Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 41

Backstepping Controller (Contd.)

Hello, everyone. In today's lecture, we'll have another example of how we can implement the backstepping control. And also, we have the MATLAB simulation for the same. So let me define the problem. Let's consider we have a nonlinear system.

$$\dot{x}_1 = ax_1^2 + x_1^3 + x_2$$

$$\dot{x}_2 = u$$

So this is quite a highly nonlinear system, and here we design control u to stabilize x_1 and x_2. We'll start solving the problem, find the controls for the system, and finally, we'll have the MATLAB code for the same solution. The process of backstepping design remains the same as what you have done in the earlier examples. So let us consider we have x2 as a virtual control, which is denoted as

$$x_2 = \lambda$$

For example, λ is here a virtual control. In the previous example, we had α_1, so here we are assuming λ, and for this, we have the system, the first subsystem. The first subsystem, which is basically x1 dot dynamics, yields to be

$$\dot{x}_1 = ax_1^2 + x_1^3 + \lambda \ \ldots Eq(1)$$

Let's consider the Lyapunov function

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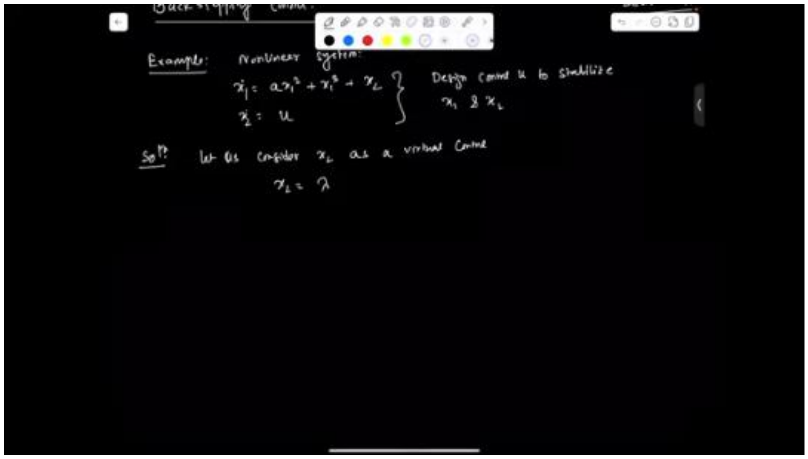

So I'm just going through some steps, the same process. I'm not following what you have done in the previous examples. So I'm using some straightforward method. So consider a Lyapunov function as

$$V_1(x_1) = \frac{1}{2}x_1^2$$

and taking time derivative we have

$$\dot{V}_1 = x_1\dot{x}_1 = x_1(ax_1^2 + x_1^3 + \lambda) \ldots Eq(2)$$

So now in this equation, we have to choose λ in such a way that $\dot{V}_1$ is negative definite

$$\lambda = -k_1x_1 - ax_1^2 - x_1^3 \quad \ldots Eq(3)$$

Here, k_1 is greater than zero, so this is the virtual control for the subsystem. Now, in this equation three, substituting equation two,

$$\dot{V}_1 = -k_1x_1^2$$

This we can say is negative definite, right? Now, substituting three in equation one

$$\dot{x}_1 = -k_1x_1$$

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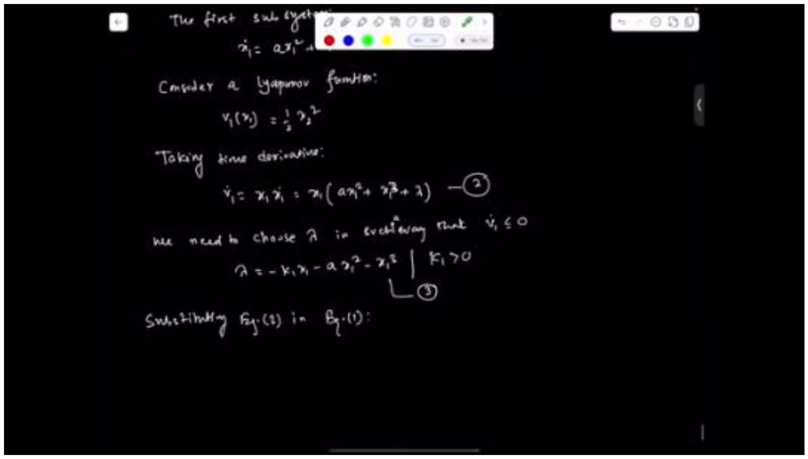

