Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 04

Example

Hello everyone, this is the lecture number 4 in this course Advanced Aircraft Control System. In today lecture, we will be taking an example through which we will solve an equation which is defined in the state space form. And also we will have the MATLAB code, how we can find the solution of the states of that system. And also we will have some concept on asymptotic stability, how the system is going to behave over time. This is very important concept in modern control or classical control techniques.

After having all this stuff, we will conclude the lecture. So, let us start the lecture. First, let us define the problem. We have the problem. So, we will take a simple ODE and also we will discuss, how the ODE can be converted into a state space form. So, we have the second order system,

$$\ddot{x} + 3\dot{x} + 2x = 2u$$

And initial condition given to us, x(0)=0 and $\dot{x}(0) = 1$. You can assume, this is the position and this is the velocity of the state. So, now here one more thing, here you can assume this is the control or maybe you can assume this is some external term, which is not the part of the natural dynamics of the system. So, the question is, one more thing is also given to us, the output equation, this is the state equation, you can assume the output equation given to us.

output equation is $y = 3x + \dot{x}$ let us assume this is the output equation. First question is, study the system in state space and second, find the state transition matrix which will transform from initial state to some other final state And third question is, find the response of the system if U is the unit step function. So, this is the problem for us, now we will be working on this problem, how we can find the state space model and also how we can find the solution of this system.

So, here the system is given to us

$$\ddot{x} + 3\dot{x} + 2x = 2u$$

So, if you notice in this equation, this is not in state space form. As we know the state space, if you are writing the system in state space form, we have to come up the first order ODE. So, we need to convert the first order system ODE s, so for that, we need to apply the change of variables, so let's assume, $x_1 = x, \dot{x} = \dot{x}_1 = x_2$ from the equation, we can write

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

So, this is basically two second first part of ODE s. So, from these two equation, we can write the state space form, and the matrix

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

So here you can simply write,

$$\dot{X} = AX + BU$$

So this is the state space form of the given system. Now, also we are given the output equation.

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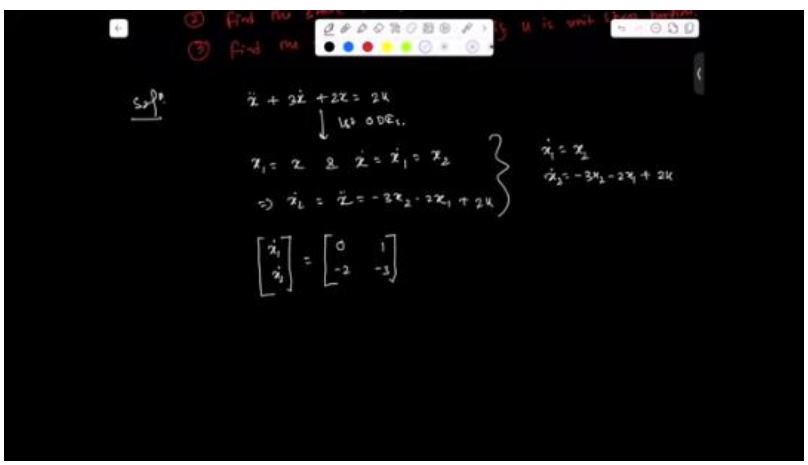

So, this is the state equation you can write and the output equation given to us,

$$y = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

So, this is the complete state space model given to us Now, to find the solution of this system, first and foremost step is, we have to find the state-transition matrix for this particular example. So, let us find that state-transition matrix. This is already we have found

$$e^{At} = L^{-1}\{(SI - A)^{-1}\}$$

$$= \frac{1}{s^2 + 3s + 2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \ldots Eq(1)$$

now we'll we have to simplify this matrix, so we have to work on the individual terms of this matrix, so now using partial fraction we can go step by step, so let's work on the first term of this matrix, let me define the equation number one, so first term is s plus three divided by s square plus three s plus four, so we can apply the partial fraction, we can write this expression as a upon s plus one and we can write Okay, also this term you can simplify further as, s+1 and s+2. So you can simplify this term. So this we can write further.

