Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 38

Feedback Linearization based control for lateral dynamics

Hello everyone, in today's lecture, we will be covering this topic: pitch dynamic motion of the aircraft, how we can design the control for the pitch dynamic motion, and also we will have the MATLAB simulation for the same example. Finally, we will discuss how we can design a control feedback optimization control for the lateral motion dynamics of an aircraft. So, let's start the example. We will take a simple example and also use the MATLAB simulation code for the same. Let us consider the pitch dynamic motion, the pitch dynamics of an aircraft:

$$\dot{\theta} = q$$

$$\dot{q} = \frac{M}{I_{yy}} \quad \ldots Eq(1)$$

So here, θ is the pitch angle, q is the pitch rate, and M is the total moment acting on the system and strain or semantics along the pitch axis. So here we can write the pitching moment, which is M, the pitching moment

$$M = M_0 + M_\alpha \alpha + M_q q + M_{\delta e}\delta_e$$

So here, these are the terms we have already defined in the last lecture, and here delta is the only control, which is basically elevator deflection to control the pitch motion. So here, the objective is to design control which makes the pitch angle θ track the desired trajectory. θ D, okay, and also the MATLAB simulation to verify the result through MATLAB simulation. So, this is the problem statement. Now, let us go to the solution part.

So, for simplicity, we are going to assume some values for M_0, M_α, M_q, and $M_{\delta e}$. These are basically aerodynamic coefficient parameters. So, those parameters can be obtained through wind tunnel testing. So, for simplicity, let's assume

$$M_0 = 0, M_\alpha = -2, M_q = -1, M_{\delta e} = 0, I_{yy} = 1$$

So, why have we taken these numerical values? Because we need to do the MATLAB simulation. To solve the MATLAB simulation, we need to have the numerical values of these parameters here. Based on these values, based on these values, our system dynamics, which is defined by equation one, equation one

$$\dot{\theta} = q$$

$$\dot{q} = -2\alpha - q + \delta_e$$

And our control objective is for θ to track θ_d as t tends to infinity. Or we can say the error tends to zero as t tends to infinity. Right. So here, the error can be defined as θ minus θ d. So here, since we are going to control θ as the parameter, we can assume the output of the system is θ. So, let's define the output as the pitch angle. Pitch angle, and we can write

$$y = \theta \ \ldots Eq(2)$$

Now, our primary objective, as you have done in the previous lecture, is to take the time derivative of this equation until the control appears in this equation.

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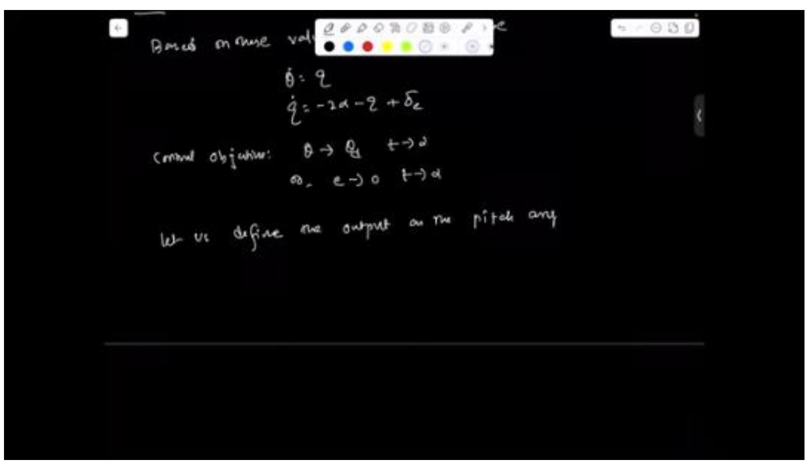

