Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 22

Stability augmentation System for short period longitudinal dynamics

Hello everyone, in today's lecture we are going to consider longitudinal short period dynamics of an aircraft, how we can find the stability augmentation system. So, let me first define the problem and the longitudinal short period So, as we have studied before in our previous course that longitudinal motion has two different modes. One is the short period. that long period, long period motion.

So, this thing with this different modes we can define based on the eigenvalues of the eigenvalues of the system matrix. So, I am not going to detail about it already I have explained in our previous course. Also in this course we had also some basics how we can define the short period and long period motion of a longitudinal motion dynamics. angle of attack and pitch rate. And the system matrix given to us

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{q} \end{bmatrix} = \begin{bmatrix} -0.33 & 1 \\ -2.52 & -0.38 \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \Delta q \end{bmatrix} + \begin{bmatrix} -0.027 \\ -2.6 \end{bmatrix} \Delta\delta_e$$

So, here if you notice we need to design the stability augmentation system which will help us how can we make the state goes to 0 it is a linear system and the equilibrium point is quite obvious 0 0 and the question is here the aircraft. The aircraft has short period handling qualities, this is the based on values of the system, we can define this is basically short period motion dynamics and design stability augmentation system such that the system follows the desired dynamics short period Eigen values

$$\lambda_{sp} = -2.1 \pm 2.14i$$

So, this is the problem and we have to design stability augmentation system and we have the desired poles. This is the desired poles. The system should follow this dynamic characteristics. So, let us find the solution for this particular problem. From the given system, we can find the augmented matrix.

Okay, one more. This problem we have taken from Nelson book. course website the book so this program taken from this book the augmented we have

$$A_{CL} = A - BK^T$$

$$= \begin{bmatrix} -0.33 + 0.027K_1 & 1 + 0.027K_2 \\ -2.52 + 2.6K_1 & -0.38 + 2.6K_2 \end{bmatrix}$$

this is the augmented matrix of the closed loop control system. Now, we can find the characteristic equation of the closed loop system we can write

$$|SI - A_{CL}| = 0$$

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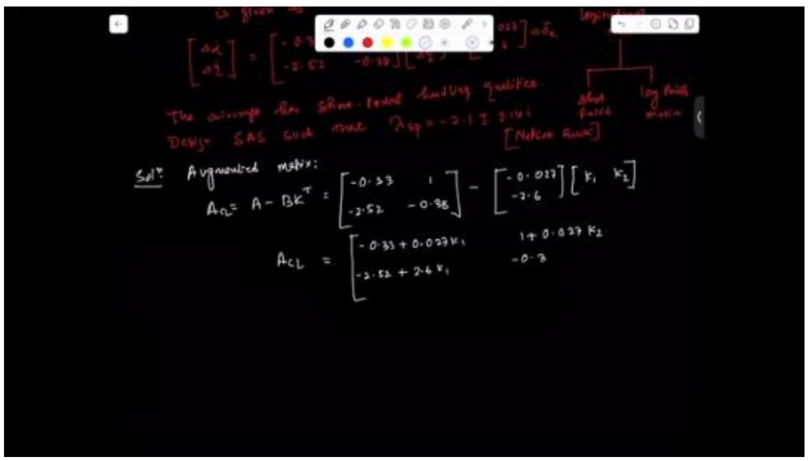

$$\begin{vmatrix} s + 0.33 - 0.427K_1 & -1 - 0.027K_2 \\ 2.52 - 2.6K_1 & s + 0.38 - 2.6K_2 \end{vmatrix} = 0$$

$$s^2 + (0.71 - 0.027K_1 - 2.6K_2)s + 2.65 - 2.61K_1 - 0.8K_2 = 0 \ldots Eq(1)$$

Now, also from the desired poles we have the desired dynamics from desired poles the Desire polynomial we can form

