Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 35

Feedback Linearization

Hello everyone, in the last few lectures, we have discussed stability and how we can study the stability of the system. From now on, we'll be starting how we can design the controls for the nonlinear system. So, our first control technique we are going to start is the feedback linearization-based control. Here, basically, if you have a nonlinear system, we're going to use some concepts which are basically feedback linearization concepts. Which help us to convert the system to a linear form. And once you have the linear system, we can design the linear control techniques. And that actually is going to help us to derive the nonlinear controls for the nonlinear system. So, this is basically the concept behind the feedback linearization-based control.

So, let me write some notes before we move to the examples. So here, the feedback linearization techniques. The feedback is a control technique or control method to transform Transform a nonlinear system into a linear one through a change of variables. This allows the use of linear control techniques to design controllers for nonlinear systems. So, this definition actually is what we have explained here. From the nonlinear system, we can use the feedback linearization technique, which helps us to convert the nonlinear system into a linear form. And for the linear form, we can design the linear control techniques, which indirectly will define the nonlinear control for the nonlinear system. So, let's start with a motivational example which will help us to understand this concept. Example. So, how does this technique work? Let's work on this. So, let's consider we have a system, the following nonlinear system, which is defined by

$$\ddot{x} + a\sin x + b\dot{x} = cu \ldots Eq(1)$$

. So here, if you notice, this is the control here, this u we are going to design, and this u is going to control the state of the variable x here. Basically, so here our main motivation is to design design u to control x. So here, basically, if you notice, the system is nonlinear, so we have to design the nonlinear control for this system. But since we are going to use this method, the feedback linearization technique so our motivation is how we can convert this nonlinear system into linear form and how we can design the linear control. So here, as for the rule what you have explained here, change of variables, so we are going to apply the change of variable for this system. So let's do it, change of variables So here we're defining

$$x_1 = x$$

$$\dot{x}_1 = \dot{x} = x_2$$

So if you apply these variables to the nonlinear system, which is defined by equation 1, so we can write the equation 1 in state space form, we can write

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a \sin x_1 - bx_2 + cu \ldots Eq(2)$$

Now, this is basically two first-order ODEs, so we have to design the control u to control state x_2, and if x_2 is controlled, we can control x_1 because x_1 depends on x_2. Right now, let us choose u equal to, for the first second equation,

$$u = \frac{a}{c} \sin x_1 + \frac{1}{c} v \ldots Eq(3)$$

So here, v we is newly defined, some variable we will define. So basically, this v will help us to design the linear control. Now, if you substitute this the expression of u in equation two, so we can write. Now, substituting equation three in equation two we can write

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -bx_2 + v \ldots Eq(4)$$

So now the interesting equation we have, this is equation four, for example. So this system is a linear system. The system, if you notice here, this system is a linear system, right?

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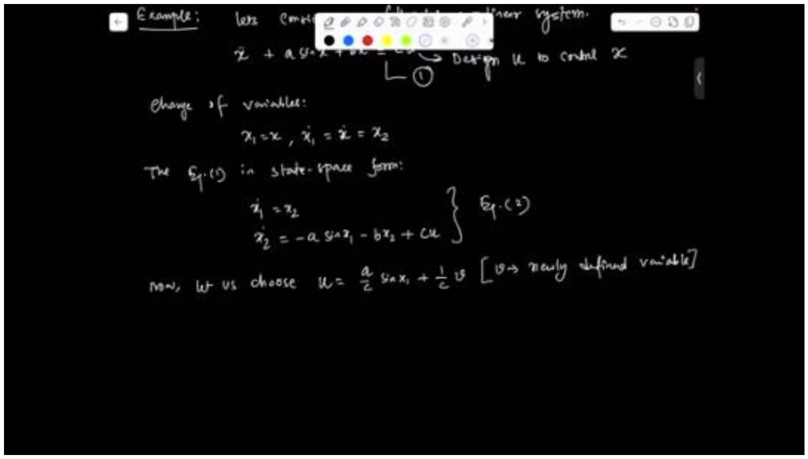

