Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 26

Altitude hold auto-pilot design

Hello everyone, in this lecture, we will be studying the altitude hold autopilot system for an aircraft. So here, zb xb is the body-fixed frame, and del alpha is basically the change in the angle of attack. V is the total velocity of the aircraft, del theta is the changing pitch angle, and del gamma is the change in the flight path angle. We can write from the figure. We can write

$$\Delta\theta = \Delta\alpha + \Delta\gamma$$

$$\Delta\gamma = \Delta\theta - \Delta\alpha$$

So, here we can find the vertical side as a function of the climb angle. So, here the rate The rate of climb or change in vertical height, we recognize

$$\Delta\dot{h} = Vsin(\Delta\gamma)$$

$$= Vsin(\Delta\theta - \Delta\alpha) \dots Eq(1)$$

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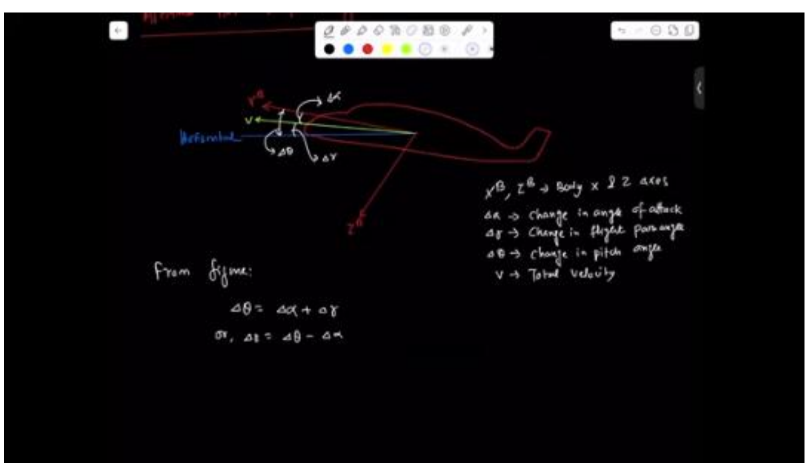

sine del gamma. So, del h is basically the changing vertical height of the aircraft. And also, let us have an assumption for small angle approximation for small angles, we can write $\sin\theta \approx \theta$. So, in that condition, if you assume this angle is very small, then we can write. Equation 1 yields, we can write

$$\Delta\dot{h} = V(\Delta\theta - \Delta\alpha) \ldots Eq(2)$$

Also, we know that

$$\Delta q = \Delta\dot{\theta} \ldots Eq(3)$$

So, these are the things we have already discussed in our first course. So, I'm not going to detail about it. If we add these two equations to the original, short-period short-period dynamics we had already derived in our first course. So, I am not going to repeat it again.

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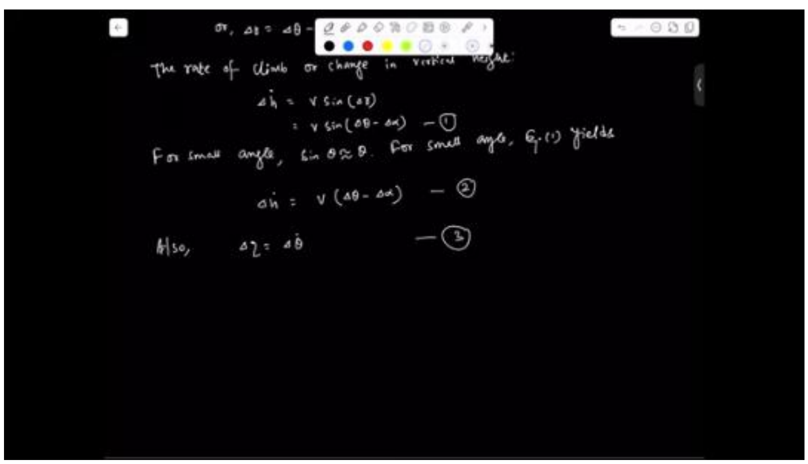