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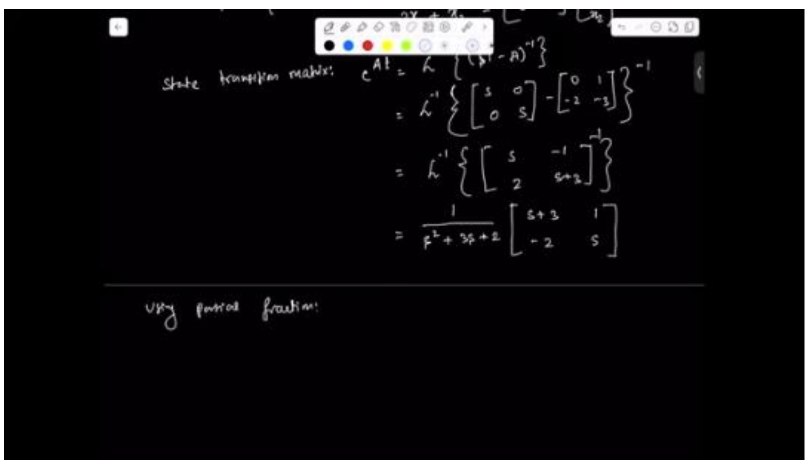

Using partial fraction

$$\frac{s+3}{s^2+3s+2} = \frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$s + 3 = A(s+2) + B(s+1)$$

When s=-2, B=-1 and when s=-1 and A=2. Similarly

$$\frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\frac{-2}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$\frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

So now we can write from equation 1. we can write,Laplace transform e to the power At which is nothing but our this expression,

$$L\{e^{At}\} = \begin{bmatrix} \frac{2}{(s+1)} - \frac{1}{(s+2)} & \frac{1}{(s+1)} - \frac{1}{(s+2)} \\ \frac{-2}{(s+1)} + \frac{2}{(s+2)} & \frac{-1}{(s+1)} + \frac{2}{(s+2)} \end{bmatrix}$$

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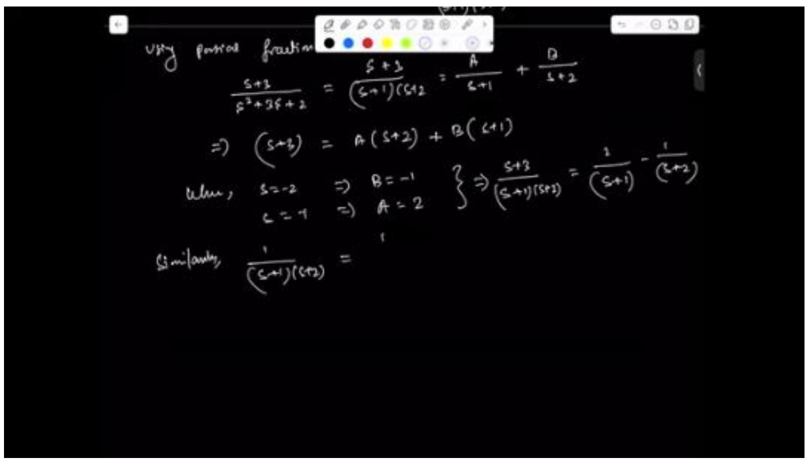

if you take the inverse Laplace transform for this equation, we can write taking inverse Laplace transform we get

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2(e^{-t} - e^{-2t}) & e^{-t} + 2e^{-2t} \end{bmatrix}$$

So, this is the matrix in time domain. Now, we will find the complete response of the system.

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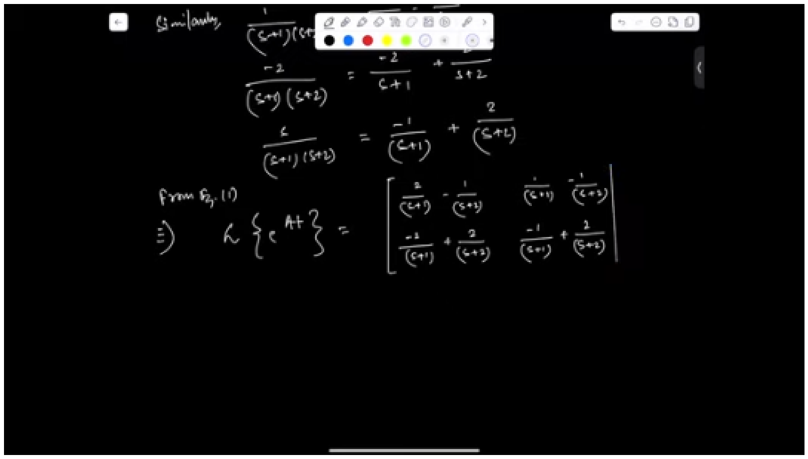

So, here is what we use So, here we can use the part from the solution of state equation, we know that

$$X(t) = e^{At}X(0) + \int_0^t e^{A(t-\tau)}BU(\tau)d\tau$$

This you already have found in the last lecture, 0 to t e to the power a t minus tau v u tau d tau this expression we have right. So, here one thing basically here our u of tau is the unit step function, unit step function, this is already given in the problem, so now and we can write further from this, so e^{At}, already you have so if you multiply Ae^{At} into X(0) is given to us, so in our case x naught is we can write 0, So, this is how we can write

$$= \begin{bmatrix} e^{-t} - e^{-2t} \\ e^{-t} + 2e^{-2t} \end{bmatrix} + \begin{bmatrix} 2\int_0^t (e^{-(t-\tau)} - e^{-2(t-\tau)})d\tau \\ 2\int_0^t (2e^{-2(t-\tau)} - e^{-(t-\tau)})d\tau \end{bmatrix}$$