and this is globally asymptotically stable since k_1 is greater than zero. Here, the steps we are following now, you can follow the same steps for designing control for the nonlinear system in case you are designing the control for aircraft or. Any other dynamical system, you can follow the same steps to design the control. Now, we'll introduce the second error between x_2 andλ, the process we followed in the previous example.

$$e = x_2 - \lambda$$

$$x_2 = e - k_1 x_1 - a x_1^2 - x_1^3 \ \ldots Eq(4)$$

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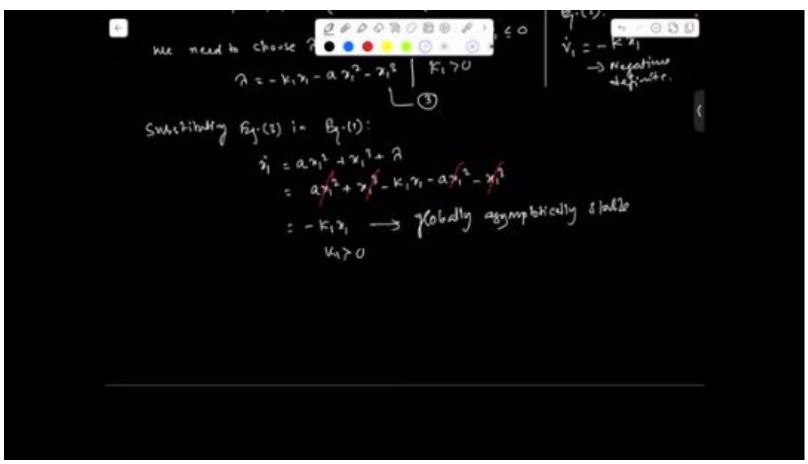

Now, we will define x_1 dynamics in terms of error, this error what you have defined here the same steps we have followed in the last example

$$\dot{x}_1 = e - k_1 x_1 \ldots Eq(5)$$

now we have to also find the second error dynamics so now now we have to find we have to find the error dynamics $\dot{e}$ in terms of u what you have done in the earlier lecture so we can write

$$\dot{e} = \dot{x}_2 - \dot{\lambda} = u - \dot{\lambda} \quad \dots Eq(6)$$

And also we know that

$$\lambda = -k_1 x_1 - a x_1^2 - x_1^3$$

$$\dot{\lambda} = -[k_1 + 2a x_1 + 3x_1^2][e - k_1 x_1] \quad \dots Eq(7)$$

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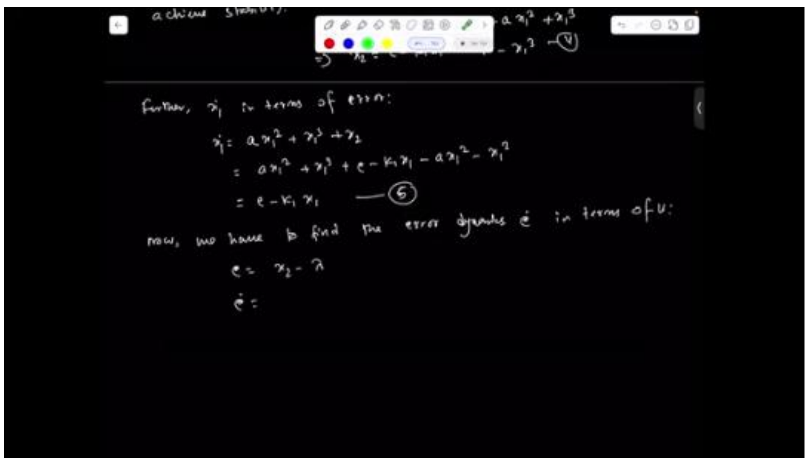

