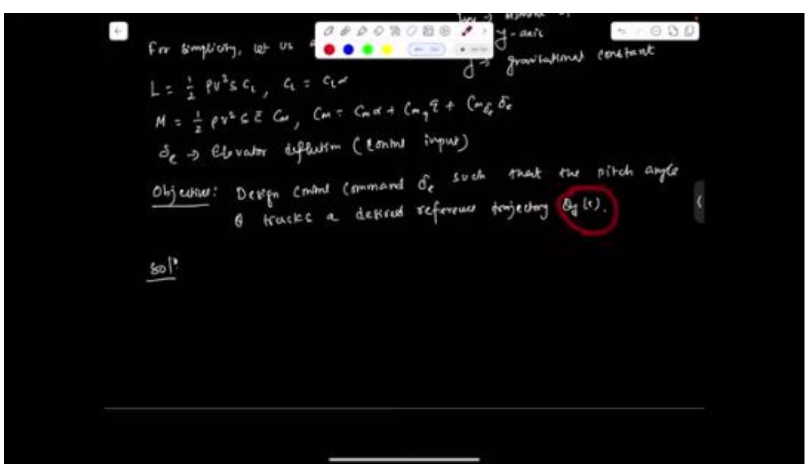

let us define q as the virtual control defined by α_1, and let us choose

$$\alpha_1 = -k_1 z_1 + \dot{\theta}_d \quad \dots Eq(2)$$

We can combine equations one and two

$$\alpha_1 = -k_1 z_1 - \dot{z}_1 + q$$

$$\dot{z}_1 = -k_1 z_1 + (q - \alpha_1) \quad \dots Eq(3)$$

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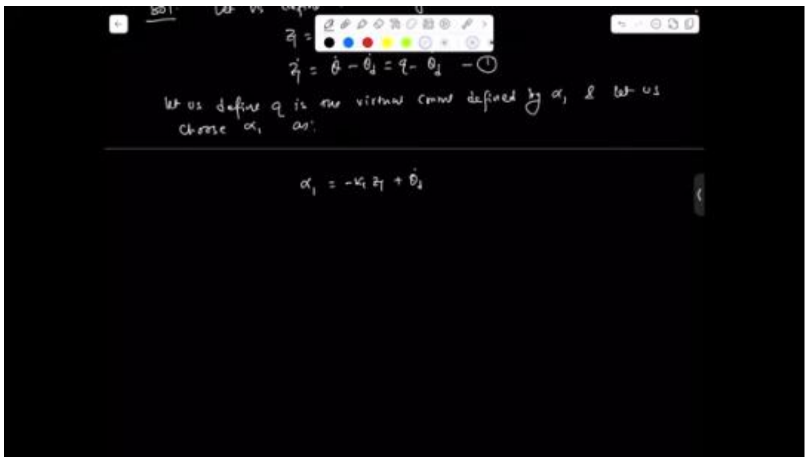

Now, here is our main motivation. Now, let us write. Let us choose a Lyapunov function. Let us choose Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2$$

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1\big(-k_1 z_1 + (q - \alpha_1)\big)$$

And if $q = \alpha_1$ we can write

$$\dot{V}_1 = -k_1 z_1^2$$

So, this is basically negative semi-definite, and if you use this condition, this condition in equation three, so I can write

$$\dot{z}_1 = -k_1 z_1$$

So, this is basically asymptotically stable dynamics. So now, we will work on the second variable, z_2. And z_2 we can define using these two parameters So, how can we take before? This is already defined in the earlier lecture because the initial values of these parameters q and α_1 are different from the different initial conditions, and due to which, we can write the error

$$z_2 = q - \alpha_1$$

$$\dot{z}_2 = \frac{\frac{1}{2}\rho V^2 S\bar{c}\left(C_{m\alpha} + C_{mq}q + C_{m\delta e}\delta_e\right)}{I_{yy}} - \dot{\alpha}_1 \quad \dots Eq(4)$$

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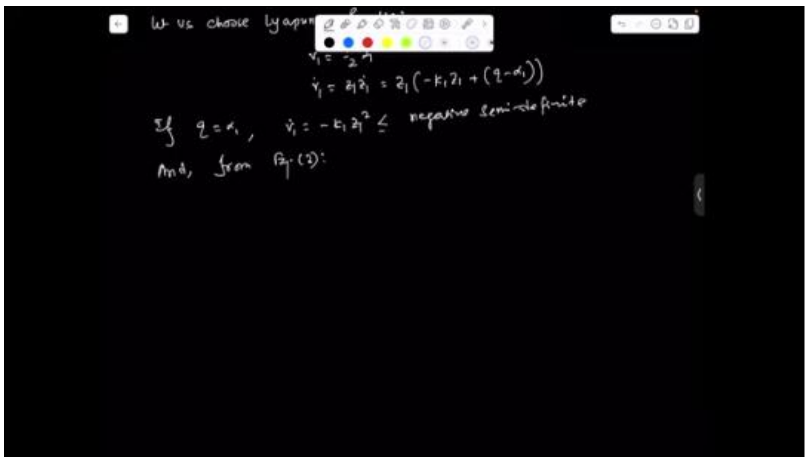