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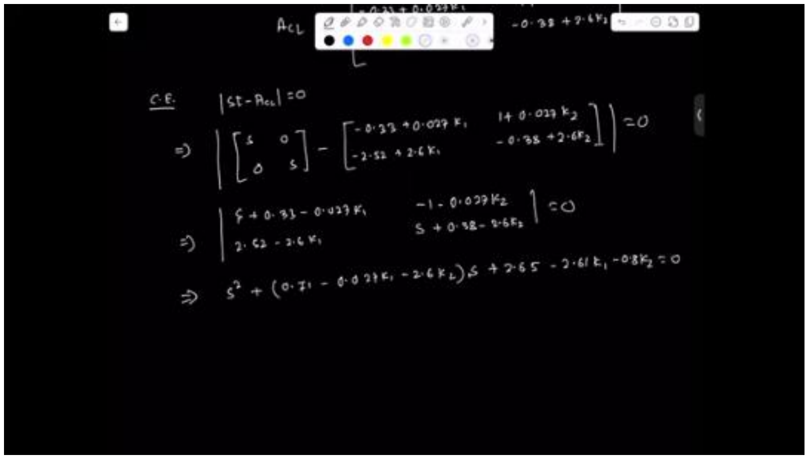

$$[s - (-2.1 + 2.14i)][s - (-2.1 - 2.14i)] = 0$$

$$s^2 + 4.2s + 8.98 = 0 \ldots Eq(2)$$

Now, comparing comparing equation 1 and 2 we have

$$0.71 - 0.027K_1 - 2.6K_2 = 4.2$$

$$2.65 - 2.65K_1 - 0.8K_2 = 8.98$$

Now, if we solve both the equation we have

$$K_1 = -2.03 \qquad K_2 = -1.31$$

And from this we can find the control algorithm

$$U = -K^T X = 2.03\Delta\alpha + 1.31\Delta q$$

So, now let us write the code in MATLAB and we see the result. So, if you notice carefully stability augmentation system nothing, but the controller design for the system and this is the MATLAB code for this example. So, if you notice the dotted line here

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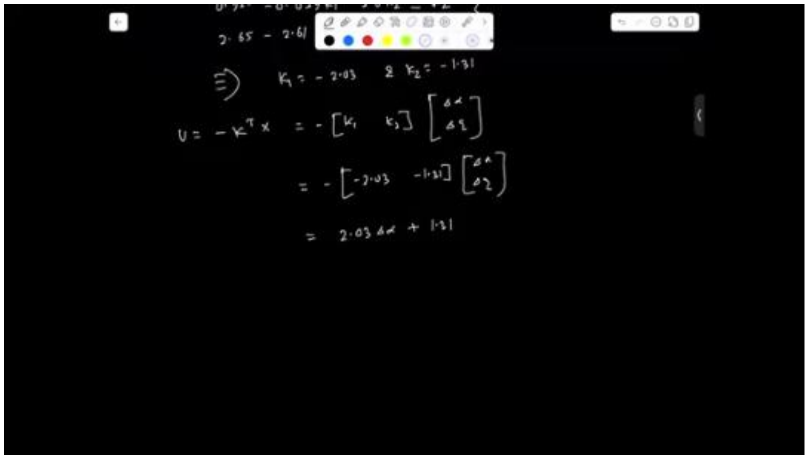

indicates the basic airplane and the solid line basically augmented airplane where we have designed the controller. So, the system dynamics system matrix is defined here and we can find the damp damp of area where we can find the omega and you know damping ratio and the poles and we can find the system into state space form. And where if you notice here carefully we have set the initial conditions of the state 5 into 5 basically 5 radian and 0. And we have the where we can find the solution of the x, y and t. This is the solution of the different states.