Given the initial conditions, we ccan easily get

$$X(t) = 1 - 2e^{-t} + e^{-2t}$$

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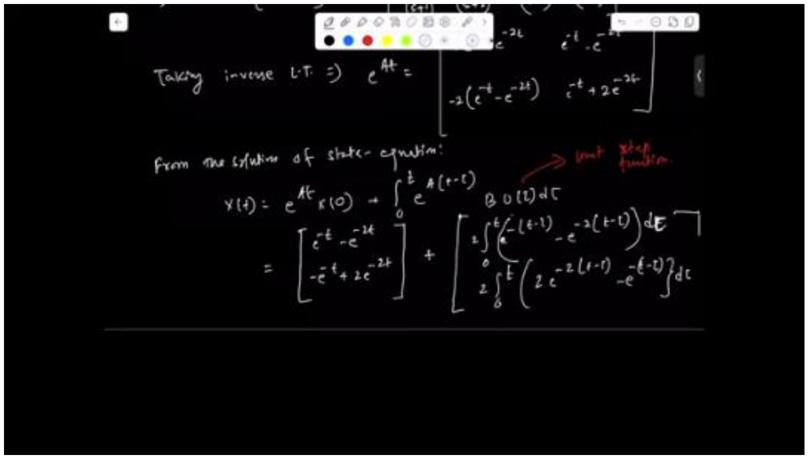

So, there is a solution of the first term in this vector, this is the solution of this.

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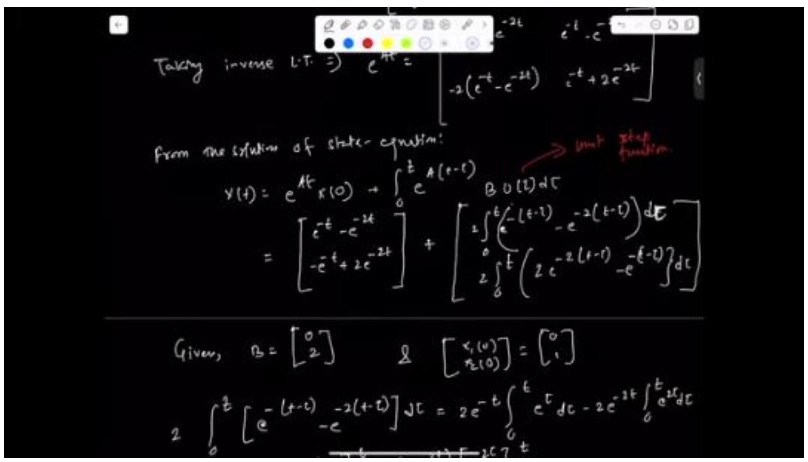

Now, we will work on the second term, similarly we can write

$$2\int_0^t (2e^{-2(t-\tau)} - e^{-(t-\tau)})d\tau = 2e^{-t} - 2e^{-2t}$$

if you substitute all the terms in our solution in this expression, so we can write,

$$X(t) = \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} = \begin{bmatrix} 1 - e^{-t} \\ e^{-t} \end{bmatrix}$$

So, this is the solution of the state equation x not equal to x plus b u. And the output equation we can write, let us have the output equation. So, y this is the state equation solution of state equations and the output equation

$$Y = CX = [3 \quad 1]\begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$$

$$Y = 3 - 2e^{-t}$$

so this is our output equation of the system, now we will go to the matlab code, so matlab code is here, so this is the matlab code, so here professor

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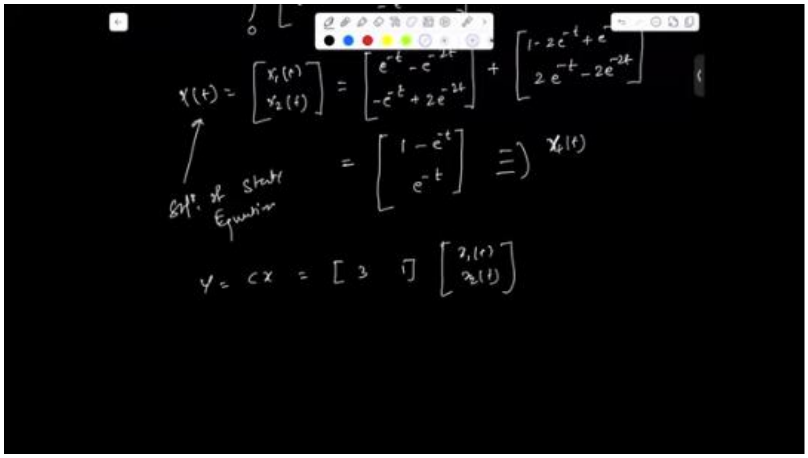

