Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 37

Feedback linearization control for longitudinal dynamics

Hello everyone. In today's lecture, we will be taking the full longitudinal dynamics of the aircraft and how we can design multiple states at a time. So let us check the problem. Example. We will take the nonlinear system of the aircraft. So let us consider the nonlinear longitudinal dynamics of an aircraft, which is

$$\dot{V} = \frac{1}{m}(T\cos\alpha - D) - g\sin\gamma$$

$$\dot{\gamma} = \frac{1}{mV}(L + T\sin\alpha) - \frac{g}{V}\cos\gamma \quad \ldots Eq(1)$$

$$\dot{\theta} = q$$

$$\dot{q} = \frac{M}{I_{yy}}$$

so this is the full long-term motion dynamics of an aircraft, and here maybe there are different in the different literature we have different models of the equations, but we are taking this equation, and this is highly nonlinear, and how we can design the nonlinear controls for the system using the feedback linearization based. So where are the parameters? Let me define the parameters for this in these equations. So here V is the airspeed and $\gamma$is the flight path angle, which is basically γequal to $\theta - \alpha$, so here θ is the pitch angle and q is the pitch rate, and T is the thrust applied to control the longitudinal motion of the system, and D is the drag.

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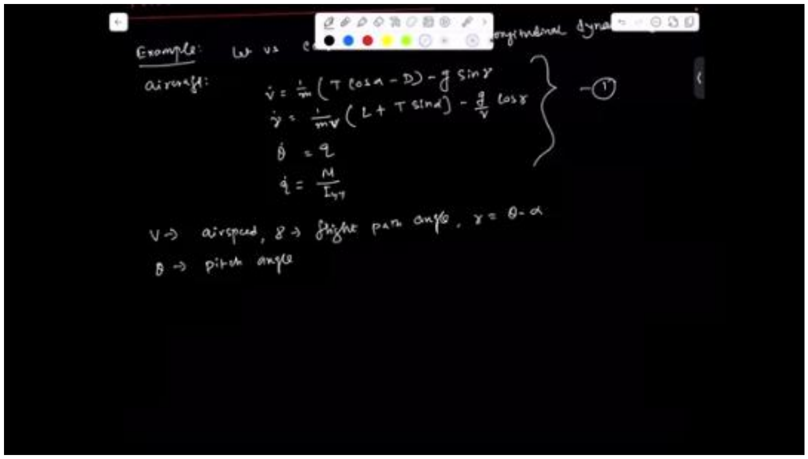

And lift, here M is the pitching moment, and small m is the mass and this is the moment here in the $\dot{q}$ equation. Small m is the mass of the aircraft, and I_{yy} is the principal moment along this axis. Some moment along each axis, and g is the gravitational acceleration. So, this is our dynamics equation, which is defined by Equation 1. And we are going to design control, which is going to help propagate the state variables, which are u, γ, θ, q. Here, we are going to control multiple states at a time for this system. So, our objective is to design the control. To control the airspeed, which is V, the pitch angle θ, and the pitch rate q to track the desired trajectories. So, we have to design a control which is going to help us For this dynamical system to follow the desired airspeed, pitch angle, and desired pitch rate. So, here we are going to have some simplification for this problem to solve it. The solution we are assuming that For simplicity, this is just an assumption to make the problem much simpler. Otherwise, you can go with the equation as it is. For simplicity, let us assume the angle of attack is small, and the small angle of attack, we can come up with some simplification for this term, cos alpha and sin alpha. So, what you're going to do is, here you can write cos alpha is assumed to be, for small angles, it's approximately one, and sin alpha is assumed to be alpha. So here, the pitching moment is also a function of the variables in the system. So, let us, this thing we have covered in our classical controls, the first course, aircraft control system. There, we have discussed all these moment and force equations. The pitching moment M Is a function of alpha, q, and the elevator deflection delta_e. So, in this case, we can write the total moment due to these parameters. We can write

$$M = M_0 + M_\alpha \alpha + M_q q + M_{\delta e} \delta_e$$

So, these are the values, basically, very specific to the particular aircraft. This terminology, so we can get these values from the wind tunnel testing. So, these are for the particular aircraft. These values are basically given, and based on that, we can come up with this relationship. And here, actually, this is our control, and based on that, we can come up with the m. So, let us, while we are solving this problem, it will be clear how we are going to do this. So here, where m naught, this is the, we can say, baseline pitching moment. And M_{α}, pitching moment coefficient due to the angle of attack alpha, and M_q , and this is the pitching moment coefficient. Coefficient are due to pitch rate q and $M_{\delta e}$, this is the pitching moment coefficient due to elevator deflection, elevator deflection right here. Deflection δ_e is the control input here, control input, and this is nothing but the elevator deflection. Yeah, so now we have to design the control input, this delta e, and this problem now, similarly. The lift and drag are the function of alpha and V, that we know.