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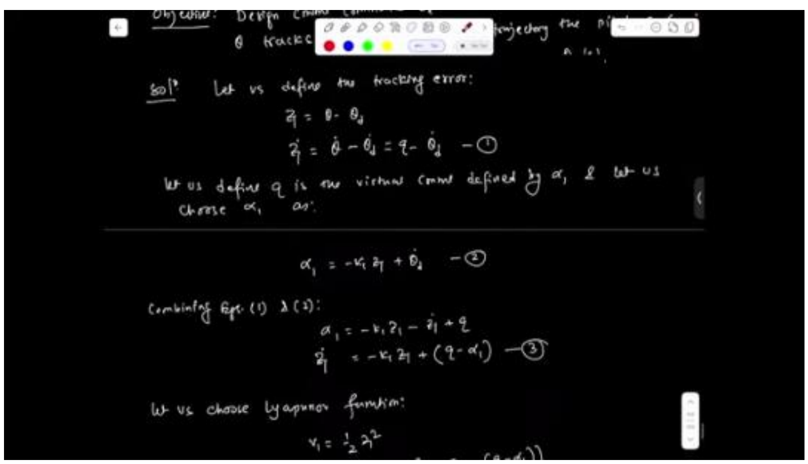

Now we have to choose in this equation delta e delta in such a way that z_2 dynamics is stable. So now choose δ_e in such a way that choose delta in such a way that which makes z_2 goes to 0 as t tends to infinity. So here we are choosing

$$\delta_e = \frac{1}{C_{m\delta e}}\left(-C_{m\alpha}\alpha - C_{mq}q + \frac{2I_{yy}}{\rho V^2 S\bar{c}}(-k_2 z_2 + \dot{\alpha}_1)\right) \quad \dots Eq(5)$$

substituting equation 5 in equation 4 we can write

$$\dot{z}_2 = -k_2 z_2$$

Now let us define a Lyapunov function. So here we've already found the controls and also we have shown this is stable. So now we will check the overall control rate of the system using Lyapunov stability theorem. So now let us choose that the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} z_2^2$$

$$\dot{V}_2 = z_1 \dot{z}_1 + z_2 \dot{z}_2$$

$$= -k_1 z_1^2 + z_1 z_2 - k_2 z_2^2 \dots Eq(6)$$

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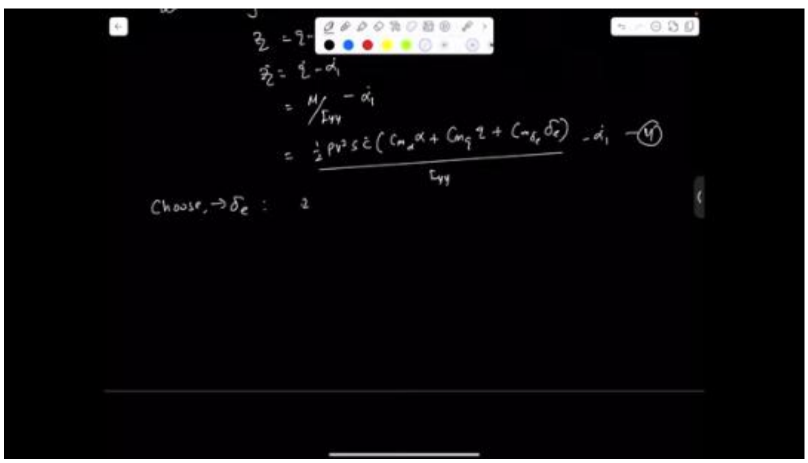

So now, using the fact, we have

$$z_1 z_2 \leq \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2$$

equation 6 yields

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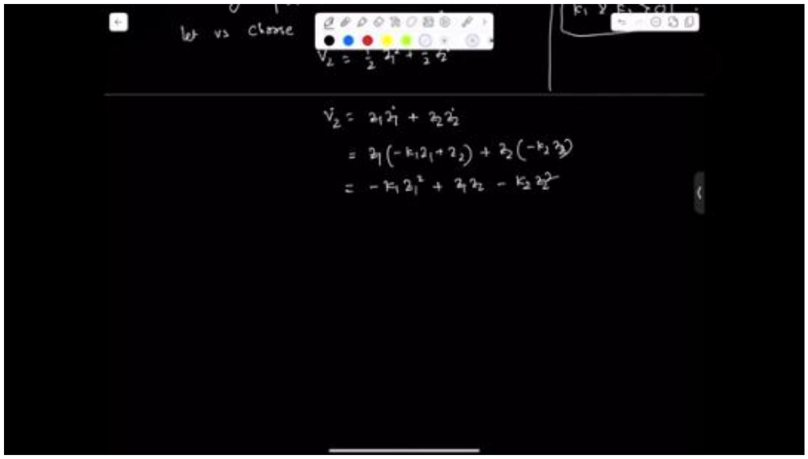

$$\dot{V}_2 \leq -\left(k_1 - \frac{1}{2}\right) z_1^2 - \left(k_2 - \frac{1}{2}\right) z_2^2$$