There are no nonlinear terms. In this equation, so in state-space form, we can write equation four. Equation four in state-space form we can write

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \dots Eq(5)$$

So if you notice this equation, equation 5, this is basically the system represented in linear form and also state-space form. So while discussing modern control, We had the state-space model in linear form, and we can design V in such a way that x_1 , x_2 can be controlled. So here, since the system is in linear form, we can apply the pole placement control technique or regulatory problem, and we can design the expression for V, and this V can control the states x_1 and x_2 . So here, we can write, this is the linear system. Linear system, we can design a linear control, what we have done already, what we have covered in modern control. The first half of this course, so now let us choose V as a state feedback control Kx, or we can write

$$v = -kx = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -k_1 x_1 - k_2 x_2 \dots Eq(6)$$

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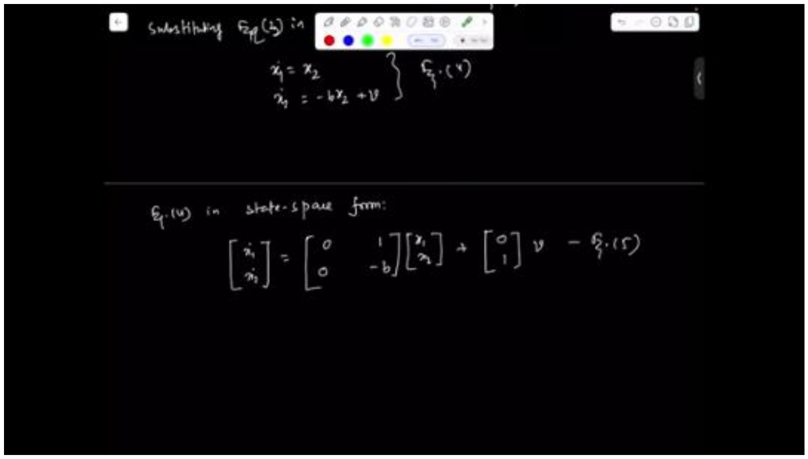

And now, if you substitute the expression of V in equation 5, equation 6, substituting equation 6 in equation 5, we are going to have

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -bx_2 - k_1x_1 - k_2x_2$$

So if you notice this equation, actually in control form, actually in closed-loop form, because V has been designed, this V has been designed using the variables x_1 , x_2, which are coming from the sensors or from the output. So this is basically a closed-loop system. And if you write in state-space form or in augmented system form,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k_1 & -(k_2+b)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

This is basically a linear system which is in closed-loop form, and the way we can design k_1 and k_2 has already been done multiple times while discussing modern control. So, k_1 and k_2 can be designed based on the desired dynamics, and we can find the expression for V. So, this is how we can design the linear control for the linear system, which is defined by equation 4. Right. Now, we can write, if you notice here, we are now taking the linear system. So, what about our original nonlinear system?

So, the control for the nonlinear system, we can write, So here, let me write some notes. Here, k_1 and k_2 can be designed based on the standard pole placement technique. So now, the overall control we can design, the overall nonlinear control for the system defined by equation 1. Now, our last part is how we can design the control for this nonlinear system. So here, what you're going to do is we'll substitute in this equation the expression of V. So, we'll get the overall nonlinear control which is going to control the whole system.

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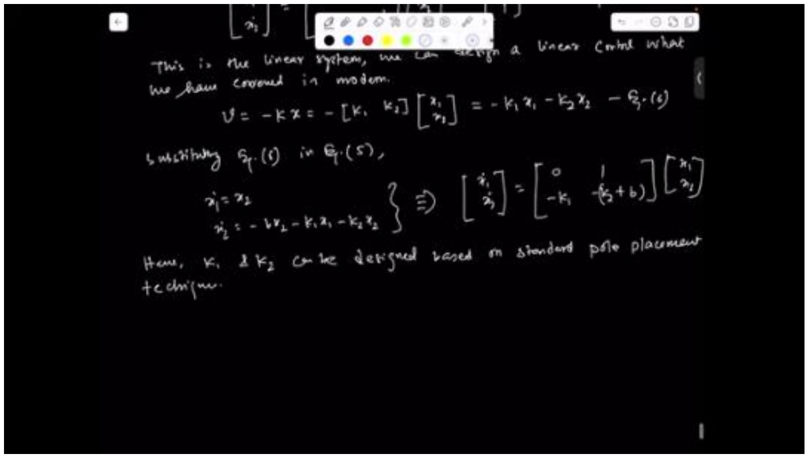

