Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 39

Introduction to Backstepping Control

Hello, everyone. Here, we are going to start a new topic on Backstepping Control. This is basically a nonlinear control design approach. In the last lecture, we discussed feedback linearization control, and we studied how we can design the control for the nonlinear system. So here, we are also going to consider backstepping control for the nonlinear system, and we will have MATLAB simulation. Later in this topic, we will have the aircraft problem, and we will see how we can apply this concept to aircraft applications. We will also have the MATLAB code. So let's first start with the concept, the main concept behind this topic. Then we will have the example and MATLAB simulation. So let me write some of this concept, how we can come up with real applications before you go to the aerial systems. Here, the backstepping control is a recursive control design method for the linear system. So what does this mean? The recursive method will come up, recursive control. So this kind of control you can apply to the particular structure of the system.

And that is basically called a recursive structure. And this system here, basically, we're going to have It is primarily useful for systems where the states can be grouped and we go into subsystems. And this subsystem actually will have the recursive structure subsystems. So here, we'll go step by step on how we can come up with the final control design for a particular system using the backstepping control. So let's first, we have the Basically, this is, for example, the first case. So here, a recursive system. So here, the system is divided into subsystems, and here, basically, each subsystem depends on the previous one. So if the system is in the structure, we can say the system is in the recursive structure. For example, if you have a system, for example,

$$\dot{x}_1 = f_1(x_1) + x_2$$

$$\dot{x}_2 = f_2(x_1, x_2) + u$$

So what is, so here, basically, U is going to be designed for x_2. And this x_2 is going to be the control variable for x_1. So for this first subsystem, we're going to have the virtual control for x_2, which will help us to control x_1. And that virtual variable, what we will be assuming for x_2 , will be used to design the overall control for x_2 dynamics. So as you proceed, we will come up with the full idea of how it is happening.

So, the next part is once you have done the next part, we'll go to the next part. So here, for each subsystem, a virtual control will be considered. Which is as a reference signal, a reference for the next subsystem. So it means, suppose for x_2, we will consider some reference signal as a virtual control for x_1 , and this reference or virtual control will be used in the for the next subsystem, for this system. Okay. Then we'll move to the next step. Define the tracking error between the actual state and the virtual control. So, for example, we can assume

$$z_1 = x_1 - x_{1d}$$

$$z_2 = x_2 - \alpha_1$$

This is the error. z_1 is the error, and z_2 is the error for the z_2 dynamics. Okay, here α_1 will be the virtual control. And finally, we will come to the last step here, basically. So here, Lyapunov function is constructed for each subsystem to prove stability. So, these are the processes we will be following for designing the backstepping control. Now, let's take an example, and we will validate this project. How we can come up with an example where we will be using all these steps.So, as an example, in this example, we will also have MATLAB code to validate our results. So, let us consider The following non-linear system is defined by

$$\dot{x}_1 = f_1(x_1) + x_2$$

$$\dot{x}_2 = f_2(x_1, x_2) + u$$

So, if you notice, this system is in a recursive structure. So here, if you notice, this $f_1(x_1)$ is the only function of x_1 . And here, this is actually a function of x_1 and x_2. So, if it is $\dot{x}_3$, then the function will be $f_3(x_1, x_2, x_3)$. So, if the system is written in this structure, we can say the system is in a recursive structure. So now, here we are, f_1 and f_2 are the nonlinear functions. Let's assume they are nonlinear functions, and u is the control input. You can say now our objective is to design u in such a way that x_1 tracks x_{1d}, (d is desired) okay? And the overall closed-loop system is stabilized. So, this is our mission objective for this problem. Here, you'll be using a backstepping-based control concept and validate all the loops you have considered. This process will move on, okay? So, the first step is step one. Solution step one So here, first, we will define the error, the tracking error for the first subsystem. We can write Let us define the tracking error

$$z_1 = x_1 - x_{1d}$$

$$\dot{z}_1 = x_2 + f_1(x_1) - \dot{x}_{1d} \; \dots Eq(1)$$

Now, step two. Here, let us define the virtual control for x2. Okay. So let us assume that this x_2 will be the virtual control for z_1 dynamics. So let us write, consider x_2 as a virtual control or virtual control input and design the stabilizing. So in this particular topic, you will have more than five or six examples, and I hope through these examples, you'll have a better understanding to design backstepping control for the nonlinear system. So, stabilizing, design the stabilizing control law for z_1 . So for this, let us choose alpha1 be the virtual control input for z_1. So here, this is how I am, how we are choosing. So here, this is k1 be the control gain parameter, which is a positive control gain. If you notice carefully, if you substitute this function,

$$\alpha_1 = -k_1 z_1 - f_1(x_1) + \dot{x}_{1d} \; \dots Eq(2)$$

So we just, we will prove this through the Lyapunov.