Now, if k_1 is greater than half and k_2 is greater than half, we can say $\dot{V}_2$ less than or equal to zero. This actually ensures asymptotic stability of the system. Now, we have completed the proof. The stability proof is done, and we also have the control. Now, we can summarize the full control. Here, control stops. We can write

$$\delta_e = \frac{1}{C_{m\delta e}}\left(-C_{m\alpha}\alpha - C_{mq}q + \frac{2I_{yy}}{\rho V^2 S\bar{c}}(-k_2 z_2 + \dot{\alpha}_1)\right)$$

$$z_1 = \theta - \theta_d$$

$$z_2 = q - \alpha_1$$

$$\alpha_1 = -k_1 z_1 - \dot{z}_1 + q$$

This is the full control setup. Now, we will go to the MATLAB code. To see how delta θ is going to achieve some desired value of the pitch angle. So, here is the MATLAB code for the same. These are the initial conditions we have taken. This is the initial condition. And k_1 , k_1 , the control gains, we have chosen as 2 and 3.

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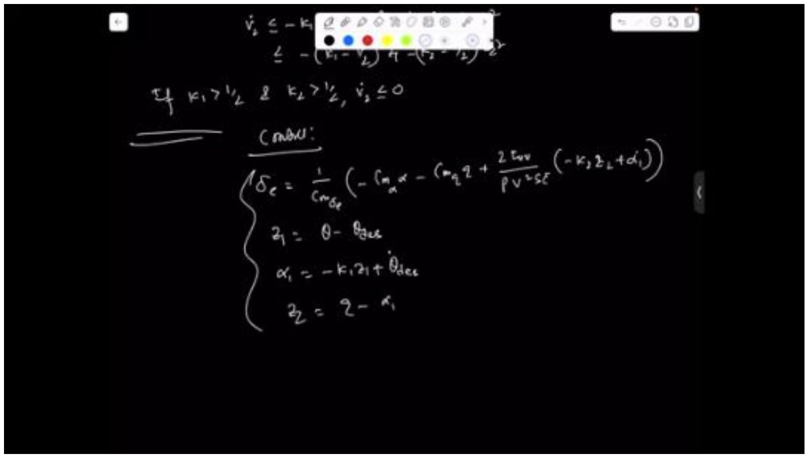

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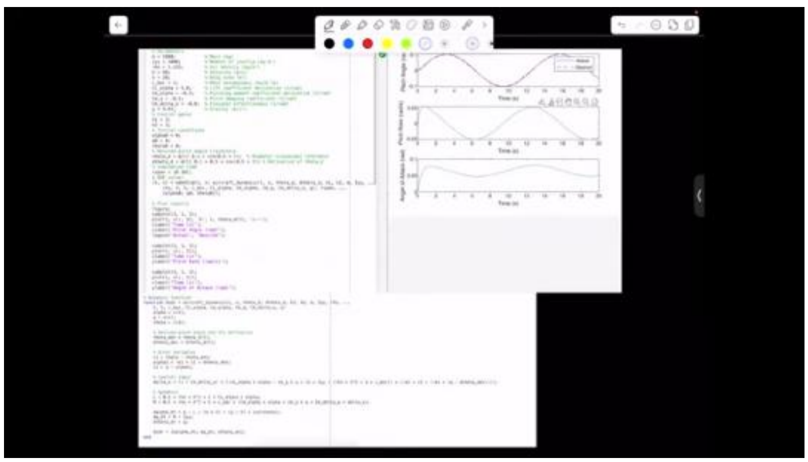

The initial conditions for alpha, q_0, θ_0 are chosen like this: 0, 0. The desired pitch angle is chosen as this. This is the θ dot, θ desired dot. The total simulation time is 20 seconds. We have used ode45 to solve the ODE. These are for plotting the results. This is the dynamics function, basically aircraft dynamics. Here, all the parameters used in the mathematical formulation are applied. This is the desired θ, and this is the error variable we have used. This is the total control we have derived through the analysis.

And this is the dynamic equation we have. And these are the derivative terms going to the function here. So, now if you simulate this code, this is the result we have obtained. And we can see that this desired θ, this is the pitch angle θ, it tracks over time to the desired θ. And this is the blue is the basically actual and this is the desired one.

And other are also controlled. If you notice other angles also control over time. So this is how we can design the backstepping-based control for the nonlinear system. What we have considered in this problem is the longitudinal motion dynamics of the aircraft. So if you have any other dynamical system, you can move like this and you can design the nonlinear control for the same.

Let's stop it here. In the next lecture, we'll have how we can come up with the control for the lateral dynamics. Thank you.