Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 21

Stability Augmentation for pitch dynamics

Hello everyone. Now we are going to start one of the most important topic in this course which is stability augmentation system. In the last few lectures we have discussed the design of controls for an aircraft at different equilibrium points and now we are going to study how we can design the control throughout the complete flight envelope. So for example take off, cruise, landing how we can come up with effective control which will help us to control the systems throughout the trajectory.

Here we are going to consider both the motion dynamics, longitudinal motion dynamics and lateral or directional motion dynamics. And we will design the controls, we will do the simulation results and also we will validate the controls what we will be designing in this topic. Here we are going to cover 2 to 3 lectures in this topic, then we will conclude this topics. The stability augmentation system is a very important part in flight control algorithm. Here if you notice in this diagram,

we are having different stages of flight, right? First is takeoff, then first climb, initial cruise, second climb, final cruise, descent and landing. So in this flight trajectory, you can see that the system has been linearized at different equilibrium point, you can see. And at this different equilibrium point, conditions, we are going to have different stability derivatives aerodynamic data.

So, for that the system matrices differ at each conditions. So, here we are going to have some kind of conditions which provide the flight cruise with an airplane that has desired handling qualities over its entire operational envelope. This achieved by this concept stability augmentation system. This is basically provides artificial stability for an airplane that has been has undesirable flight characteristics. So, to understand how this stability and augmentation system can be implemented for a parallel aircraft system in this following example.

This is very important example how the stability damping or system parameters can be adjusted so that the desired handling qualities of the aircraft can be maintained. The example, Now, this example we have taken from the book Nelson, this is very important I mean very good book how to study the control system and dynamics of the aircraft. Considered a jet aircraft, jet aircraft which has poor short period short period long period we already discussed in this course also in our previous course ah how can define best and eigen values of the system and the equation of motion for for a constrained constrained pitching dynamics pitching motion is

$$\ddot{\theta} + 0.071\dot{\theta} + 5.49\theta = -6.71\delta_e \ldots Eq(1)$$

So, here the control variable is theta and delta u is the control input by the elevator. Here design stability augmentation system augmentation system which has damping ratio 0.3 So, we have to design a stability augmentation system which will have the damping ratio 0.3. Now, so first we have to find the damping ratio of this current system, right. So, we have to find the damping ratio of this system solution. our original dynamics

$$\ddot{\theta} + 0.071\dot{\theta} + 5.49\theta = -6.71\delta_e$$

Now, let us assume since we will be discussing this problem based on the transfer function for classical control method to understand the stability augmentation system. Let us assume initial condition initial conditions for $\dot{\theta}$ and θ to be 0. Now if you take the Laplace transform of this equation taking Laplace transform will have

$$s^2\theta(s) + 0.071s\theta(s) + 5.49\theta(s) = -6.71\delta_e(s)$$

and if you write in transfer function form

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-6.71}{s^2 + 0.071s + 5.49}$$

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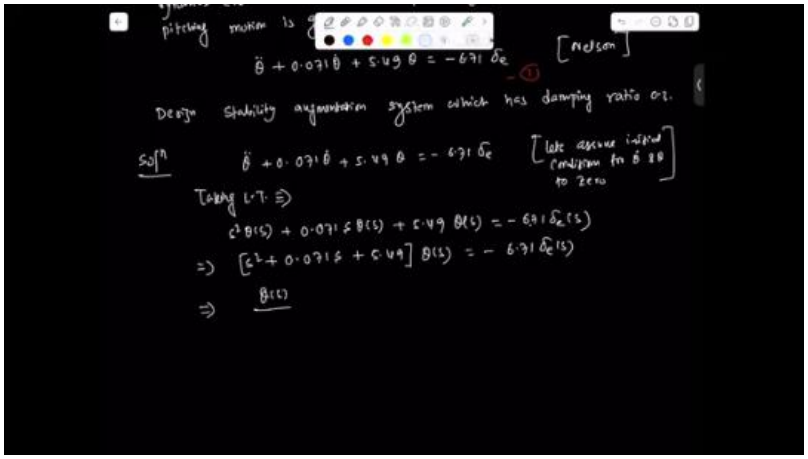

This is the plant transfer function and the characteristic equation we can write characteristic equation

$$s^2 + 0.071s + 5.49 = 0$$

Now, this is a second order system for this particular open loop system. Now, if you compare compare comparing with the standard second order systems

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

if we compare here

$$\omega_n = 2.34\, rad/s$$

$$\xi = 0.015$$

So, if you notice here the damping ratio is quite low. But we need to find, we need to design the control where the damping is going to be 0.3. Now, how to achieve this?