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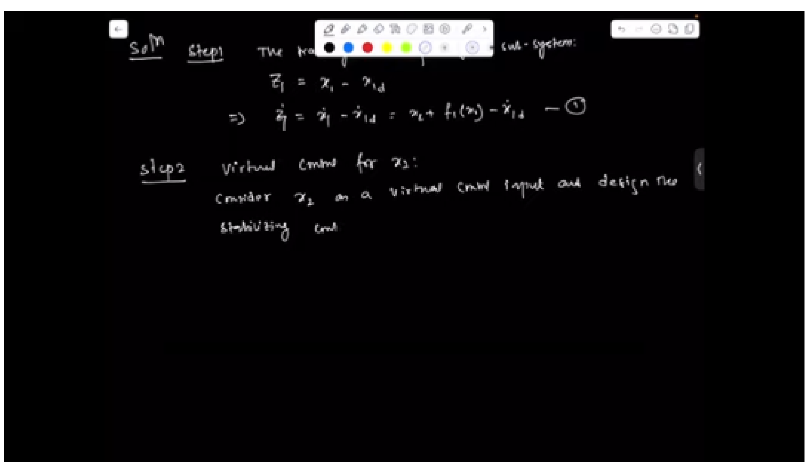

Okay. Now, Combine equation 1 and equation 2, we can write

$$\dot{z}_1 = -k_1 z_1 + x_2 - \alpha_1$$

So if you notice here carefully, once these two parameters are equal, then this becomes a stabilizing function. This stable dynamics, right?. So this is what we have done, right? So in place of x_2 , we are choosing α_1 to be the stabilizing function for z_1 dynamics. So now, step 3: we will prove this through Lyapunov. Step 3: let us define our Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2$$

So this is the dynamics equation we are having. And for this, we are choosing the Lyapunov function in the structure. Now from this, we can write

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1(-k_1 z_1 + x_2 - \alpha_1)$$

So now if $x_2 = \alpha_1$, we can say this is the virtual control, right, α_1. So we can write

$$\dot{V}_1 = -k_1 z_1^2 \leq 0$$

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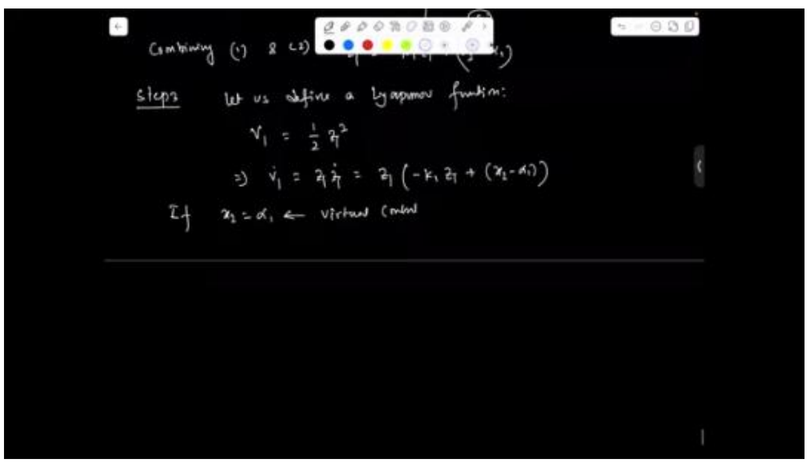