Prabhjeet Singh helped me a lot in preparing the MATLAB code for this course. He is also an instructor of this course. So, he will be covering the 6 DOF model and also some control algorithms for this course. So, now we are having the solution. Here we are having this solution, right?

This is our solution and we have two state equation, X_1 and X_2. So, we will plot these expressions here. so we are simulating this code for 0 to 10 second and here if you notice, this is our 0 to 10 second and this is the expression for X_1, this is the expression for X_2 and if you notice here, this code can can be useful for you, youcan write the code and test and verify in your matlab window, so here if you notice here X_1 of t goes to 1, right, sum, this goes to 1, why? Because e to the power minus t is t tends to infinity, e to the power minus t goes to 1, this is e to the power minus t goes to 0, because if the t is going to be infinity, it is going to be 0, so X_1 goes to 1,

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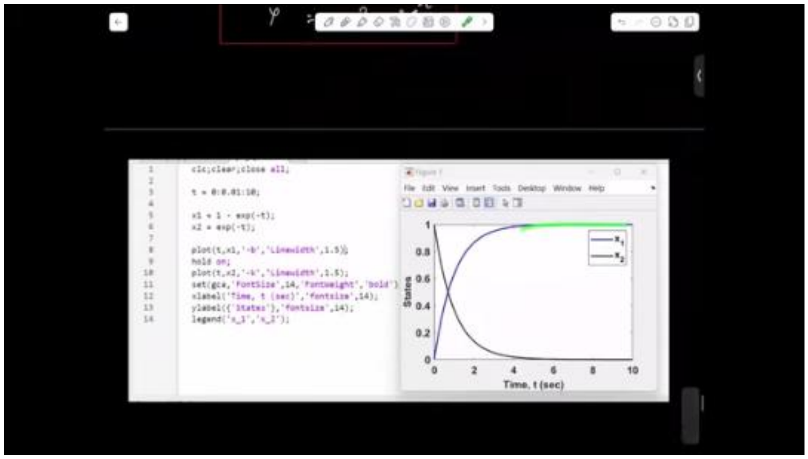

and this goes to zero as t tends to infinity, so here this is the same thing, we can see from this figure, so here the important concept is asymptotic stability, so here we can say that X_1 this is the plot for X_1, this is the plot for X_2, but X_1 instead of going one, it is instead going to zero, it is going to one, but X_2 is going to zero So, X_1 of t is not asymptotically stable. But what is the definition of asymptotically stable? If the system state goes to 0 as time tends to infinity, does not matter where it is starting. But in the X_1, it goes to non-zero than 1.

So, now I will define what is asymptotically stable. Let me write asymptotically stable. so let's assume, we have two region, let's assume we have two region here for example, this is some parameter x, it's just t and let's say, we have having two region, one region let me define, this is the delta minus delta is there, another region let us assume in the control system book, it is already explained so there is another region r epsilon minus epsilon, so we can assume there are two circles, one is the smaller circle, another is a bigger circle, and let us assume sorry t, i can say this is the time axis and this is the X(t), so does not matter where it is starting, if it is starting in some point for example here, some point here and over time if it can go any way but over time, it if it is supposed to go to zero, we can say if it is

it can go like this and go like this, it should go to 0, it should go to 0. So, it means I can say if X(t) naught is starting somewhere within less than delta or less than epsilon, let us assume that I am adding some value c maybe and if limit t tends to infinity X(t) is close to 0 then we can say the system is asymptotically stable this is how can define. So, in this figure, so our X_2 is asymptotically stable, but X_1 it is not asymptotically, but over time it is not going to 0, but X_2 is going to 0.

So, I can say this is asymptotically stable. So, this is how we can find the solution of the system which is defined in state space form and how we can find the output equation or state equation, I hope this example is very useful for you and also we have given the MATLAB code, how to find the solution, sorry how to write the code of a equation and also we have discussed what is the concept of asymptotic stability, over time we will come up with different terminology on stability, we will explain on that particular example, so let's stop it here and in the next lecture, we'll come up with some new topic, if the system is coupled and how can we find decoupled form using some concept which is called linear transformation concept, so we'll discuss that part in the next lecture, thank you.

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