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\end{bmatrix} = \begin{bmatrix}\dfrac{Z_\alpha}{u_0} & 1\\ M_\alpha + \dfrac{M_{\dot{\alpha}}Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}}\end{bmatrix}\begin{bmatrix}\Delta\alpha\\ \Delta q\end{bmatrix} + \begin{bmatrix}\dfrac{Z_{\delta e}}{u_0}\\ M_{\delta e} + \dfrac{M_{\dot{\alpha}}Z_{\delta e}}{u_0}\end{bmatrix}\Delta\delta_e \ldots Eq(4)$$

We are not going to explain these individual terms inside the A matrix and B matrix. So, this thing we have already discussed in our first course. So, now if we add this equation 2 and 3 to equation 4, we have the state space model,

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{u_0} & 1 & 0 & 0 \\ M_\alpha + \frac{M_{\dot{\alpha}} Z_\alpha}{u_0} & M_q + M_{\dot{\alpha}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -u_0 & 0 & u_0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} \frac{Z_{\delta e}}{u_0} \\ M_{\delta e} + \frac{M_{\dot{\alpha}} Z_{\delta e}}{u_0} \\ 0 \\ 0 \end{bmatrix} \Delta\delta_e$$

So, here basically let us consider, let us consider $u_0 = V$, the assumptions we have considered. Now, we will be designing autopilot, how we can control this day length over time, this is our, how can we maintain the First, define the problem for this is basically the natural dynamics, this is basically the natural dynamics of the system in the linear domain, of course, and these are the parameters we vary from aircraft to aircraft, these are the parameters for the particular flight regime, we can get these parameters from the wind tunnel testing. And these are basically stability derivatives, moment and force derivatives, so this varies from aircraft to aircraft, now, example use state feedback control to design to design And altitude fold from the system from the system, assuming assuming forward speed forward speed

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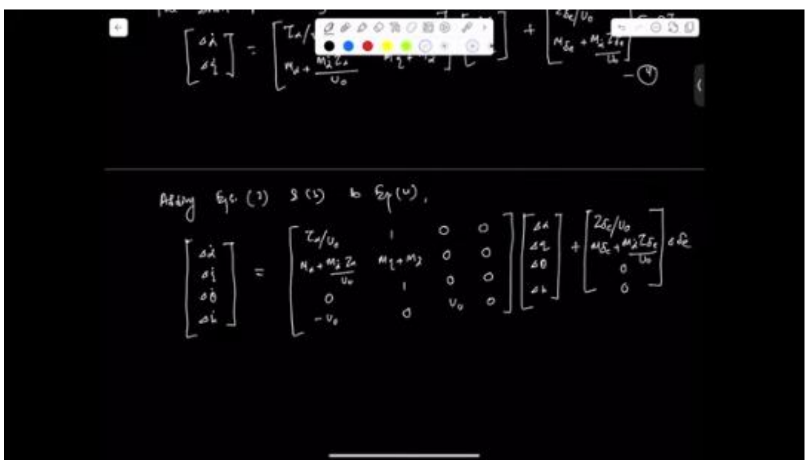

Of the aircraft is held constant constant by by a separate separate velocity control system and further neglect the control surface actuator dynamics, ok. The short period, the short period approximation For a particular flight means this is a for a particular flight, the state space model

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -1.5 & 1 & 0 \\ -4 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} -0.2 \\ -8 \\ 0 \end{bmatrix} \Delta\delta_e$$

Assume we are assuming the forward velocity, assume $u_0 = 61\,\text{m/s}$, Determine state feedback gain If the closed loop system, closed loop system eigenvalues, closed loop systems eigenvalues are located at. So, these are located at, these are desired