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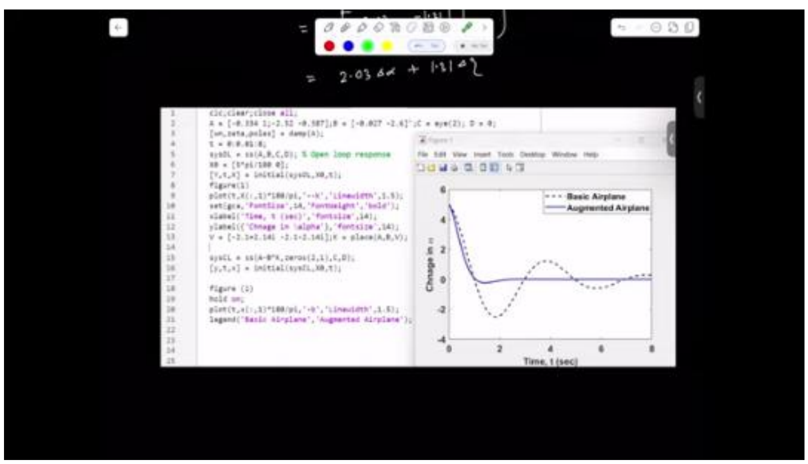

And this is the figure. And in the figure, if you notice carefully, if you have the augmented system where we have designed the controller and over the response, there is less overshoot, undershoot and it goes straight to the zero equilibrium point. But in the case of basic airplane, the convergence at after 8 second it is not exactly goes to the 0. So, here we can say that augmented airplane is performance better compared to the basic airplane and this is basically we have designed the stability augmented system for the augmented airplane.

like if we consider the longitudinal motion dynamics of the aircraft, the equation is actually fourth order, right? But once the system becomes to, the order of the system increases, it is difficult to solve sometimes numerically. So, in that case, we have different methods to solve this kind of So if I write here, if I write as the order of the plant increases, it is difficult to find the closed loop gain of the closed loop system.

closed loop of our control system. There is another method we can use which is called bass gura method. This is so to avoid we can use numerical method we can say, we can use numerical method. one of the numerical methods it is called bass gura method. So, here we can find the control gains or we can say control matrix, control matrix we can use simple formula

$$K = [(PW)^T]^{-1}[\alpha - a]$$

So, here P is the controllability test matrix, W is the triangular matrix and α is the coefficient of the desired polynomial and A is the coefficient of the open loop system, system polynomial. So, now here we can form the W matrix

$$W = \begin{bmatrix} 1 & a_1 & \dots & a_{n-1} \\ 0 & 1 & \dots & a_{n-2} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

so we can form the a matrix for

$$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$$

This is already we have found before how to find the other parameters in this equation. Let us stop it here. We will take an example how we can use this method.

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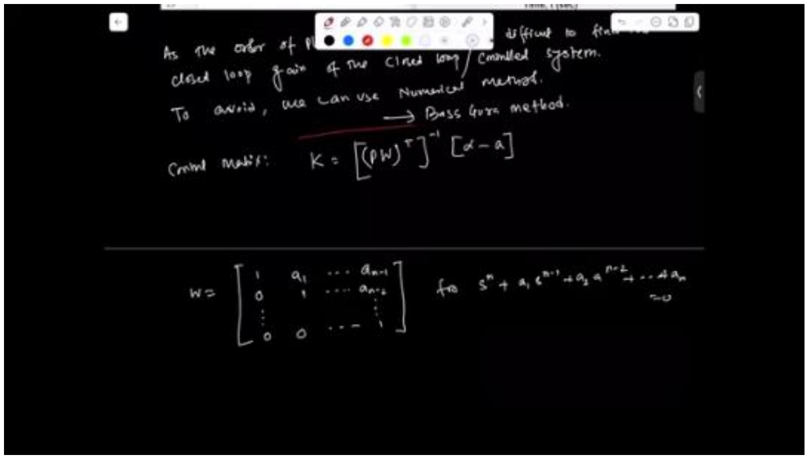

to solve or to design the control algorithm for the aircraft system. So, we will be is we will be considering the longitudinal motion dynamics of the system and how we can design stability and maintenance system using bass gura method. Thank you.