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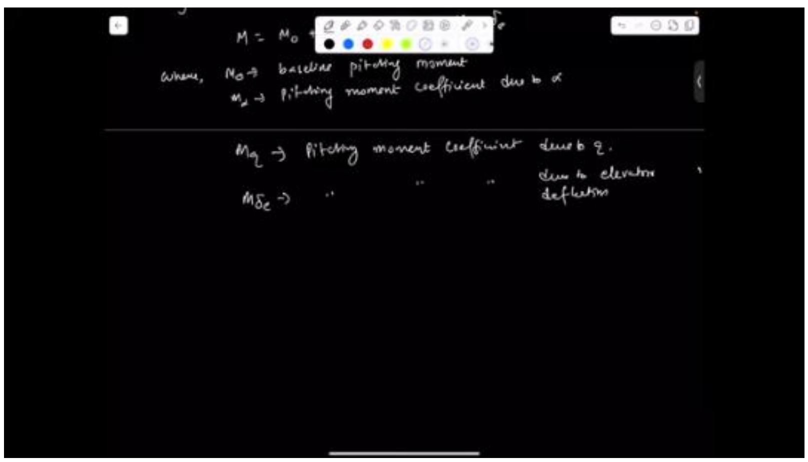

$$L = \frac{1}{2}\rho V^2 S C_L(\alpha)$$

$$D = \frac{1}{2}\rho V^2 S C_D(\alpha)$$

So here, basically, ρ is the air density, about the air density, and S is the wing area, and $C_L(\alpha)$, and $C_D(\alpha)$,, lift and drag. Coefficient, okay. So, as per the problem, what we have, as per the objective, so our control objective is control objective. Process we are following to solve this problem, the same process we can follow for other dynamical systems. So, these are the basic examples of how we can design the control for this particular system. So, if you have another dynamical system in a different form, we can follow the same steps. And here, actually, we are going to use the input-output feedback linearization, what you have done in the last example in the last lecture. So here, the control objective is. V should track v desired velocity, and θ should track θ desired, and q should track q desired. So, this is our mission objective. So now, let us define a state vector, since you are going to control multiple states in the system, so let us define.

$$X = \begin{bmatrix} V \\ \gamma \\ \theta \\ q \end{bmatrix} \qquad Y = \begin{bmatrix} V \\ \theta \end{bmatrix}$$

So, these are the variables coming from the system, and these are the states going to be used to design the control. So here, our main motivation, if you remember in the last lecture. We are going to use this output equation to find the control. Because we will take the output equation and we will do the derivations or differentiation for this equation, and we will come up with the derivative of these states, which will be a function of the controls. So, here what we are going to do is, let me write here, we need to. Apply differentiations to the output vector until the control input appears. So, here what we are going to do is we will take the first variable v, the variable, there are two variables in the output, V and θ. So, we will take the first variable.

$$\dot{V} = \frac{1}{m}(T\cos\alpha - D) - g\sin\gamma \ldots Eq(2)$$

So here, if you notice here, we are having control T in the system. Because in this equation, there are two control inputs. One is the T, and another is the δ_e. Here delta, right? So delta is going to provide the moment which is going to control the angle, the pitch rate basically, and our thrust is going to control the velocity of the aircraft. So there are two control inputs for this dynamics, so we need to design these two controls. So in this equation, you can notice the T is up here, so we can write here. Hence thrust appears in $\dot{V}$, and so we can stop here, right? So now we will take the second state in the output equation which is θ. So here we can write $\dot{\theta} = q$. So here if you notice here, this moment. Or the control which is nothing but the m actually means function of the delta e. So this does not appear in this equation, the control does not appear. So we have to do again, we have to take the time derivative

$$\ddot{\theta} = \dot{q} = \frac{M}{I_{yy}} \ldots Eq(3)$$

So here M appears in the output equation, so we can write here. The elevator deflection δ_e appears in $\ddot{\theta}$. So now we can start the feedback generation concept what you have done in the last lecture. We can follow the steps and we will do the, we can find the control laws for. T and δ_e. So now let us choose u_1 and u_2 as the new variables, new control inputs.