So, before that, we can define the error, the tracking error. Tracking error, we can write,

$$e = \theta - \theta_d$$

So, now we will take the time derivative of this equation, equation 2. Taking the time derivative of equation 2, we write

$$\dot{y} = \dot{\theta}$$

If you notice here, $\dot{\theta}$ equal to q, so in the q equation, there is no control. Only the $\dot{q}$ equation has the control δ_e. So now, we have to take the second derivative here,

$$\ddot{y} = \dot{q} = -2\alpha - q + \delta_e$$

Now, let us assume, so here our main motivation is how we can control this q. So, let me write, to linearize the system, To linearize the system, the control input, I can write the new control input such as

$$v = -2\alpha - q + \delta_e$$

$$\delta_e = v + 2\alpha + q \quad \dots Eq(*)$$

Now, let us define the error dynamics. So, we can write, we can come up with the system in error dynamics form. So, let us go from the error equation. Or we can write from $e = \theta - \theta_d$. We can write

$$\dot{e} = \dot{\theta} - \dot{\theta}_d$$

$$\ddot{e} = \ddot{\theta} - \ddot{\theta}_d = v - \ddot{\theta}_d \quad Eq(2)$$

And from this, we can write Now we have to choose, we have to choose v, this v in such a way that e is going to zero, right? So if e is going to zero, so you can say θ is going to track θ to zero, right? Because e is the difference between θ and θ to zero.

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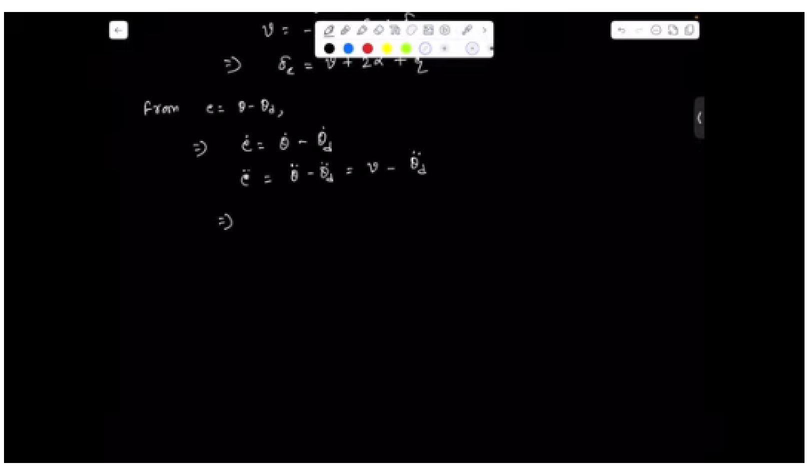

So our aim is to choose v in such a way that e tends to zero as t tends to infinity. So from this, we can write we can choose v as

$$v = \ddot{\theta}_d - k_1 e - k_2 \dot{e} \quad \dots Eq(3)$$

So this is basically PD control, where k_1 and k_2 are greater than zero. Now, if you notice here, if you substitute this v in here, we'll have the aerodynamic equation, which is basically stable. Now, substituting equation 3 in equation two, we can write

$$\ddot{e} + k_2 \dot{e} + k_1 e = 0 \quad \dots Eq(4)$$

So this is very easy. since k_1 and k_2 are greater than zero, this equation four, Equation 4 is a stable dynamics. So stable dynamics means the eigenvalues of this equation are negative. So we can say that e tends to 0 as t tends to infinity for positive values of k_1 and k_2. Right, so now we can find the overall control δ_e from this analysis. So what we'll do is we'll substitute this expression here. Okay, so let us define this equation star, for example. Now, substitute equation 3 in equation star, so you can find the overall control for our dynamic system, this system. Now, substituting substitution 3 in equation star, we can write about the finite nonlinear control, we can write

$$\delta_e = \ddot{\theta}_d - k_1 e - k_2 \dot{e} + 2\alpha + q \quad \dots Eq(5)$$

So this is our overall control for this system. So let us this is equation five. So equation five is going to stabilize the system, which is defined by equation one, meaning system. And this control is going to track θ_d as time proceeds. Now, let's have the MATLAB code for this example. This is the MATLAB code for this system. So whatever values have been taken in this example, we have taken the same values here. And these are the desired trajectory of θ and θ d. And this is the initial condition for θ and q because to solve any differential equation, you need to have the initial values of the states. And this is we are going to simulate this program for 0 to 10 seconds. And this is the ODE. We have used the ode45, and this is the function defined here. And this is the code for the plots. And this is our tracking error, whatever you have taken in the example. And this is the overall control input you have obtained in this equation five. So this same equation is defined here.

