Advanced Aircraft Control Systems With MATLAB / Simulink

Prof. Dipak K. Giri

Department of Aerospace Engineering

Indian Institute of Technology Kanpur

Lecture 20

Example of Linear Quadratic Regulator design

In today's lecture, we will be taking an example of how we can design the optimal control. What we have explained in Lecture 19, we will use that concept for our system. First, let us set the problem example. We will also be showing the MATLAB code for this particular example. So here, consider A twin-engine, this is basically a jet aircraft. The longitudinal motion is given by

$$\dot{u} = -0.007u + 0.012\alpha - 9.81\theta$$

$$\dot{\alpha} = -0.128u - 0.54\alpha + q - 0.06\delta_e$$

$$\dot{q} = 0.69\alpha - 0.51\dot{\alpha} - 0.48q - 12.6\delta_e$$

$$\dot{\theta} = q$$

If you notice, this equation is linear, but the original dynamics can be non-linear. So the system being linear is at a particular equilibrium point, and you could get these linear equations. This is what we have learned so far, how to study the linear system. The question is, design the optimal control, optimal feedback, is k. So, this k is going to find $U = -KX$, and this control is actually optimal control, right? So, here if you notice in the dynamics, here we have one top

So what you have to do is, we have to substitute this $\dot{\alpha}$ here so that on the right-hand side there will be no derivative term of the state. And before that, the terms which are very conventional, use the velocity, alpha is the angle of attack, use the pitch rate, it is the pitch angle, and here only the control input is δ_e. Only one control input is working. So now To make the system in good form, we have to substitute in the $\dot{q}$ equation in place of $\dot{\alpha}$, we need to substitute this expression. So, we can write $\dot{\alpha}$ equation substituting in the $\dot{q}$

$$\dot{q} = 0.0652u + 0.96\alpha - 0.99q - 12.29\delta_e$$

So, we can now form the state equation, the state space model we can form.

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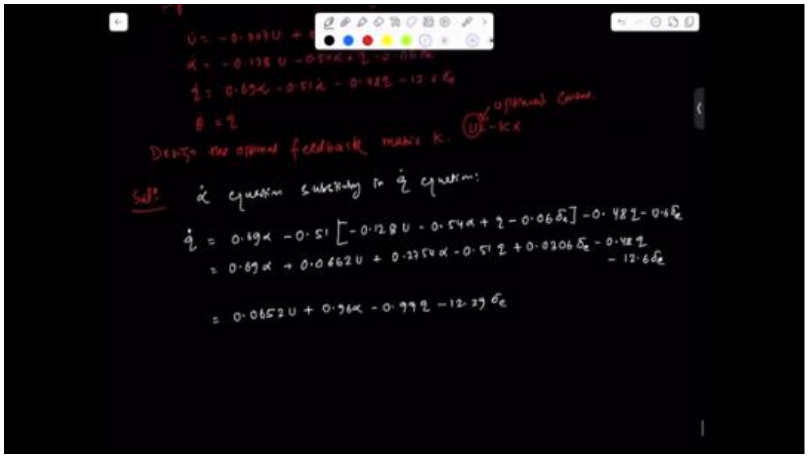

So, here we can write $\dot{X} = AX + BU$ form where

$$X = [u \quad \alpha \quad q \quad \theta]^T$$

$$U = \delta_e$$

Now, A matrix can be found

$$A = \begin{bmatrix} -0.007 & 0.012 & 0 & -9.81 \\ -0.128 & -0.54 & 1 & 0 \\ 0.0652 & 0.96 & -0.99 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -0.06 \\ -12.29 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

so we are assuming all states are available for feedback. So, we can say here all states are observable. This is the condition based on this condition we are forming the C matrix. Since we are going to design. State feedback control using optimal value of the K matrix we need to check the controllability whether all the states in the system is controlled or not so we can use the command use command you can form the P matrix ok let's define since you are using. Another term.

Let's define S.

$$S = [B \quad AB \quad A^2B \quad A^3B]$$

$$r = rank(S)$$

$$r = 4$$

So this is the full rank system matrix or a system 4 plus 4. So we can say the system is controllable. Okay. And now if you are going to use optimal control, if you notice, we have two different equations. If you go back to our previous course, we had one equation and this another equation, right? So we have to choose some Q and R, and from this equation, the only unknown will be P. That you have to find, and that you will come to this equation, and we will find K. So, for that, we have to choose Q. So, before choosing Q and R, let us say something.

