**A Advanced Aircraft Control Systems With MATLAB / Simulink**

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**Lecture 36**

**Feedback Linearization (Contd.)**

In this lecture, we'll be discussing how we can design feedback linearization-based control for the system, which cannot be defined in the standard form

$$\dot{x} = Ax + B\gamma(x)[u - \alpha(x)] \dots Eq(1)$$

as you have done in the last lecture, that in the system if you can write in this form, we can design the state feedback linearization-based control. But if you can't write in this form, then how can we design the control for that system? So let's take an example of how we can tackle this problem. The following system:

$$\dot{x}_1 = a \sin x_2$$

$$\dot{x}_2 = -x_1^2 + u \dots Eq(2)$$

Here, our motivation is how to design the control u. Which is going to control the states $x_1$ and $x_2$. So here, if you notice carefully, this system cannot be represented in this form. It is difficult. So how to tackle this kind of system if you have it? So now, we can solve this problem using feedback linearization-based control. So if you consider the following state transformation, like if you consider, let us apply the state transformation as we are defining a new variable $z_1$

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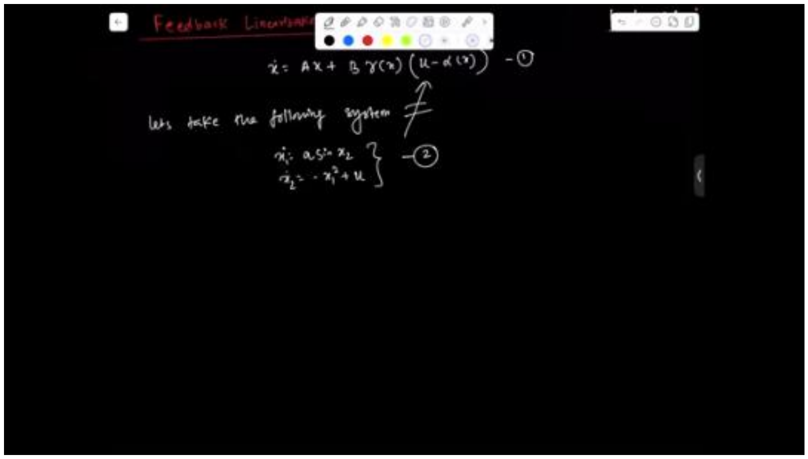


$$z_1 = x_1$$

$$z_2 = a\sin x_2$$

So we are defining two variables $z_1$ and $z_2$ , which are basically functions of $x_1$ and $x_2$ , right? So from this transformation, we can write

$$\dot{z}_1 = \dot{x}_1 = z_2 = a\sin x_2$$

$$\dot{z}_2 = a\cos x_2 \dot{x}_2 = a\cos x_2 (u - x_1^2) \;...Eq(3)$$

Equation 3 can be represented as equation 1.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a\cos x_2\,(u - x_1^2) \;...Eq(4)$$

So here, this is my A matrix, and this is the z vector. This is B, this is $\gamma$, and this is $\alpha(x)$. So now, the control, as you have done in the last lecture, the control law, the control law can be defined as if you can represent the system, this system in this form, then we can design the control. For the system. So here we can write

$$u = \alpha(x) + \beta(x)v$$

$$= x_1^2 + \frac{1}{a\cos x_2} v \;\;...Eq(5)$$

so this is the control u, which is basically non-linear control. But if you substitute this u, in this expression, so we can write the system into linear form. So how can we do it? Now substituting

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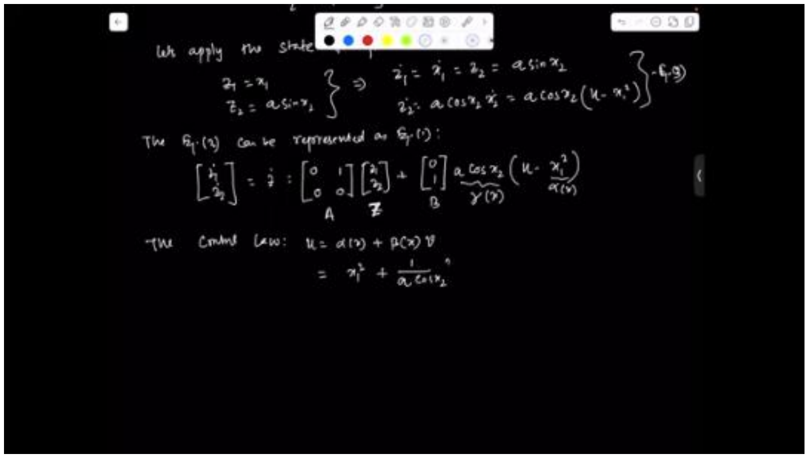