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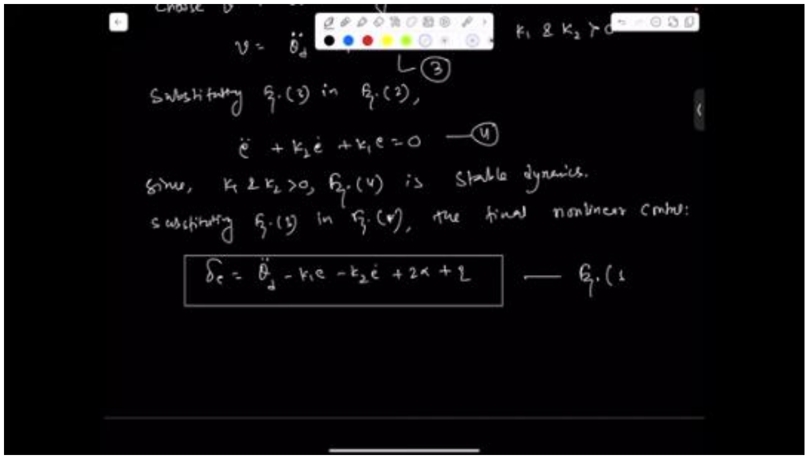

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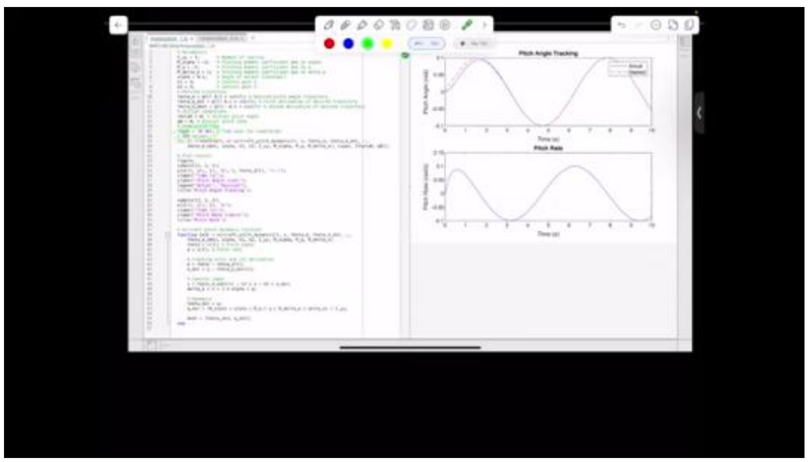

And this is our dynamic system. What we have taken is the dynamic system here, and these are the states going to our function here. If you notice, this system is controlled using the design control that we have done here. If you notice here, the actual values, which are different, which is the blue line, are tracked to the desired state, the desired value of the pitch angle, and this is the pitch rate. So, this is how we can simulate our nonlinear system using the proposed design control, using the feedback linearization concept. So, if you have another example of an aircraft system, you can follow the same steps to design the control. Now, we'll go to another example for our lateral motion dynamics of the aircraft. So, first, let us define the example that we're going to solve.

So, this is the lateral motion dynamics problem example. So, let us consider the following nonlinear motion dynamics, lateral motion dynamics, motion dynamics of an aircraft. are given as β dot. So, these are the states that are going to be involved in the lateral motion dynamics:

$$\dot{\beta} = \frac{y_\beta}{mV}\beta + \frac{y_p}{mV}p + \frac{y_r}{mV}r + \frac{y_{\delta a}}{mV}\delta_a + \frac{y_{\delta r}}{mV}\delta_r$$

$$\dot{\beta} = f_1(\beta, p, r) + \frac{y_{\delta a}}{mV}\delta_a + \frac{y_{\delta r}}{mV}\delta_r$$

$$\dot{p} = L_\beta\beta + L_p p + L_r r + L_{\delta a}\delta_a + L_{\delta r}\delta_r$$

$$\dot{p} = f_2(\beta, p, r) + L_{\delta a}\delta_a + L_{\delta r}\delta_r$$

$$\dot{r} = N_\beta\beta + N_p p + N_r r + N_{\delta a}\delta_a + N_{\delta r}\delta_r$$