So this is, we can say, negative semi-definite. And this is, through this, we can say that the system is stable. So how are we checking stability? So if you substitute, so for this condition, from this equation, we can write

$$\dot{z}_1 = -k_1 z_1$$

so this is basically globally asymptotically stable. This definition we have done in our previous lecture. What is a globally stable system? And here, why is it stable? Because here, k_1 is always positive. Now, we will be designing the overall control u for the entire system. So now, let me write one note to help understand better how we can find the error for the z_2 dynamics. To achieve this, this means this asymptotically stable system, we can write the state x_2 must be equal to the virtual control. α_1, but since x_2 and α_1 start from different initial values, we must must design the control force for x_2 so that, or you can write here, control force to track x_2 , or we can write further for x_2 to track α_1. So this is, so here in this case, we can write uh step four, step four define the error between x_2 and α_1. So in this case, we can write

$$z_2 = x_2 - \alpha_1$$

$$\dot{z}_2 = u + f_2(x_1, x_2) - \dot{\alpha}_1 \dots Eq(2)$$

So now we'll be designing the overall control u for the entire system. So here's step five So now we have to design u control to stabilize z_2 dynamics. So now let us choose, let us choose u as

$$u = -k_2 z_2 - f_2(x_1, x_2) + \dot{\alpha}_1 \;\dots Eq(4)$$

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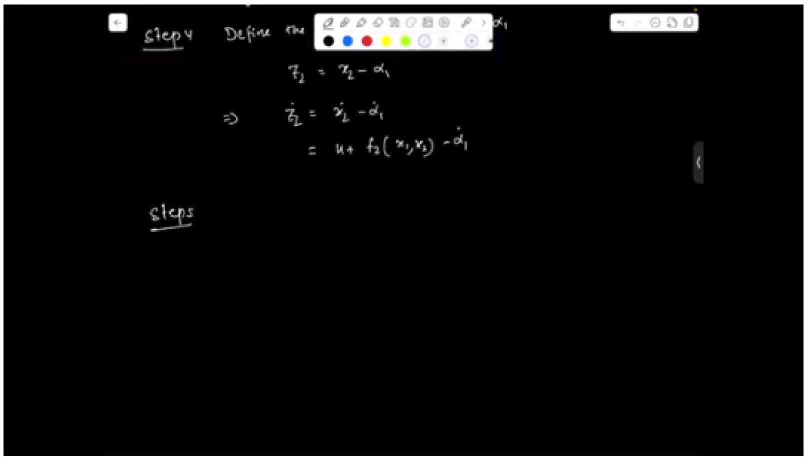

So if you notice here, in place of u, if you substitute this expression, so we will get $\dot{z}_2$ equal to this part. So it is also stable dynamics. So because k_2 , so here we can write k_2 is a, greater than zero positive control gain parameter. Now substituting 4 in equation 3, we can write

$$\dot{z}_2 = -k_2 z_2$$

So this is our stable dynamics. Now we will prove the overall stability of the system step six. So in this stability proof, we will come up with some condition further Control gain parameters will help us to choose the specific value of the control gains for practical examples. Now, let us define the Lyapunov function for the entire system:

$$V_2 = V_1 + \frac{1}{2}z_2^2$$

$$\dot{V}_2 = z_1\dot{z}_1 + z_2\dot{z}_2$$

$$= -k_1z_1^2 + z_1z_2 - k_2z_2^2$$

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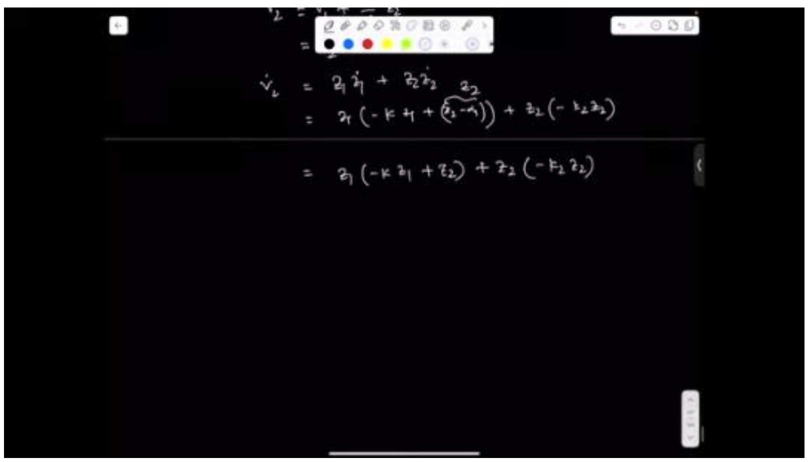

But we know that using the fact we can write

$$z_1z_2 \leq \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$$

$$\leq -\left(k_1 - \frac{1}{2}\right)z_1^2 - \left(k_2 - \frac{1}{2}\right)z_2^2$$