Equation five in equation four, we are having, we can write

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = v$$

Simply, you can write this expression. So if you substitute this expression, you can try if you substitute this expression u in this expression, we can have this form. So this is basically a linear step, and we can design the control for v, which is going to control $z_2$, and this $z_2$ is going to control $z_1$. So now, if you choose, let

$$v = -kz = -k_1 z_1 - k_2 z_2$$

So if you substitute this V here, Substitute this V here, so we can write the augmented system. We can write

$$\dot{z}_1 = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} z$$

This is our Closed-loop system. So here we can have the desired dynamics, what we have done in modern control, desired dynamics. And we can compare the characteristic equation

$$\begin{vmatrix} 0 & 1 \\ -k_1 & -k_2 \end{vmatrix} = 0$$

and we can compare both the equations and we can find the value of $k_1$ and $k_2$. So we can design the control for this system. Now we can find the value of u, which is going to control the overall system, the system. So now, you can be defined as the control, the non-linear control u for the system, for the system equation 2, we can write

$$u = x_1^2 + \frac{1}{a \cos x_2}(-k_1 x_1 - k_2 a \sin x_2)$$

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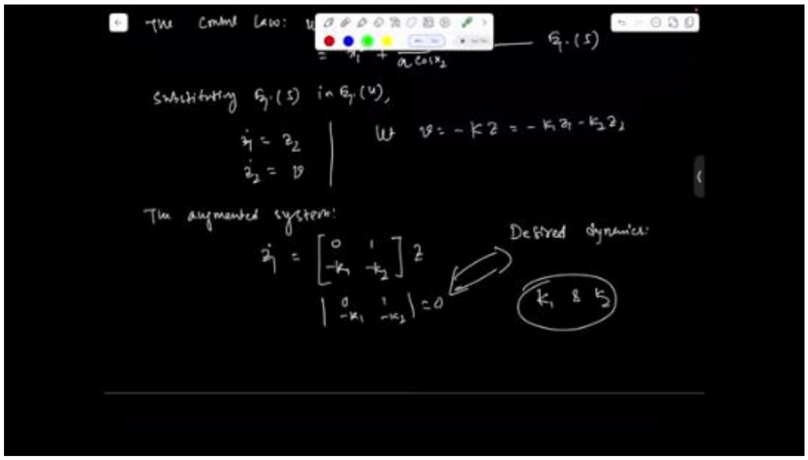


so this is the nonlinear control this is the nonlinear control which is going to control this system defined by equation two so this is how we can use feedback linearization to design the control for that nonlinear system this is the simplest method in nonlinear control techniques now we'll be taking how we can design the feedback linearization for the system so there are two types of feedback basically so the feedback linearization are two types one is full state linearization and second is input output linearization the full state linearization actually we have done so whatever stuff we have covered in the previous lecture and in this lecture as of now as of now these are actually part of the full state linearization because we are using the full state in designing control so here we are going to start the input output utilization how we can design the input output innovation for the system so we'll be taking an example of the aircraft system and how we can apply this concept for the system so let's take an example so it will be easy to understand if you have an example so let's consider we have an example

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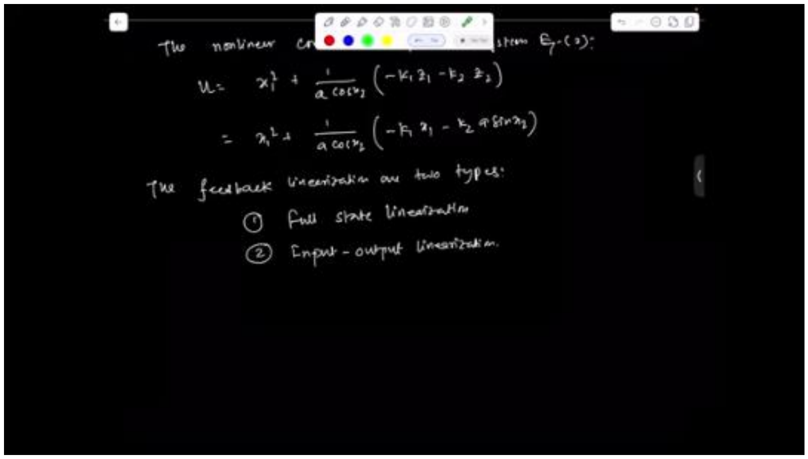