$$\dot{r} = f_3(\beta, p, r) + N_{\delta a}\delta_a + N_{\delta r}\delta_r$$

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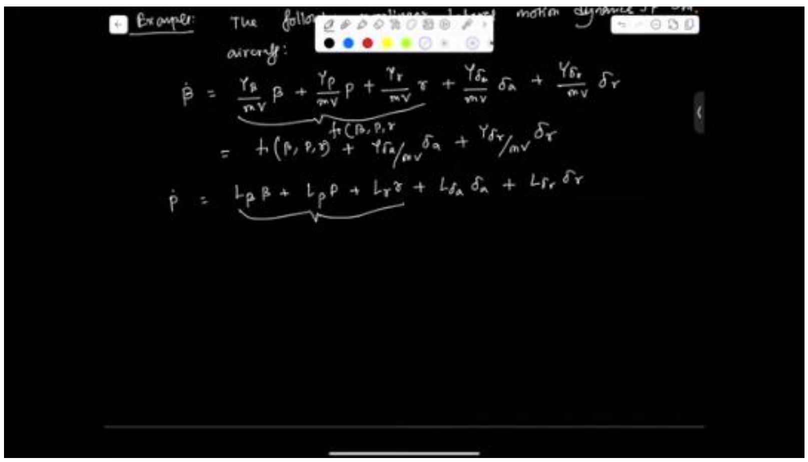

okay let me define what these parameters are here so here and this whole system we can uh this is basically the lateral motion dynamics of an aircraft system right so β equal to here β is nothing but side slip angle and r is roll rate p is sorry r is yaw rate basically here yaw rate p is roll rate and δ_a aileron deflection and δ_r deflection and y_β, y_p, y_r, $y_{\delta a}$, $y_{\delta r}$. This is aerodynamic derivatives for side force. And so these are the parameters we get from some testing and for a particular aircraft, these data are very important.

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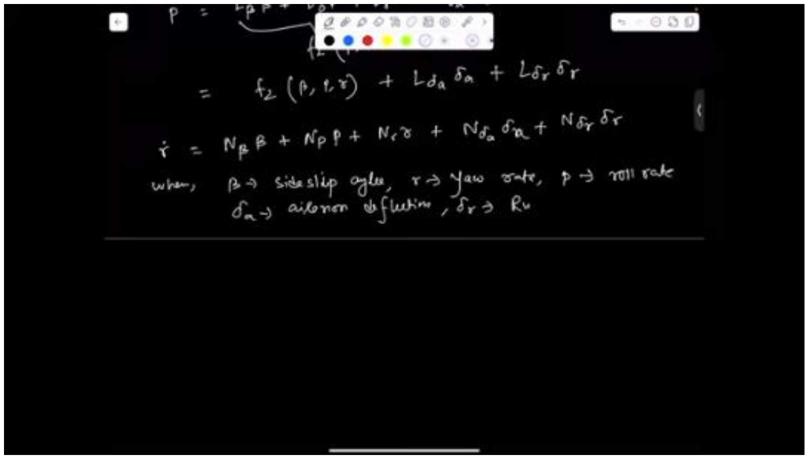

So most of the industries, they keep these things confidential for a particular aircraft. And $L_\beta, L_p, L_r, L_{\delta a}, L_{\delta r}$, this is basically aerodynamic. derivatives for rolling moment. And N_β , N_p , N_r , $N_{\delta a}$, $N_{\delta r}$ are aerodynamic coefficient for rolling moment, aerodynamic coefficient, or we can write aerodynamic derivatives, aerodynamic derivatives for yaw moment.

