

# **Advanced Aircraft Control Systems With MATLAB / Simulink**

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**Lecture 27**

**Introduction to Nonlinear Systems**

Hello everyone, this is the first lecture on nonlinear controls for the aircraft system. Let's assume we have a nonlinear system which can be defined as a first-order ODE form, and for this system, we can design linear and nonlinear controls. Linear controls we have done so far; we are designing different types of linear control techniques for the system. Let's assume this is the aircraft system. To design the linear controls, the steps we follow are: first, we form the equilibrium points, and around the equilibrium point, we linearize the system. Right? Then this is the condition for small perturbation in the state variable due to the perturbation existing in the system. And also, we assume the small perturbation theory to ignore the coupling terms, which are basically perturbed variable terms. And once we have the linear system, we can design control such that  $\Delta x$  goes to 0 as the detention period. And for designing control, we have conditions on controllability.

And once both conditions are fulfilled, then we can design an observer controller. In the controller, we have designed state feedback control and optimal control. We have designed to validate the control. And also, we have done a lot of simulations in MATLAB. But in the case of linear control synthesis, it is different.

So here, we work more on the energy of the system. So here, we define the energy contained in the system, and we need to design a controller in such a way that the total energy should be decayed or minimized. And here, we are going to use some energy function, which is basically in terms of the state variable and the system parameters. And we're going to design some control which helps us to minimize that energy. So here, the controls we can come up with different types of control like feedback linearization, and also we have no best control. We have no best control, so how can we minimize the total energy of the system? That is nothing but the Lyapunov function. We refer to the Lyapunov function and make the Lyapunov function derivative of the Lyapunov function to be negative. Suppose this is the Lyapunov function, and our main goal is this should be

zero.  $\dot{V} = 0$  or it is minimized. It is the rate of change of  $V$  that goes negative, so it means the energy is decreasing. So if you come up with some function like...

Some conditions are set in such a way that this is fulfilled. Then, we can design the Lyapunov best control, and also we can design feedback linearization. Additionally, we have sliding mode control, sliding mode control, and backstepping control. If any of the parameters in the system is adapted, if it is time-varying, changes over time, how can we design some adaptive control to tackle those kinds of situations? This is also something we are going to cover in the content of this course. Now, the next question is: why do we have to study nonlinear controls over linear controls? So, as we know, all practical systems are nonlinear in nature.

So, When you are designing linear controls, you are ignoring some terms, but practically, the system contains a lot of nonlinear terms. That is one reason why we have to study nonlinear controls. And the dynamics of the linear system are not well established enough to describe many common practical world problems. So, that is another reason why we have to study nonlinear controls.

In a linear system, the system stability depends on the system parameters. Suppose this is our system equation, which is defined in linear form. And the stability can be defined based on the eigenvalues of the  $A$  matrix. So, the terms inside the  $A$  matrix actually contain the system parameters. So, if the system parameters change, the eigenvalues also change.

So, we define the stability of the system based on the eigenvalues of the system. But in the case of a nonlinear system, the stability depends on the initial condition, input signal, and also system parameters. And also, another reason is due to the existence of discontinuous functions like backlash, dead zone, etc. The nonlinear system may not allow linear approximation. But for the starting of linear synthesis, control synthesis, we assume the linear system is assumed to be linearized. This is the condition we do, and based on that, we design the control synthesis. These are the reasons why you have to study nonlinear controls over linear controls. This is preferred, actually, preferred. Now, let us take an example of the aircraft dynamics, the original. Let me rewrite the equation we had before. So, we have translational dynamics;