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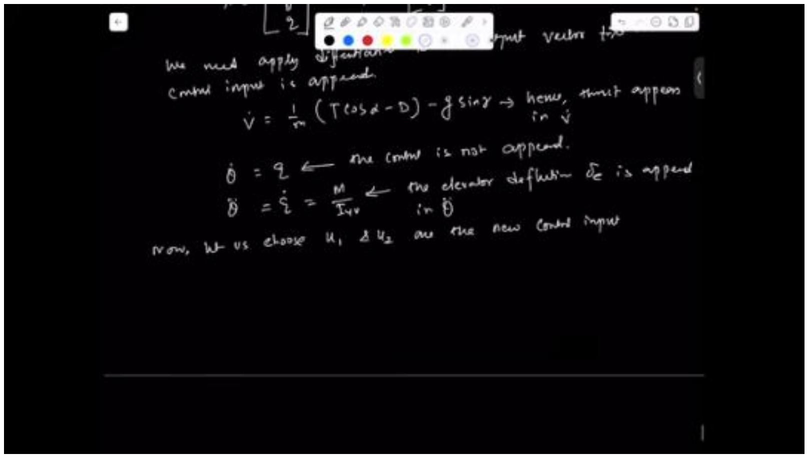

Okay, this is three equations. Now, these new control inputs, u_1 , u_2, for $\dot{V}$ and $\ddot{\theta}$ okay? So, in this case, we can write

$$\dot{V} = u_1 \Rightarrow \frac{1}{m}(T\cos\alpha - D) - g\sin\gamma$$

$$T = \frac{m(u_1 + g\sin\gamma) + D}{\cos\alpha}$$

So, this is how we can find the control T which is the function of some control, linear control here. This u_1 is a linear control because this is a linear system, right? So now this is how we can design the thrust, which will provide the dynamics to control B. Now we'll go to the second equation, sorry, the third equation here, this equation. So here we'll design control for $\dot{\theta}$, which is defined by u_2. So let's, Similarly, you can write for equation 3, you can write θ double dot equal to u_2. And from this, you can write

$$\ddot{\theta} = u_2 \Rightarrow \frac{M}{I_{yy}} = u_2$$

And as you know, you have already defined the expression for M here, this expression. So, we can substitute this M in this equation, in this equation. And from this, we can write delta E equal to

$$\delta_e = \frac{I_{yy}u_2 - M_0 - M_\alpha\alpha - M_q q}{M_{\delta e}}$$

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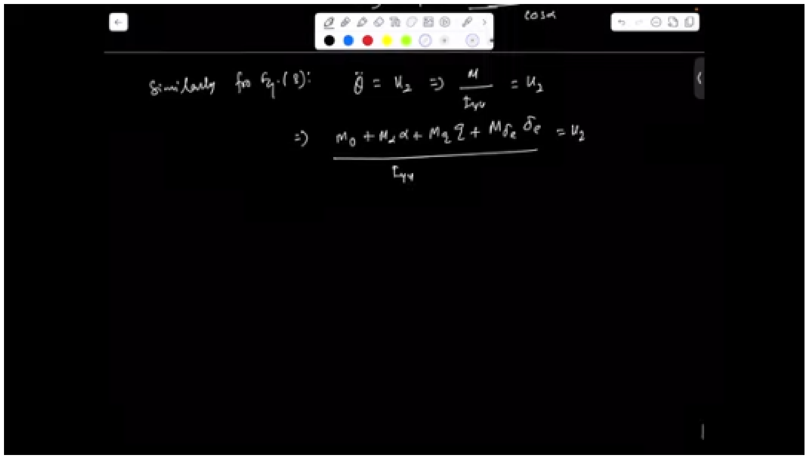

from this analysis, we can write, so now let us design, let us design

$$\dot{V} = u_1$$

$$\ddot{\theta} = u_2 \dots Eq(4)$$

So we need to design the control for this system. This is basically a linear system, and we are going to design linear controls for v and linear control for u_2 for θ. Right? So here we are going to choose this u_1, u_2, which are to stabilize. Now, to stabilize equation 4, u_1 and u_2 are chosen as,

$$u_1 = \dot{V}_d + k_{v1}(V_d(t) - V)$$

$$u_2 = \ddot{\theta}_d + k_{\theta 1}(\theta_d - \theta) + k_{\theta 2}(\dot{\theta}_d - \dot{\theta})$$

it represents a time series. So instead of that, I can explain how it is happening here in this particular case. So here, basically, what you are doing is we have the system here, for example, this is our system. Okay, and let me first define this problem, this equation, okay, how it is happening. So here, we have V_d and we have the controller here. And from the controller, we are getting u_1, and this is our plant. So in this case, the plant is actually $\dot{V} = u_1$ v dot equal to this is our plant, right? This is our plant $\dot{V} = u_1$, okay, and this is the output. Right, and this is feedback. This is the feedback. This is negative, and this is positive, and this is the output variable actually from this plant is p, right? So now, if you notice here, for this particular, this is the error, right? This error. So here we can write, so here our error is $e = V_d - V$. So based on this, we are actually defining this control, okay? This is what we have done in our modern control techniques. So our this expression, you want actually this, you can write this is the same. This is the control actually, the control expression we are getting from this error variable. So this is how we can design control for to control V, and similarly, the same way, we can define the control input for the θ. So here, basically, we have used the PD control, and here actually, we have used the proportional control.