So now, we can write u equal to, which is our u expression actually. We can write

$$u = \frac{a}{c}\sin x_1 - \frac{1}{c}k_1 x_1 - \frac{1}{c}k_2 x_2$$

So, this is the nonlinear control. Which is going to control the nonlinear system, equation 1. So this is how we can design a control algorithm or control law for the non-linear system, for this system, using the feedback linearization technique. Now the question is, whether the system in our daily life, is it possible to design feedback linearization-based control? So how to tackle this? This question, so now the question is, is it always possible to obtain feedback linearization technique? Or the dynamical system.So the answer, the answer is if the system is represented in the form of, in the form of

$$\dot{X} = AX + B\gamma(x)[u - \alpha(x)] \ldots Eq(1)$$

If we can write any system in this form In this form, if you can write, then we can design feedback linearization-based control. So here we can write here, then we can design feedback linearization-based control here. Okay. So here, let us define. So this is our new topic. We can write here. So let us define. This is equation one. And the control for equation one, we can write

$$u = \alpha(x) + \beta(x)v$$

$$\beta(x) = \gamma^{-1}(x) \ldots Eq(2)$$

so here the main thing is if you have a nonlinear system And if you can define in this form and in this form, the control u will be in this form and where $\beta(x)$, this is basically nothing but the inverse of this function. So if you can write any dynamical system in this form equation one, we can design the feedback linearization based control. So now if you substitute, so let us define We'll take an example for this concept.I've got this method 2. If you substitute equation 2 in equation 1, we have

$$\dot{X} = AX + Bv \ldots Eq(3)$$

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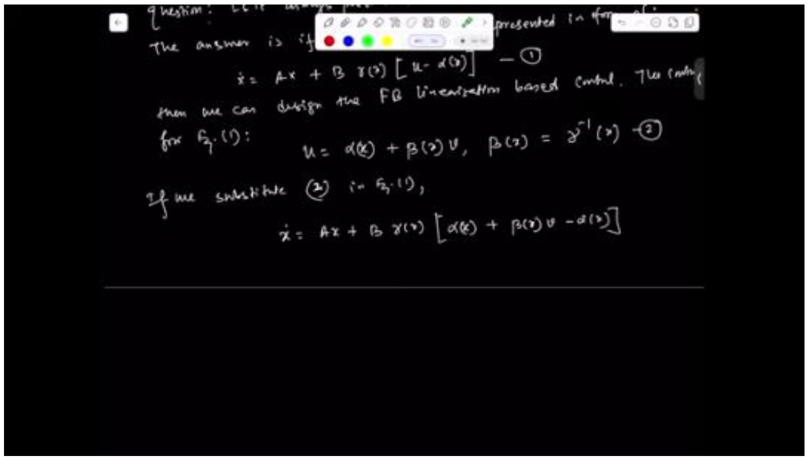

if you notice here our system has been converted to linear form so our system is A will still be linear form and you can design the linear controls for this linear system. But to design the linear control for the system, as you know, the A and v, the system should be controllable. So now we can choose, let us choose

$$v = -kx \ldots Eq(4)$$

So this is basically the way we have taken. To design control for v, what you've done in modern control, so now let us define this as equation four or this should be equation three and four. Now substituting, Eq 4 in 3, we are having

$$\dot{x} = (A - BK)x$$

So to design the linear controls for the system, we have to consider some desired dynamics. And you can compare the augmented matrix characteristic equation and you can design control using the same approach you have followed. The condition is we have to consider here $A - BK$ is Hurwitz. Hurwitz means the eigenvalues of the system of the matrix are negative. So here basically all eigenvalues of $A - BK$ lie in the left half plane of the s-plane. Okay, so now we'll go back to the same system you have considered previously, this example.