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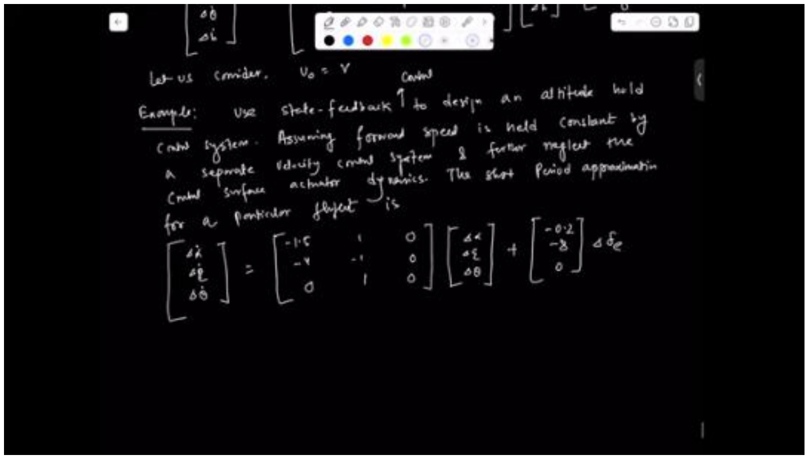

Equation right.

$$S_{1,2} = -1.5 \pm 2.5i$$

$$S_{3,4} = -0.75 \pm 1i$$

Four digital poles are given, so we have to come up with the system into four-dimensional space. So, let's start the problem solution. Let's first write the state space form including the altitude dynamics. So here, if you notice in this equation, we don't If you include altitude dynamics, including altitude dynamics, we have the state space model:

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} -1.5 & 1 & 0 & 0 \\ -4 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -61 & 0 & 61 & 0 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \\ \Delta\theta \\ \Delta h \end{bmatrix} + \begin{bmatrix} -0.2 \\ -8 \\ 0 \\ 0 \end{bmatrix} \Delta\delta_e$$

Now, here we are going to use the Bass-Gura method to find the control gain parameters. Basically, the system complexity is that the system is complex. There are multiple variables to be controlled. So, we can use the numerical method, which is basically the Bass-Gura method, as we have explained before. So, the Bass-Gura method lets us use the Bass-Gura method to find the control parameters for the gain parameters, which is nothing but

$$K = [(PW)^T]^{-1}(\alpha - a)$$

So, now I think you can design the control the same step as you have already. So, first, we have to check if the system is controllable or not. This is the first criteria to design the controller. So, first, we can find

$$P = ctrb(A, B);$$

$$r = rank(P);$$

$$r = 4$$

here one more thing: we are assuming all states are available for feedback, so states are measurable, ok. And then only you can design the state feedback-based control. This is, so it means the system is fully controllable. The system is controllable. And if you, if you remove this semicolon, we can also find the P matrix.

$$P = \begin{bmatrix} -0.2 & -7.7 & 20.3 & -8.5 \\ -8 & 8.8 & 22 & -103 \\ 0 & -8 & 8.8 & 22 \\ 0 & 12.2 & -183 & -704 \end{bmatrix}$$

This is the P matrix, and we can also check whether the system is stable or not. For that, we need to use system stability. For that, we can use

$$[\omega_n, zeta, poles] = damp(A);$$

$$\omega_n = \begin{bmatrix} 0 \\ 0 \\ 2.34 \\ 2.34 \end{bmatrix}, \quad zeta = \begin{bmatrix} -1 \\ -1 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad poles = \begin{bmatrix} 0 \\ 0 \\ -1.25 \pm 1.98i \\ -1.25 \pm 1.98i \end{bmatrix}$$

So if you notice carefully, There are two poles at the origin. So if the system has two poles at the origin, we can say the system is marginally stable. So in that case, we need to consider, we need to design the control.

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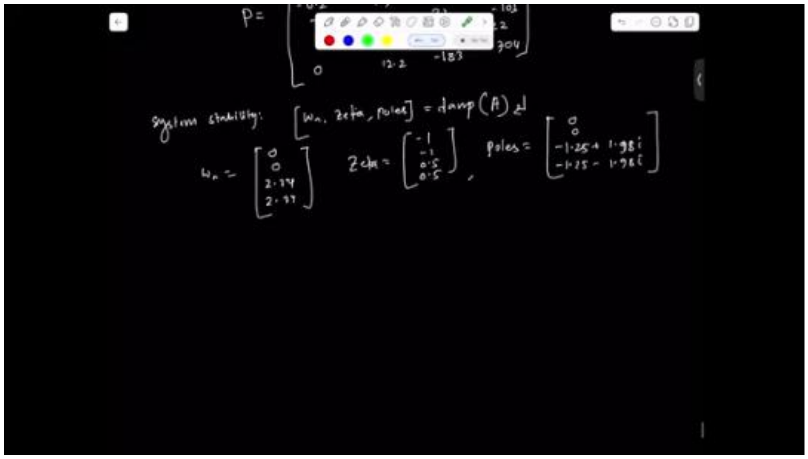