$$\dot{u} = rv - qw + \frac{F_x}{m} - g \sin \theta$$

$$\dot{v} = pw - ru + \frac{F_y}{m} + g \sin \phi \cos \theta$$

$$\dot{w} = qu - pv + \frac{F_z}{m} + g \cos \phi \cos \theta$$

and if you write that rotational dynamics, we have

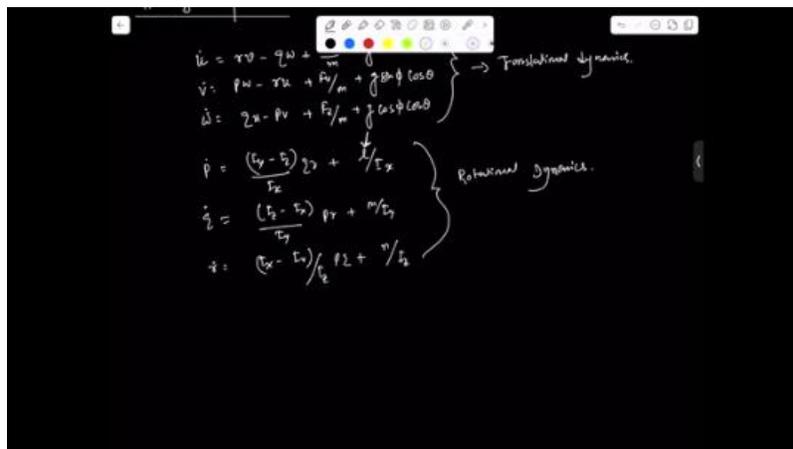
$$\dot{p} = \frac{I_y - I_z}{I_x} qr + \frac{l}{I_x}$$

$$\dot{q} = \frac{I_z - I_x}{I_y} pr + \frac{m}{I_y}$$

$$\dot{r} = \frac{I_x - I_y}{I_z} pq + \frac{n}{I_z}$$

So, if you notice here carefully, we have the coupling terms. So here, we can say these terms are kinematic coupling. These are the terms, basically, you can write here, kinematic. Now, also, you have the coupling terms here, if you notice carefully.

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We have  $qr$ ,  $pr$ ,  $pq$ . So, these are, we can sometimes refer to as the So, here also, we have these terms:  $\sin \theta$ ,  $\cos \theta$ . So, these are the terms that also exist in the system, and this is also making the system nonlinear. So, if you look, this equation is highly nonlinear.

So that's why it is very preferable to design a nonlinear control for this rotational dynamics. But if you are starting the linear system or linear control synthesis, if you apply the small perturbation theory, we generally ignore these terms, these coupling

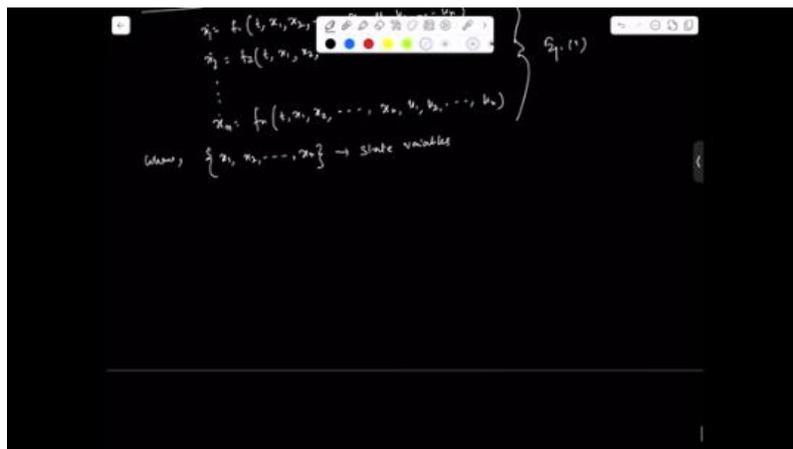
terms, and we are making the system a very simplified form. So that is maybe sometimes not feasible if the system demands high accuracy. So, to avoid this kind of problem, we generally study the nonlinear controls for the nonlinear system where the accuracy is very important and demanding for the comfort of the passenger, if you consider the aircraft system, the passenger aircraft system. Now, let us look at how we can come up with some closed-form of the nonlinear system equations. So, So, let us assume we have the first to the only of the nonlinear system defined as

$$\begin{aligned}\dot{x}_1 &= f_1(t, x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n) \\ \dot{x}_2 &= f_2(t, x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n) \\ &\vdots \\ \dot{x}_n &= f_n(t, x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_n) \dots Eq(1)\end{aligned}$$