So, since the damping is low, hence this is the short period characteristics and the aircraft has poor flight, flying qualities. So, we can improve the damping characteristic by adding some pitch damper, pitch damper So how to do that? So we are modifying the system in such a way that the damping can be improved and we can maintain the linear stability of the system. So here basically we can, if you add, so let me write the, let me draw the figure then I can explain.

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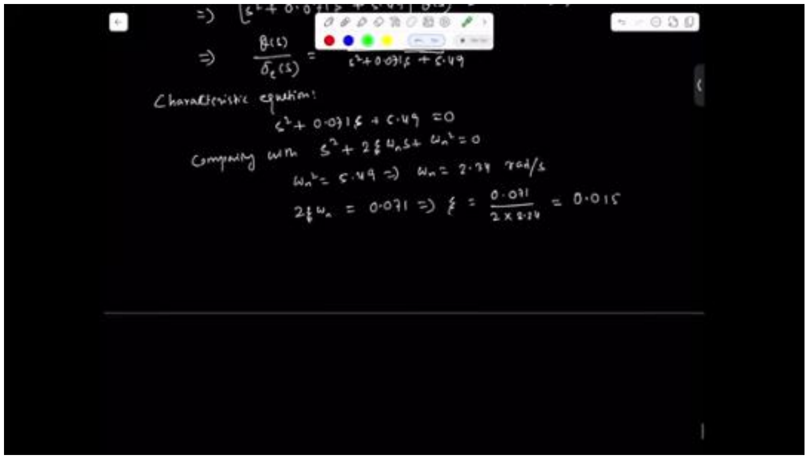

So here we have the summing point for this particular system so we have the q desired that we need to maintain and here we have the pilot pilot this is nothing but the pilots provides the command this is controller and the controller output goes to summing point so here we have the to the plant which is $\dot{X} = AX + BU$ for example or you can write

$$\ddot{\theta} + 0.071\dot{\theta} + 5.49\theta = -6.71\delta_e$$

and we have the feedback so what you are doing here is we are adding here another block which will help us to cover the desired damping the system so here we are adding k so k we are adding here is plus and plus and this is the outer loop feedback And this is here if you notice this is actually we are doing as in this case this is actually nothing, but the stability augmentation system stability augmentation system. So, this is the term k, which we need to provide the desired damping to be added to the system. So, here we can modify the control here now. So, this is the del time control which needs to provide to the system to propagate the dynamics. So, in this particular case we can write

$$\delta_e = \delta_{ep} + Kq \ldots Eq(2)$$

Why you are writing this? Because this is the control δ_{ep} is given by the controller and this is the if you because here we are multiplying q with k. So, the output from this block we can write k q. So, this total δ_e is sum of this and this you can write right. So, here basically we can use a rate gyro to basically find that Q here. Now, if you substitute this delta E in our original dynamics is δ_e, if you substitute in the original dynamic system here, so we can write let us define this equation, Now, substituting equation 2 in equation 1 we have