So now, if k_1 is greater than half and k_2 greater than half, $\dot{V}_2$ is less than or equal to zero. So, this ensures the stability of the overall system. Now, we have proved the overall backstepping process for this particular example. Now, from the above analysis, we can come up with the main control structure, which would be useful for designing the MATLAB code. So, from the above analysis, the controls are

$$u = -k_2z_2 - f_2(x_1, x_2) + \dot{\alpha}_1$$

$$\alpha_1 = -k_1z_1 - f_1(x_1) + \dot{x}_{1d}$$

$$z_1 = x_1 - x_{1d}$$

$$z_2 = x_2 - \alpha_1$$

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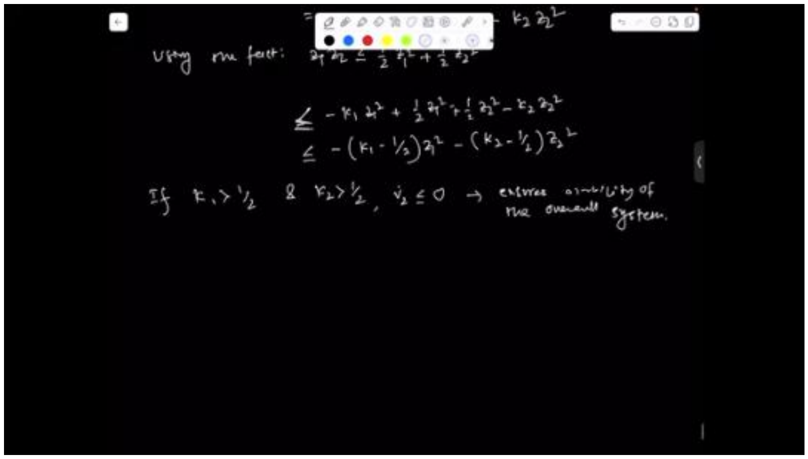

Now, we'll go to the MATLAB code and see how we can validate the whole concept and verify the result, okay? So here, whatever process we have done in this lecture, it is there. So we have taken, as per the stability condition, these are the values we have considered. And in this case, k_1 and k_2 should be like this. And these are the initial conditions for x_1 and x_2 (0) And this is the desired dynamics for x_1 , and the derivative of $\dot{x}_1$ is this. The total time span of the simulation is 20 seconds, and we have used ode45 to solve this problem. These are the MATLAB codes for the figure, and here we have defined two variables, x_1 and x_2 , like that. And these are the nonlinear functions we have introduced in this example:

$$f_1 = 0.1x_1^2$$

$$f_2 = 0.2x_1x_2$$

So these are the nonlinear functions we have considered in this example, and we have This is the error variable we have chosen, z_1 Z1, and α_1 , whatever we have taken from the analysis, is written here,

$$z_2 = x_2 - \alpha_1$$

and this is the overall control, which we have obtained here, this expression, and this is the derivative part of x_1 and x_2. So now, if you notice here, the result,

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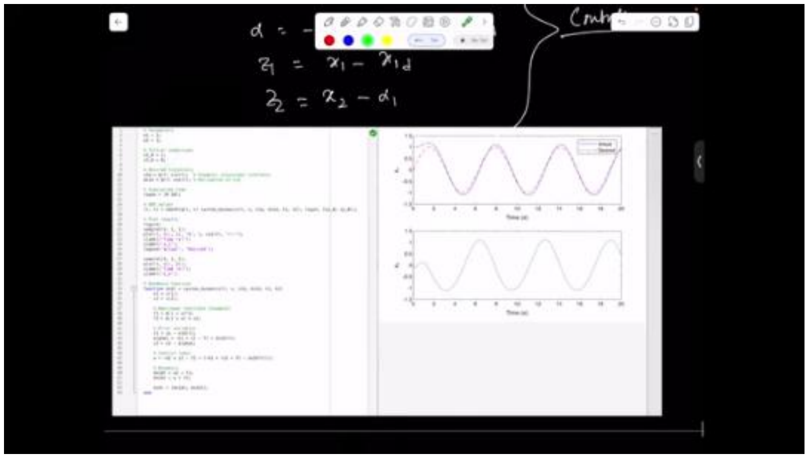

x_1 tracks the desired, which is the dotted line with red color. And x_2 is also stabilizing. So this is how we can design the backstepping-based control for this nonlinear system. In the next lecture, we'll come up with different examples and how we can also solve them through MATLAB. Thank you.