Substituting Eq 7 In Eq 6,

$$\dot{e} = u + [k_1 + 2a x_1 + 3x_1^2][e - k_1 x_1] \quad \dots Eq(8)$$

So here we have the two dynamics: $\dot{e}$ dynamics and $\dot{x}_1$ dynamics. So you can use equation 6 and equation 8 for the stability analysis using Lyapunov. So let us consider the Lyapunov function as

$$V_2(x_1, e) = V_1 + \frac{1}{2} e_2^2$$

and from this we can take the time derivative

$$\dot{V}_2 = x_1 \dot{x}_1 + e\dot{e}$$

$$\dot{V}_2 = -k_1 x_1^2 + e[x_1 + (k_1 + 2a x_1 + 3x_1^2)(e - k_1 x_1) + u] \quad \dots Eq(9)$$

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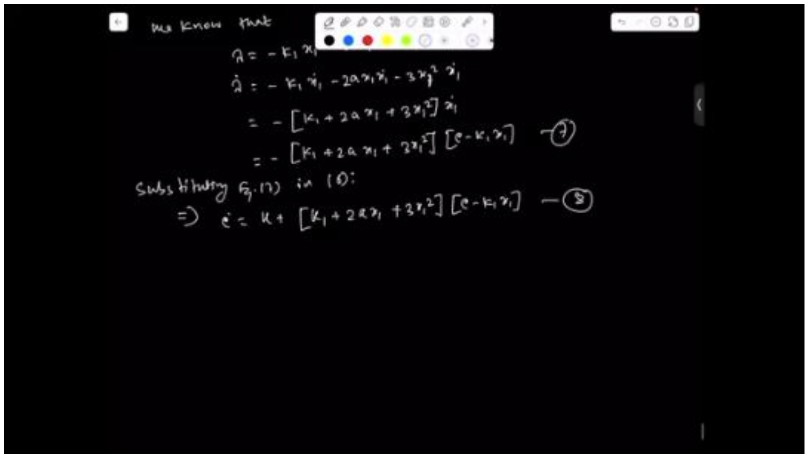

so let us define Eq 9 such a way that the $\dot{V}_2$ is negative definite. So for this, we can choose u in equation 9 which makes $\dot{V}_2$ less than or equal to 0. So to achieve this condition, we have to modify this u accordingly. So we can choose as

$$u = -x_1 - (k_1 + 2ax_1 + 3x_1^2)(e - k_1x_1) - k_2e \quad \ldots Eq(10)$$

Where $k_2 > 0$, Substituting equation 10 in equation 9, we have

$$\dot{V}_2 = -k_1x_1^2 - k_2e^2$$

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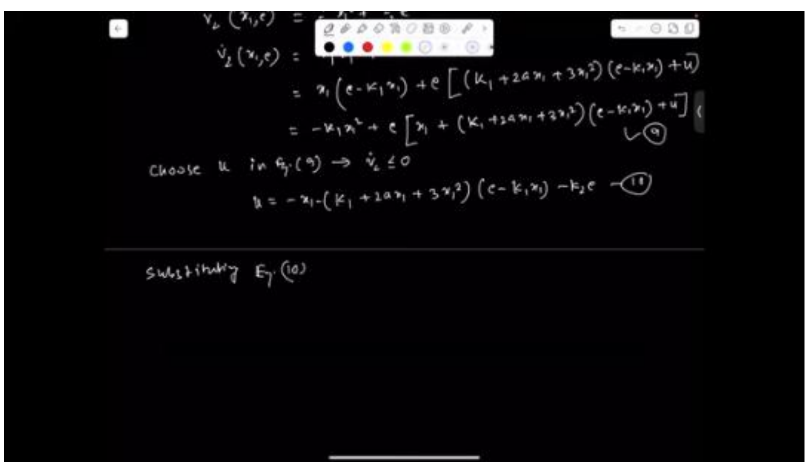

Yeah, we have found our—so this is basically, you can say, uh, negative, sorry. negative definite, and for this condition, we can write hence $x_1(t), e(t) \rightarrow 0$ as t tends to infinity for all initial conditions $x_1(0)$ and $e(0)$ right? So, also, we can write here now if you put as in this condition, the condition for what we have found.