And here mass M is the aircraft mass and V is the cruise airspeed. Now our control objective is control objective for this particular problem and design control so if you notice here this system is quite complex because in each equation we have different control part delta in each equation delta and delta are in equation so designing control for this particular system and may be challenging but we'll make the problem very simplified so let's work on this design control for δ_a and δ_r such way that by that β and r can track can track β desired and r desired so this is your control objective so here only we are controlling β angle uh β this β and r these two equations okay so we can take these two equations to design the control so here let's start the problem solution.Let us define the output equation. Since you are going to use the input-output feedback generation-based control, here, since this is our objective, from this, we can find the output equation, right? So, the output equation y,

$$Y = \begin{bmatrix} \beta \\ r \end{bmatrix}, \quad Y_{des} = \begin{bmatrix} \beta_{des} \\ r_{des} \end{bmatrix}$$

Now, based on this, we can find the error. So, based on this, we have

$$e_1 = \beta - \beta_{des}$$

$$e_2 = r - r_{des}$$

Okay, and our control goal, our objective, since we have found the error, we can say e_1 and e_2 go to 0 as t tends to infinity. So, if this condition is satisfied, we can say β can track β_desired, and r can track r_desired once this condition is satisfied, right? So, now we'll design the control and see how we can do these things, okay. So, here, let's start with the first equation, this equation, and we'll design the control, and then we'll come up to the second part. So, here, let us define this as equation number one, and this is two. From one, we can write

$$\dot{e}_1 = f_1(\beta, p, r) + \frac{y_{\delta a}}{mV}\delta_a + \frac{y_{\delta r}}{mV}\delta_r - \dot{\beta}_{des} \quad \dots Eq(3)$$

So, if you notice here, in the $\dot{\beta}$ equation, we have the control here. Here, you see, in the β_dot equation, we have the control. So, no need to go to the second derivative. In the first derivative itself, we have the control signal appearing in the $\dot{\beta}$ equation. So, from this, we can write, we can substitute here the $\dot{\beta}$ equation. So, here, since our main aim is to get to zero, for that, we can choose some desired control for $\dot{e}_1$ so that this can be achieved, okay. So, here, what you want to do is let

$$\dot{e}_1 = u_1$$

This is basically to control the error, okay? u_1 error u_1 , and from this, now let us choose u_1 in such a way that this can be achieved, okay. So, for that, let us choose

$$u_1 = -k_1 e_1 - k_2 \dot{e}_1 \quad \dots Eq(4)$$

So, we can write, substituting equation 4 in 3

$$\frac{y_{\delta a}}{mV}\delta_a + \frac{y_{\delta r}}{mV}\delta_r = -k_1 e_1 - k_2 \dot{e}_1 - f_1(\beta, p, r) + \dot{\beta}_{des} \quad \dots Eq(5)$$

Okay, now what we'll do is we'll take the second error equation, this equation. So from the first equation, this is our, we have these two equations, five Now, we will go to the second part, this part. Okay, the equation 2.

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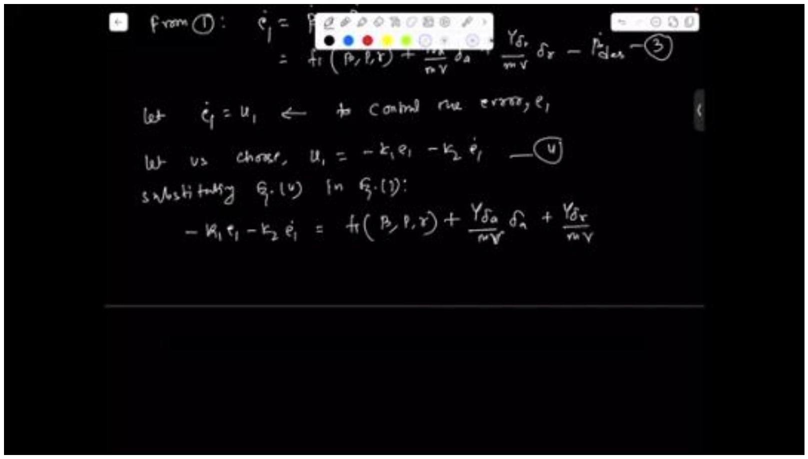

So, let me write from equation 2, we can write

$$\dot{e}_2 = f_3(\beta, p, r) + N_{\delta a}\delta_a + N_{\delta r}\delta_r - \dot{r}_{des} \ldots Eq(6)$$

let's choose $\dot{e}_2 = u_2$, so which is going to control To control the error e_2, right? So to control e_2 , we have to choose u_2 to be stable dynamics, right? Some error, some control, which is basically going to make e_2 to 0 over time. So for that, let's choose, let's choose

$$u_2 = -k_3 e_2 - k_4 \dot{e}_2 \ldots Eq(7)$$

simple PD control. Now we'll substitute this u_2 here, and this $\dot{e}_2$ we can substitute here, and we can come up with the function of this and this part, right, as a relation.