This is equation number one where we can where basically you can say where  $\{x_1, x_2, x_n\}$  these are basically state variables. And  $\{u_1, u_2, u_n\}$  are the controller inputs, controller inputs in the system, ok. And if you write the states in vector forms or matrix form we can write

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}, \quad f(t, X, U) = \begin{bmatrix} f_1(t, X, U) \\ f_2(t, X, U) \\ \vdots \\ f_n(t, X, U) \end{bmatrix}$$

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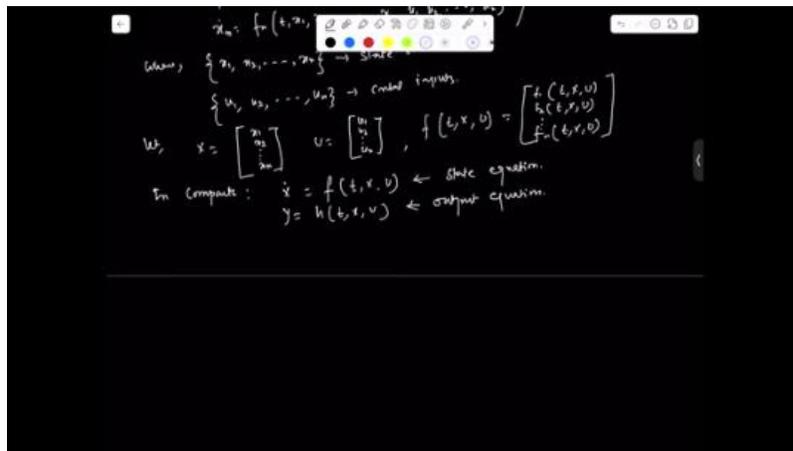
So, now if we write in compact form, in compact form we can write the above terms in this form

$$\dot{X} = f(t, X, U) \dots Eq(2)$$

This is we called in nonlinear system, sometimes we define as state equation. And output equation also you can define as  $Y = h(t, X, U)$ . This is called. So, these are the terminology we will be using while we will be studying the control synthesis system. Now, with the system does not explicitly. When the system does not depend explicitly on control input Equation 2 can be written as

$$\dot{X} = f(t, X) \dots Eq(3)$$

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So, here the system does not depend on the external inputs or the perturbation input anything. Once this condition happens and if the system can be written in this form, we call it an unforced system. This is does not get affected by any external force or torque, then you can say there is an unforced system. And another condition when an unforced system

is independent of time, it means  $\dot{X} = f(X)$ , and this when we have this kind of system, what we call a time-invariant system, invariant system, this is very very important. We also see that the system, since the system does not depend on time, the system also we can call an autonomous system. The system is autonomous. If dependent on time, So, these are the definitions very important while discussing the non-linear system.

Now, we will go to some important part based on this equilibrium point because this is one of the most important points in starting the non-linear system. Equilibrium point, equilibrium point, a point is said to be equilibrium if it has the property that whenever the state of the system  $X^*$  at the equilibrium point, it will remain at that point, so I can write

here A point, this part we already discussed in our first course introduction to aircraft control system, but still, I am repeating this thing because this thing is very important for studying the non-linear systems. A point  $x^*$ , for example, this is the equilibrium point, is said to be an equilibrium point if it has the property that whenever the state of the system of the system. So, we can say the current state  $X$ ,  $X$  starts at  $X^*$ , and it will remain at  $X^*$  for all future time. This is basically a desirable property when designing and implementing a control system. This is very, very important.

This is the desirable property for designing and implementing a control system. So, let us consider we have the non-autonomous system, let us consider. Let us consider the above number system, and let us consider the system is not equal to  $f(X, t)$ . For example, the point, the point  $X = X^*$  is an equilibrium point at time  $t$  equal to  $t_0$  if the following condition holds.