So first, let's find the initial response. First, let's find the open-loop system response, how the system is going to behave in open loop. So, in that case, we have if you open-loop response, I will come in the code to the end of this lecture, but the open-loop response is found to be open-loop response is found to This is we are assuming the aircraft is this is an altitude under the time, and this is in meters. So it is actually going like this, going like this. So it means at the altitude from this plot altitude, we have the code for this how the response is coming will At the end of the lecture, I will show the MATLAB code of this figure.

So, as expected, altitude is continuously decreasing because there is no control applied in the system. So, automatically, you can decrease the altitude over time. Now, we will use the state feedback control. Based on the given desired poles, the system is the desired poles, and how we can maintain the altitude hold of this particular example. So, for that, let us form the closed-loop poles, closed-loop desired poles. We can define

$$V = [-1.5 + 2.5i \quad -1.5 - 2.5i \quad -0.75 + 1i \quad -0.75 - 1i]$$

So, this is the desired pole locations. So, using the place command, we can find the control gains, right?

$$K = place(A, B, V);$$

$$K = [0.77 \quad -0.26 \quad -1.57 \quad -0.01]$$

Through the control gains of the system. So, we have the K, we have A, B, and also we can assume some initial condition also. So, we can find the solution.

So, the augmented system matrix will be the augmented system matrix, $A_{CL} = A - BK$ you can find. All the matrices are known. We know A also, B also, K. We already found here. So, now we can write the code for this particular example. So, this is the code.

This is the code for this example. So if you notice in the first case without control, State feedback control brings the aircraft to a particular altitude. This is how we can control the altitude. It means the autopilot quickly brings the airplane back to the designated altitude using the state feedback control we have here. This is basically how you can design altitude hold. of an aircraft. This is the altitude hold autopilot system, and this code you can use in your MATLAB to find the response.

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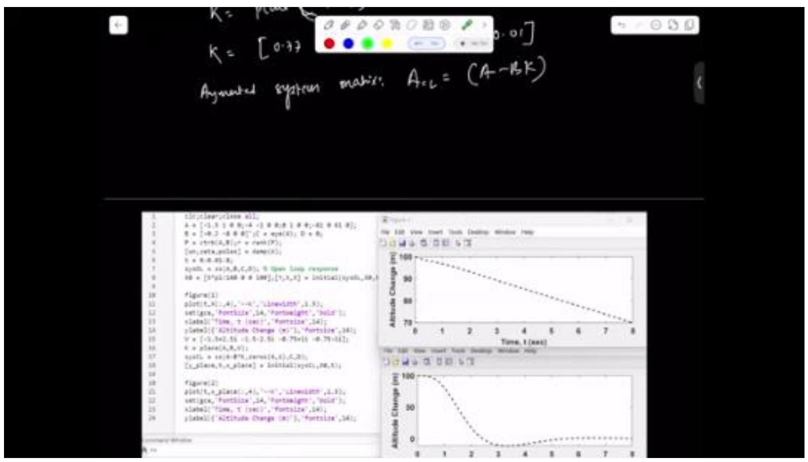

This is the response for the open loop, and this is the response for the closed loop system. We hope it is useful to design an autopilot for your aircraft. So as of now, we have done many things, such as how to design control for unstable systems based on eigenvalues. You can define the system stability, and if the system is found to be unstable, you can design control. We have come up with optimal control, state feedback control, and also how to develop the stability augmentation system for state space model systems. We are almost at the end of the control synthesis in the linear domain, and how to come up with different control concepts. Now we are going to start a new topic on how we can design non-linear controls for the aircraft. We will also have a lot of MATLAB code and examples. So let's stop here and continue from the next lecture on the new topic: how we can design non-linear controls for an aircraft. Thank you.