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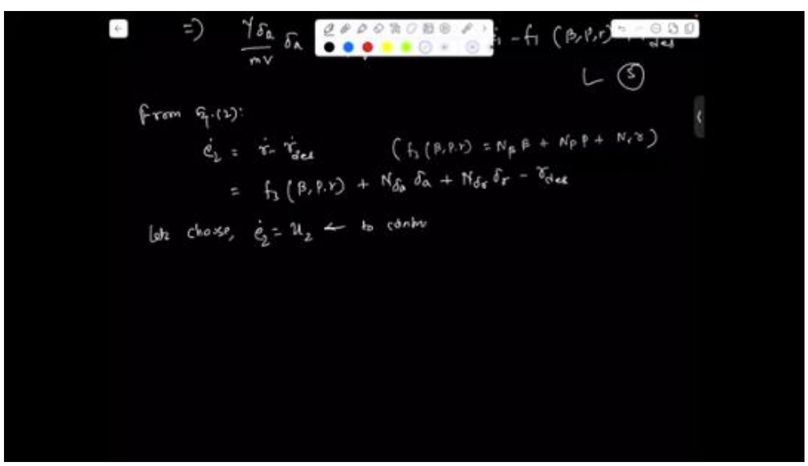

Now substituting equation 7 in equation 6, we can write

$$N_{\delta a}\delta_a + N_{\delta r}\delta_r = -k_3 e_2 - k_4 \dot{e}_2 - f_3(\beta, p, r) + \dot{r}_{des} \ldots Eq(8)$$

Now from equation 5, and equation 8, we can come up with some relation in matrix form. So we can write from equation 5 and 8, we can write

$$\begin{bmatrix}\delta_a \\ \delta_r\end{bmatrix} = [B]^{-1}\begin{bmatrix}-k_1 e_1 - k_2 \dot{e}_1 - f_1(\beta, p, r) + \dot{\beta}_{des} \\ -k_3 e_2 - k_4 \dot{e}_2 - f_3(\beta, p, r) + \dot{r}_{des}\end{bmatrix}$$

So this is our nonlinear control to control the dynamics what we have defined here. This is for the system.

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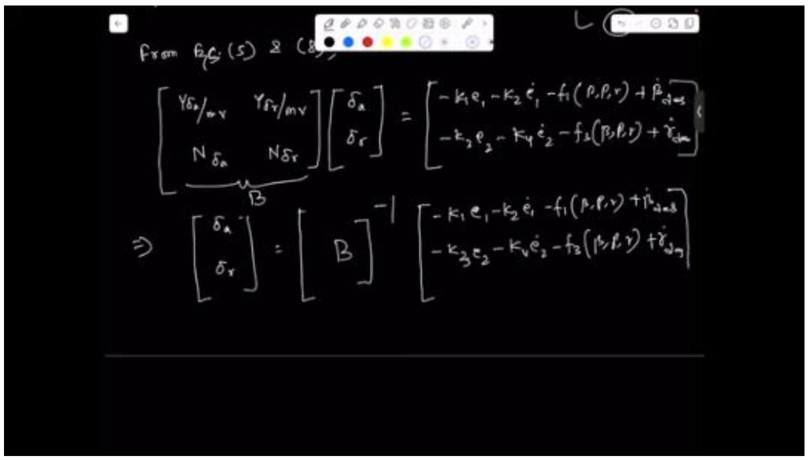

And this control is going to fulfill our mission objective. This is our objective. So this is going to help us to track δ_a, sorry, β and R2, β desired and R desired. So this is how we can design the control, linear, equitable linear synthesis control for the nonlinear system. And I hope you can design your control for other nonlinear systems in aircraft dynamics or any other dynamical system.

This is the last lecture on this topic. From the next lecture onwards, we will have another topic on backstepping-based control for aircraft applications. Thank you.