The pitch dynamics of an aircraft can be modeled as

$$\dot{\theta} = w$$

$$\dot{w} = f(\theta, w) + g(\theta, w)u \ldots Eq(1)$$

So we are considering some general equation for the pitch dynamics of the aircraft. And here we'll be designing the control for this system. So let's consider this is the pitch dynamics of the aircraft. So here theta is the pitch angle and omega is the pitch rate. And u is the elevator deflection, which is basically the control input. This is also the elevator deflection, which controls the pitch angle of the aircraft. And a theta w, this is what we are assuming, is the non-linear aerodynamics. Forces and moments, and z theta w, we are assuming control coefficients. So now we are going to design the control u, which is going to control the theta only. So how can we control the pitch angle of this system? Okay, so our control objective is to design a control u, which can help. Which helps theta to track theta desired, or you can say the trajectory. This is a very interesting problem: how can we design control for the system which can help? To track the desired trajectory of the system, so this is our control objective for this example. So we start solving this problem using the input-output linearization. Here, this concept will apply. So here, let us define an error function

$$e = \theta - \theta_d$$

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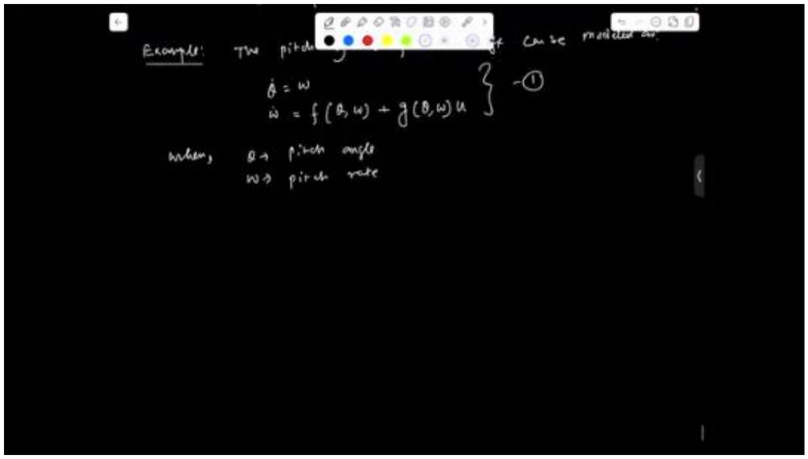


So in this case, the goal is e tends to 0 as t tends to infinity. When e tends to 0, it means when theta d0 goes to theta or theta s goes to theta d, right? When the error goes to 0, so this part is going to 0. So here we can assume theta d0 can be tracked. So here, before proceeding to this problem, we need to understand some important topics in this case. So here, one concept we have to discuss is the concept since we are going to use this method. So here, we need to come up with some rules which can help us.

For designing the control, the number of times the error equation and the number of times the error equation. e of t, which is the equation e of t, let's assume this is the e of t error, must be differentiated before the control input. Appears in the error dynamics. So here, the thing is we have to take the derivative of this expression until we get the control input in this dynamics. Okay, so here. If you notice here, okay, let me write here. We can take, we are taking the output equation here, taking the output equation as, or you can write. To find the control. So, from the title of the concept, input-output linearization, here we're going to take the output equation, and that output equation will be differentiated until we get the control expression in the output equation. So here, since we have the error equation defined, So, if you take the first derivative of e(t), you can write

$$\dot{e}(t) = \dot{\theta} - \dot{\theta}_d = w - \dot{\theta}_d$$

So, in this expression, if you notice here, the control is not appearing, right? So, you can write the control is not appearing. So now, we have to take the time derivative again.