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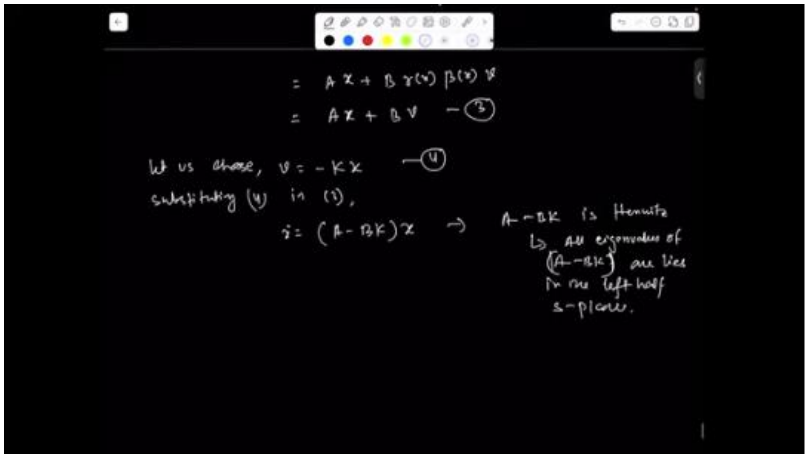

Same example here, and we'll see whether we can apply this method or not. So we'll take the same example here. Okay, so now the system, let's consider the same system:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a \sin x_1 - b x_2 + cu \ldots Eq(5)$$

Okay, now in state space form, we can write. We shouldn't say state space form; we have to represent that equation in this form, in this form, right, in equation one form. So this equation we should represent equation one form. Now this is equation five, representing equation five in the form of equation one. We can write

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} c \left(u - \frac{a}{c} \sin x_1 \right)$$

so if you notice here the system we have this system in this form in this form so here basically we can write this is basically we can say this is a this is X vector this is A this is γ this is B and this is $\alpha(x)$ right so now we can write

$$\dot{X} = Ax + B\gamma(x)[u - \alpha(x)]$$

$$= \frac{a}{c} \sin x_1 + \frac{1}{c}(-k_1 x_1 - k_2 x_2)$$

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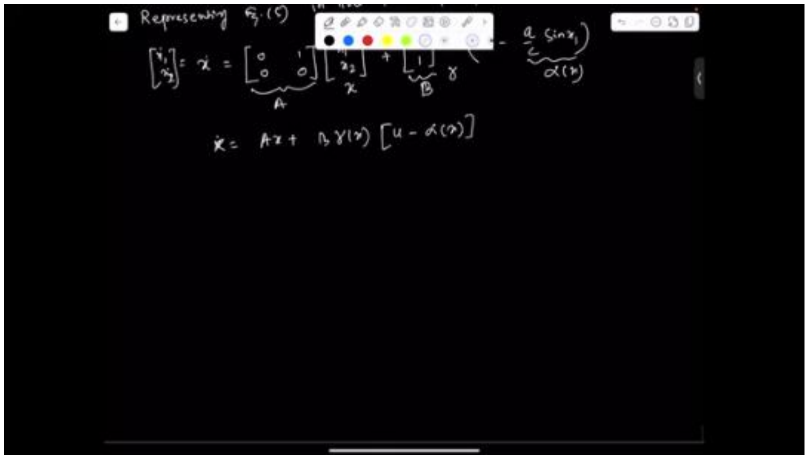

So this is our control, nonlinear control. So if you notice here, the same expression what you had before here also we are getting the same control expression here so this is how we can design the non-linear control for the nonlinear system using the feedback linearization based control and let's stop it here from in the next lecture we'll be taking some aircraft examples and how we can design this concept for the aircraft applications thank you.

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