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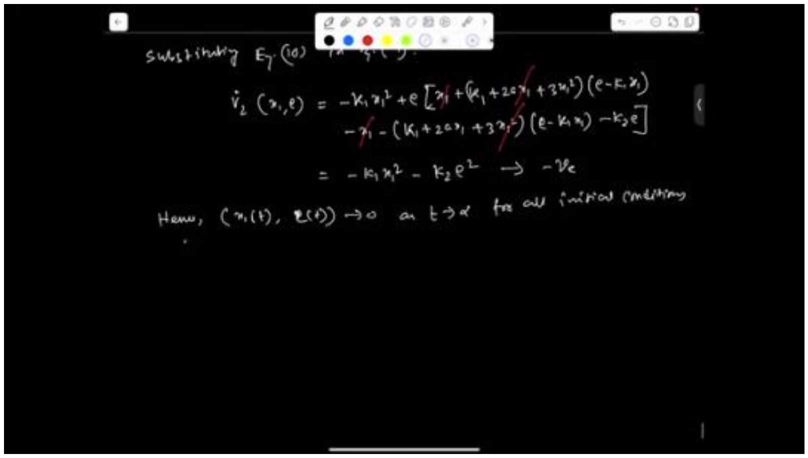

Also from Eq (4) as $x_1(t), e(t) \to 0$, as for this condition, $x_2(t)$ also goes to 0 because the equation we have written, equation 4, which is a function of $x_1(t)$, and $e(t)$. So, we have found our controls for this particular example. Now, we'll go to the MATLAB code. This is the code, this is the MATLAB code, and so whatever the function we have considered in this example, it is here. So, this is the initial condition, and this is the initial values. This is the time period for simulation, 10 seconds. ode45 to solve the ODE, and this is the function where we have taken a equal to 2, $k_1 = k_2 = 2$. In some examples, why these values we have considered, we have also explained in the first few lectures in this topic. And this is the lambda expression, and this is our control.

And this is our dynamics: $\dot{x}_1$ and $\dot{x}_2$, and these are the figures to plot the figure, and these are the results. So, we can see that as time tends to infinity. So, if you can see that as time tends to infinity. the response of these plots goes to zero. So, the system is stable. This is how we can design the control for the nonlinear system. In the next few lectures—two lectures—we will be considering this concept for the aircraft system, how we can design the control for the longitudinal dynamics and lateral dynamics of the aircraft.

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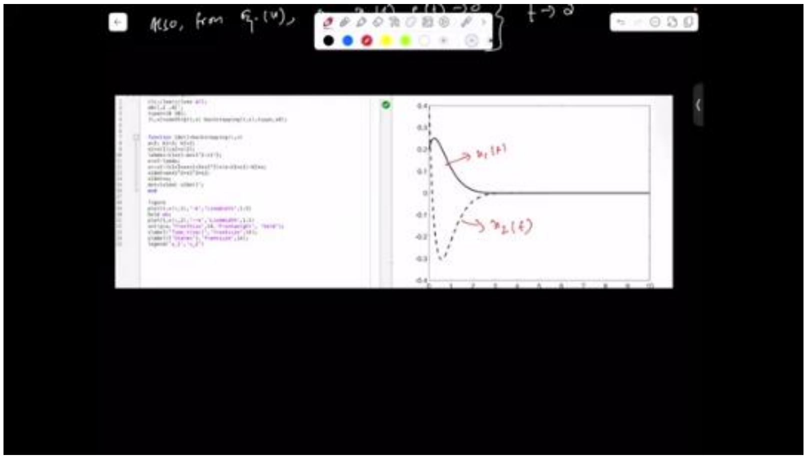

Thank you.