$$\ddot{e}(t) = f(\theta, w) + g(\theta, w)u - \ddot{\theta}_d$$

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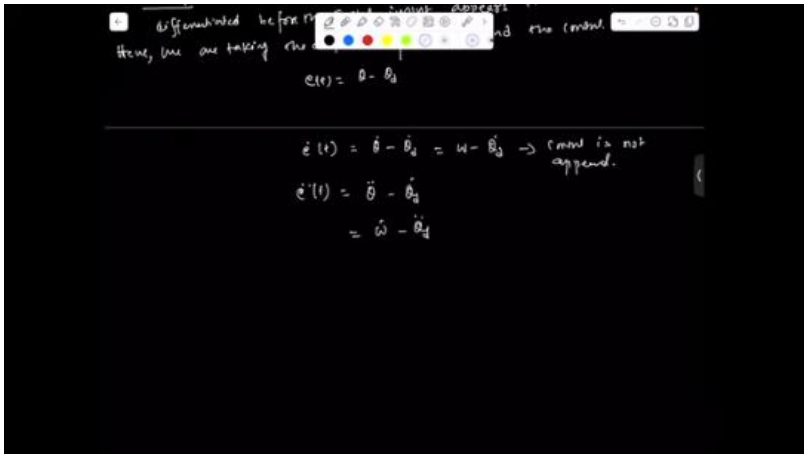


So, you can see that the control appears. So here, what you're going to do is to linearize the system. To linearize the system, what we're going to do is we are taking

$$\ddot{e}(t) = v \dots Eq(1) \quad or \quad *$$

So, this is what we are assuming to design v in such a way that, over time, e(t) goes to zero. So, what you're going to do is, this is basically nothing but the linear system. So, we can write here, this is the error dynamics the aerodynamics is linear. Yeah, the aerodynamics is linear. So now, let's design the control input for v so that e of t is stabilized. Yeah, so here, how we are assuming Now, this can also be transformed into a similar step, the step you followed in the last lecture. The same steps you can follow. So, here you can consider

$$e(t) = x_1$$

$$\dot{e} = \dot{x}_1 = x_2$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = v$$

This you can also write, right, from this equation one. So now, we are defining the control for this dynamics. The control, let's consider the control v,

$$v = -k_1 e - k_2 \dot{e} = -k_1 x_1 - k_2 x_2 \dots Eq(2)$$

substituting equation 2 into equation 1, we can write

$$\ddot{e}(t) = -k_1 e - k_2 \dot{e}$$

$$\ddot{e}(t) + k_1 e + k_2 \dot{e} = 0$$

So here, actually, $k_1$ and $k_2$ are positive, so in this case, this system is a stable system, right? So over time, e of t goes to zero, so this is basically a stable system, right? So now what you're going to do is we'll substitute this v here. We can substitute this v here. This v, the expression of v here, this is also v because $\ddot{e}(t)$ we have as into v, so this is also v equal to this whole expression. So now we can write The control for the system, let us define, okay, this should be, let us define now star (*), okay.

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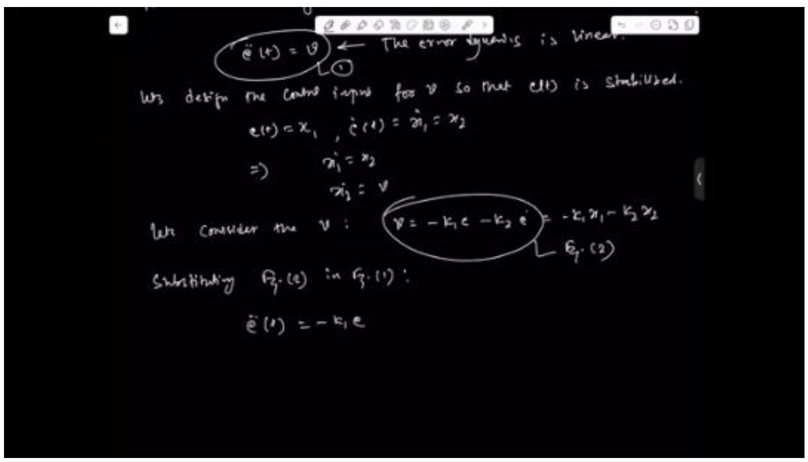


The control for star, we can write V equal to, we can write this full expression here. This expression here we can write, right? So let me copy it. So from this, we can write or we can write

$$v = f(\theta, w) + g(\theta, w)u - \ddot{\theta}_d$$

$$u = \frac{1}{g(\theta, u)}\left[-f(\theta, w) + \ddot{\theta}_d - k_1(\theta - \theta_d) - k_2(\dot{\theta} - \dot{\theta}_d)\right]$$

So this is our control expression for our pitch dynamic system for this system. So this is how we can design The nonlinear control for the nonlinear system using input-output linearization. So this is a very, very powerful technique, how we can design a control for the nonlinear system using this concept. Let's stop it here. We'll have another aircraft example in the next lecture, and then we'll wind up this concept. Thank you.

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