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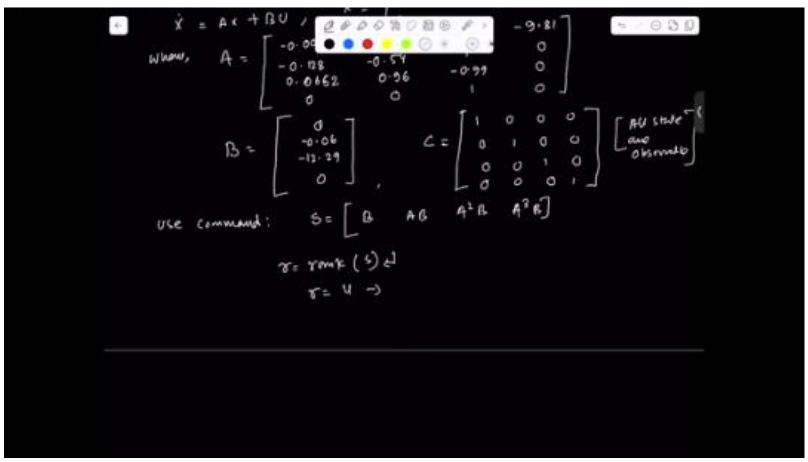

Here, actually, basically, if you notice, the pitch rate and angle of attack are important and which can be controlled by the elevator deflection. So, in that, for this, we can choose

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So if you notice here, we are getting more penalty for controlling the pitch rate and angle of attack because these are the only important variables for the longitudinal motion dynamics, which can be controlled by the elevator deflection. So now, let us choose R equals to 5. We are not using R to be a matrix because here we use a scalar only, δ_a, right? Only delta is here, so we are using. R equal to 5 and this choice can vary if you want to choose a different value, it is fine, but we are choosing these values now. Based on these values, we can say whether the system is stable or not. The current system, because before we design the control, we can say whether my system is stable or not. So for that, we need to find the poles of the system, open up the system. So, let us look at how to find the poles. So, here we can use the common DAMP we used before also,

$$[\omega_n, \xi, poles] = damp(A);$$

$$\omega_n = \begin{bmatrix} 2 \\ 0.8 \\ 0.8 \\ 0.6 \end{bmatrix} \qquad \xi = \begin{bmatrix} 1 \\ 0.1 \\ 0.1 \\ -1 \end{bmatrix} \qquad poles = \begin{bmatrix} -2 \\ -0.09 + 0.8i \\ -0.09 - 0.8i \\ 0.65 \end{bmatrix}$$

If you notice in the poles, the pole location, there is one pole plus, so there is one pole on the right-hand side of the s-plane. So this pole is actually making the system unstable. This pole is actually making the system unstable. This is the open-loop system, right? So now we need to design optimal control or LQR control. How can we come up with optimal values of k? So for that, in the MATLAB command, we can use LQR to find to find optimal values of the E matrix and E. So here, we can use the command

$$[K, P, Eg] = lqr(A, B, Q, R);$$

$$K = [0.4742 \quad -0.3880 \quad -3.2256 \quad -5.353]$$

$$P = \begin{bmatrix} 1.36 & -0.65 & -0.18 & -7.25 \\ -0.65 & 8.75 & 0.11 & -3.57 \\ -0.18 & 0.11 & 1.31 & 2.19 \\ -7.25 & -3.57 & 2.19 & 90.22 \end{bmatrix}$$

$$Eg = [-0.76 \pm 0.79i \quad -0.77 \quad -39.89]$$

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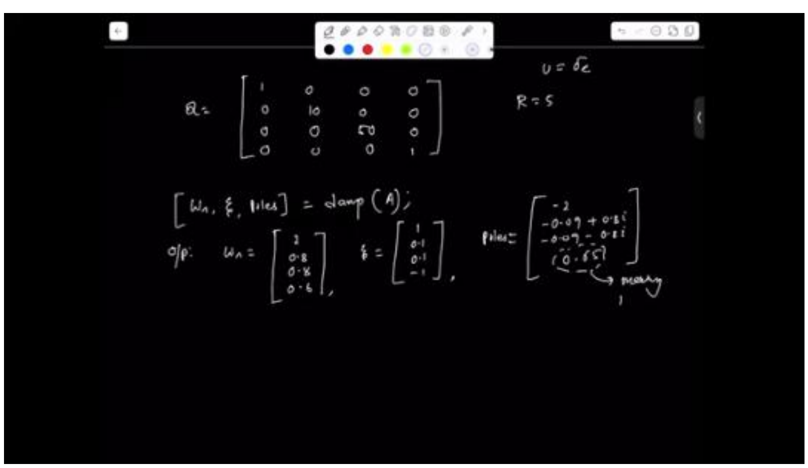

So, here if you notice, very interesting because it is giving the eigenvalues as negative for the system $A - BK$, right. The closed-loop system of these are, this is the closed-loop control system, and this is the augmented matrix, and the eigenvalues of these are negative. So, the closed-loop system is stable. Now, we are going to, okay, one more thing if you notice here, if you go back to the earlier lecture, we assumed the P matrix to be the C matrix. What is the C matrix?