$$\ddot{\theta} + (0.071 + 6.71K)\dot{\theta} + 5.49\theta = -6.71\delta_{ep}$$

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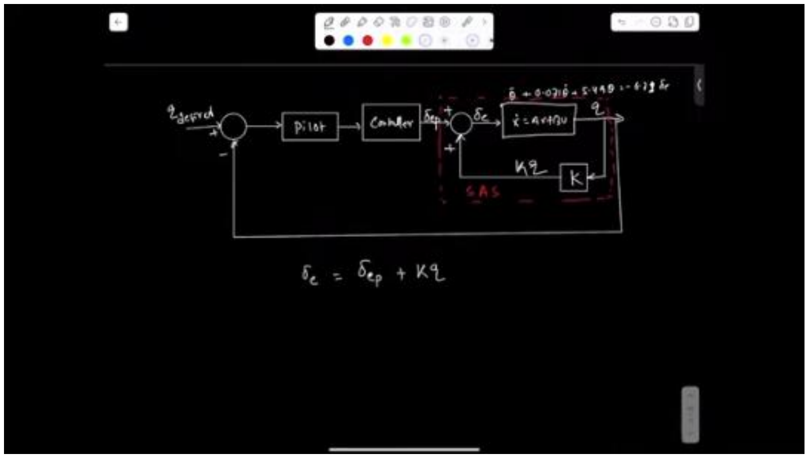

Now, let us take the Laplace transform to the system above we have

$$\frac{\theta(s)}{\delta_{ep}(s)} = \frac{-6.71}{s^2 + (0.071 + 6.71K)s + 5.49}$$

and characteristic equation we are getting

$$s^2 + (0.071 + 6.71K)s + 5.49 = 0$$

Now, if you compare this equation with standard characteristic equation, standard characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

we can get we can find

$$\omega_n = \frac{2.34rad}{s}$$

$$\xi = 0.0151 + 1.433K$$

So here, if you notice here, our damping ratio, it's very interesting equation, damping ratio is function of K, here.

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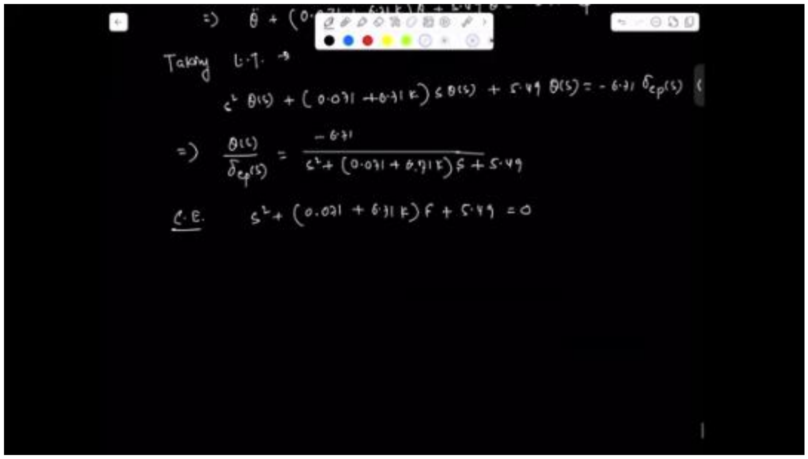

So now, gain can be selected, this gain can be selected in such way that the damping can be, damping can be improved. which can provide the good handling qualities to the pilot so this is basically the problem if you look how we can provide damping in the system so that the in the different flight regime we can come up the desired flight or handling qualities of the system so now damping ratio is function of k now In this case, if you, since in the problem, if you notice, we have to design the stability augmentation system where the damping ratio is 0.3. So, now, if we substitute in this equation

$$0.3 = 0.0151 + 1.433K$$

$$K = 0.198$$

So, this is basically the value of k which improves the damping characteristic of the system. So, this is how we can come up the stability augmentation system for a particular flight envelope. So, this is we have taken for the aircraft. In the next lecture, we will take how we can improve the flight condition or stability augmentation system for the longitudinal motion of the aircraft. So, we will take the full dynamics of the longitudinal motion and how we can and in state space form and how we can improve the flight condition in different flight. Thank you. Let us stop here.

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C.E. $s^2 + (\dots)$

S.C.E. $s^2 + 2\dots$

$\Rightarrow \omega_n = 2.34$ rad/s

$2\xi\omega_n = 0.071 + 6.71K$

$\xi = 0.0151 + 1.433K$

$0.3 = 0.0151 + 1.437K$

$\Rightarrow$ $\boxed{K = 0.198}$