So, an equilibrium point we can write 0 for all  $t$  greater than  $t_0$ . So, what does it mean? Because we have a system here, for example, we have the system

$$\dot{X} = f(X, t) = 0$$

So, to find the equilibrium point, we will put this equal to 0. Why? Because the equilibrium point is at some point where there is a constant value, right? So, at that point,  $dx/dt$  So, this is also equal to 0. So, if you solve this equation, we will get some  $x$  terms, and these are basically the equilibrium points.

And here in this definition, we are defining that  $x$ , that  $x$  is the  $x^*$  equilibrium point, which will satisfy this condition. And So, this is for the non-autonomous system, but if it is an autonomous system, we can say. So, if it is an autonomous system with the condition, the condition for the equilibrium point becomes independent of the initial. So, you can write

$$f(X^* = 0)$$

So, this is how we can get the point, and the equilibrium point, the function goes to 0. So, here if you consider, if the equilibrium point is not equal to 0, how does it happen? So, in most cases, we can let me write something that is very important. We consider or we shall only consider systems with equilibrium at the origin, which is  $X^* = 0$ . This is the favorable condition for designing a control system. If you have the equilibrium point in this kind of condition, it will be easy to determine the stability of the system. But if the equilibrium point is not zero, we can change the system to another domain where the

equilibrium point will be zero. So let's look at how we can do this. So, for example, let us consider the following system,

$$\dot{Z} = g(Z, t) \dots Eq(4)$$

And let us assume the equilibrium point for this system is not equal to 0,  $Z = Z^* \neq 0$ . So, let us define the equilibrium point, for example, equilibrium point of 4, we can write  $z$  equal to 0. Not equal to 0 at  $t = t_0$  ok.

So, now let us consider the change of variable, the change of variable  $X = Z - Z^*$ . This is the conditional requirement. But one thing, do not forget, this is we are assuming to be a constant value, right. In that case, then we can write

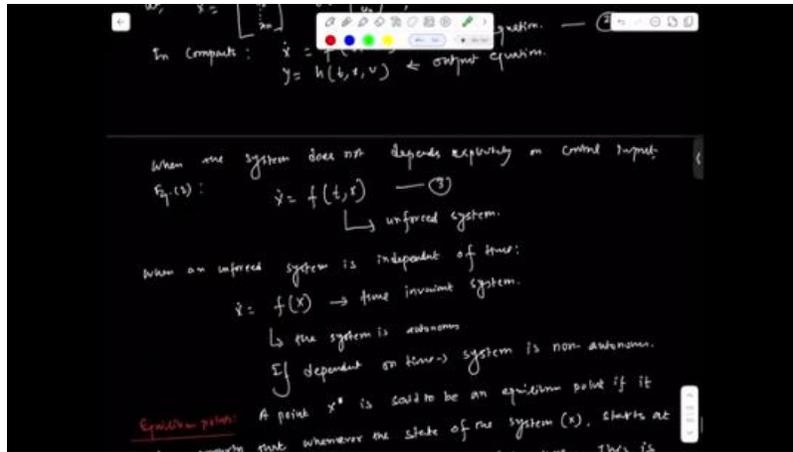
$$\dot{X} = \dot{Z} = g(X + Z^*, t)$$

So, now defining

$$f(X, t) = g(X + Z^*, t) \rightarrow \dot{X} = f(X, t)$$

has the equilibrium point at  $X = 0$  at  $t = t_0$ . So, this is how we can say that the system has, if the system does not have the equilibrium point zero. We can make another system. We can come up with another system with the change of variable, and where we can come up with the new system, the equilibrium can go into zero. So, this is how we can define the system, how we can define the equilibrium point of the system, and how we can come up with the complete form of the system if there are multiple Equations which are only in form, and how we can come up with this form, and for this particular form, how we can come up with the different concepts of forced system, unforced system, and what is the definition of the equilibrium point, and at the equilibrium point, the function solution goes to zero And even if the equilibrium point is not equal to 0, how can we make a new system where the equilibrium point will be 0?

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So, this is how we can study the equilibrium point of a nonlinear system. Let us stop it here. Next lecture, we will be studying the stability of the system. So, how can we come up with the system stability by considering the equilibrium point? That is very, very important for studying the lecture. Thank you.