Uh, because we are having here only one state, one derivative partial, that's why this part is coming, and here actually, we are going to a second-order system, and we are using double dot, the desired dynamics of the θ_d, and this is the proportional part, and this is the derivative part. So this is how we can use some of the linear controls for tracking our time-varying and desired values. Okay, so now we can go back to our problems. So if you substitute one more thing here, here $k_{v1}$are positive, and $k_{\theta 1}$and $k_{\theta 2}$are also positive. Now, from these two equations, from these two equations, if we substitute in this expression, so we can, not this expression actually, in this expression, And this expression, if we substitute, we can come up with the control.

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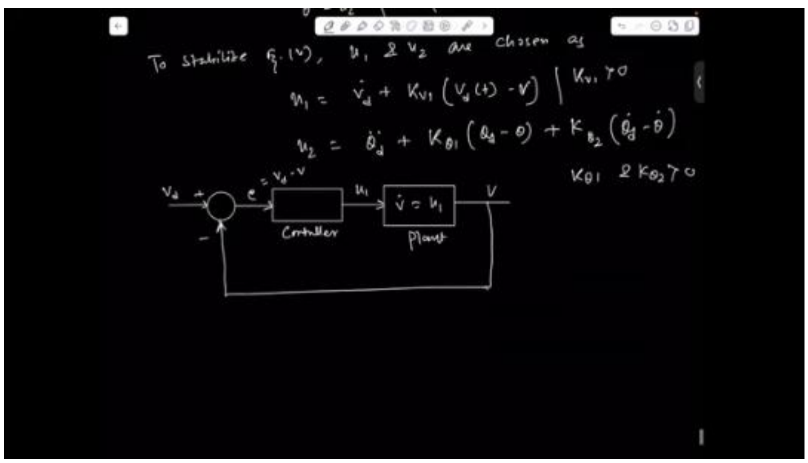

So now the expression for T and δ_e, we can write

$$T = \frac{m\left(\dot{V}_d + k_{v1}(V_d - V) + g\sin\gamma\right) + D}{\cos\alpha}$$

$$\delta_e = \frac{I_{yy}\left(\ddot{\theta}_d + k_{\theta 1}(\theta_d - \theta) + k_{\theta 2}\left(\dot{\theta}_d - \dot{\theta}\right)\right) - M_0 - M_\alpha \alpha - M_q q}{M_{\delta e}}$$

so these are the nonlinear controls these are the nonlinear controls for our nonlinear system defined here so the expression T will be we can fit here and in the place of so here we are going to provide delta here we will get the moment and that moment going to control these two variables and here T is going to control these two variables so this is how we can design the nonlinear controls using feedback linearization technique for this nonlinear systems.

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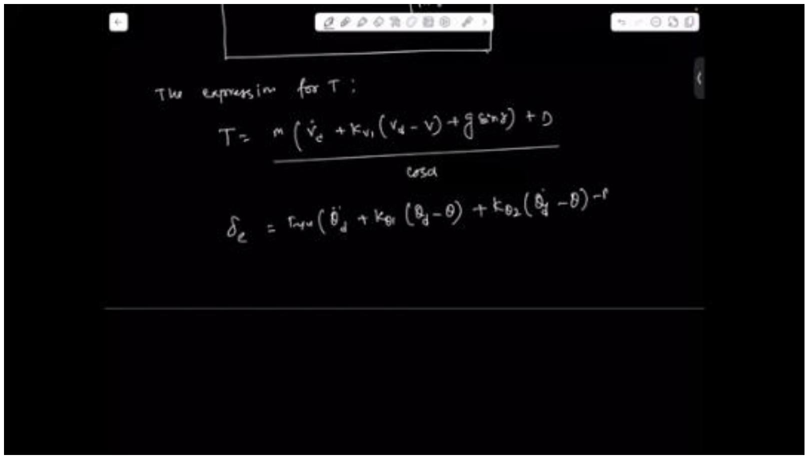

So, let us stop it here. In the next lecture, we will have the MATLAB simulation for different systems and we will wind up this topic. Thank you.