If you, this is the diagonal elements, and the upper triangle and lower triangle are the same. The lower triangle and upper triangle are the same. So, that is why I can say it is a symmetric matrix system. And now, since we will be doing the simulation in MATLAB, to solve any dynamical system, you need to have initial conditions to find the solution of the system. So, for that, we are choosing the initial values of the state x initial, initial values of x,

$$X_0 = [0 \quad 1 \quad 0 \quad 0]$$

so this is nothing but $u_0, \alpha_0, q_0, \theta_0$ right? Now we can go to the MATLAB port. This is the MATLAB port. If you notice, the system over time all goes to 0 here, right. So, it is a stable system, and this is also a regulator problem because the desired value should be 0.

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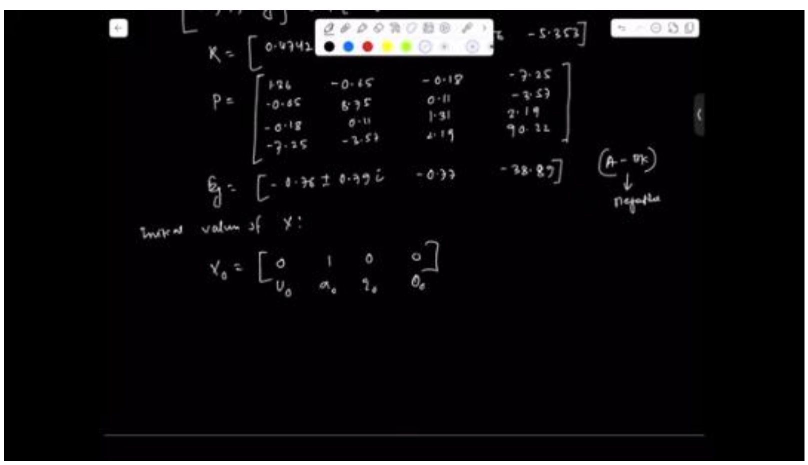

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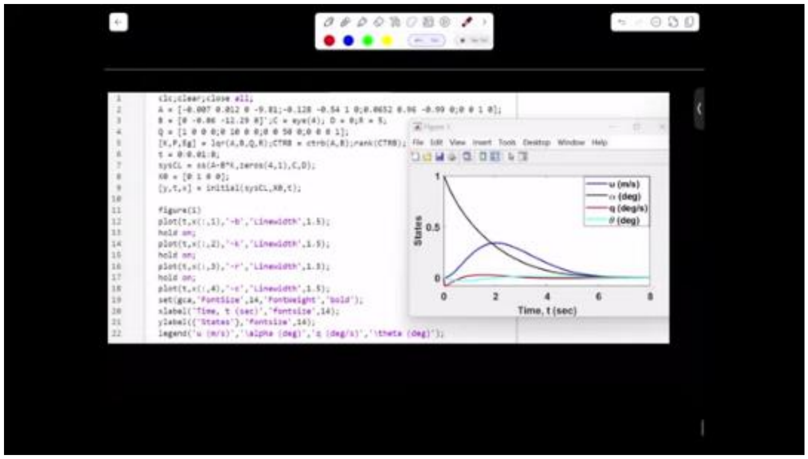

So, it is going to 0. So, this is our A matrix, and this is the B matrix. K has been chosen like this. And also, we have the—sorry, Q is chosen like this, and R is 5, which you have taken in the problem. You can use this LQR command to get K, P, and eigenvalues. Also, you can check the continuity of the system and the rank of the system. These are for finding the condition of whether the system is controllable or not, and we are doing the simulation for 8 seconds. This is the closed-loop system—the closed-loop system in state-space form. This is the initial condition we have considered, and this is how we can solve the system. We can also consider another condition using 'lsim,' which also gives you the solution. This is the command for how to plot, so it is basically.

The code for the optimal control of this aircraft system, and you can check how, if you vary Q and R, we can come up with different conclusions for this particular example. You can verify this. So, this is the homework for you: how we can also design the same control concept for other dynamic systems, which you have done before. You can design the code and see the results. How optimal control is going to give a better response compared to the control concept we have designed for the systems we have considered before. So, you can verify the result and come up with some conclusions that, yes, LQR control provides better results over other control commands or design dynamics we have considered before in the previous lectures. So, this is how we can design the optimal control MATLAB code. I hope it is useful for you.

So, this is still the only example we are considering for optimal control design. If you want more examples, you can reach us. From the next lecture onwards, we will come up with a new topic, which is basically the stability augmentation system. That is a very powerful concept in modern control systems, especially in aircraft systems. So, we will discuss how we can implement stability augmentation control for different dynamical systems, especially for aircraft systems.

Thank you.