Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 02

Basics of Linear Algebra

Hello everyone, this is the second lecture in this course. In today's lecture, we will be discussing how the state-space model can be derived from the series of equation of motion of the aircraft and how we can look at the closed loop control system for the aircraft system and this will be followed by the basics of linear algebra, which will be useful for designing the control algorithms. So, let us start the lecture. Here this is the aircraft equation and this is being derived from the real system. So, this is our aircraft and this is the six DOF equation motion.

So, first three equation here is the rotation, sorry translation motion of the aircraft and this three equation talks about the rotational dynamics of the aircraft and last three equation talks about the actual kinematics of the aircraft. So, here if you notice, this equation is highly non-linear, and coupled and as you have done in our previous course in classical controls for the aircraft system So, if you remember we had two equations of motion, one for the longitudinal equation of motion and another was the lateral directional motion. And in longitudinal motion, we are having the state variable u w q and theta and in lateral dynamics we are having the variable v p r and phi.

These are the states which are involved in the different motions. Now, since the systems are non-linear in practically. So, what we are doing is, we are going to design linear controls for state-space-based control. So, our system should be written in state-space form, so that we can design the state-space-based control. So, for that, first you should have the equilibrium point.

So, for this particular problem, we are considering the trim level flight condition and these are the equilibrium conditions for the level flight of the aircraft. And what we are doing is, we are linearizing the aircraft systems, longitudinal and lateral directional motion of the aircraft. around this equilibrium point. So, we are applying the linearization technique around the equilibrium point for small perturbation of the variables. So, those perturbed variables we are denoting it by $\Delta u, \Delta w, \Delta q$ and $\Delta \theta$.

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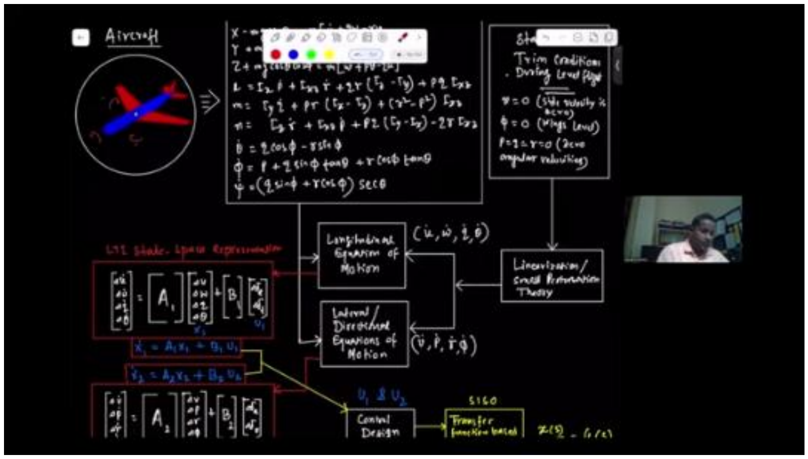

And for the lateral directional motion, the perturbed variable are $\Delta v, \Delta p, \Delta r$ and $\Delta \phi$. So, how this perturbed variable is coming? I will explain here. So, suppose the equilibrium point is x_0. So, due to the disturbance or external perturbations in the system, this equilibrium point being shifted to some Δx.

So, this is the x, you can see. So, our main aim is we have to design a control algorithm, which will help us to mitigate this perturbed variable over time. So, our main aim is even in the presence of disturbance, we need to design some control techniques, which will mitigate this perturbed variables. So, the system will be always at the equilibrium point. So, that is why the system, if you notice here, our state space model are written in perturbed variable form in both the cases.

Now, for lateral directional motion, the control inputs are δ_a and δ_r. This is the aileron control and this is the rudder control. And for longitudinal motions, we are having two control inputs. One is elevator, another is the thrust. So, these are the control input and state vector are, this is my state vector, this is the state vector and this is the system matrix which talks about the system stability, very important point, A_1, A_2. So, this talks about the natural system properties, how the system is going to behave.

So, based on the eigenvalues of the A_1, A_2, we can comment whether my system is stable or not. so if it is unstable, we need to design some control algorithm, even it is stable we can make it better, if you have better control algorithms, so if you take particular example for like longitudinal motion of the aircraft and here for example, we are going to control

u_1, we are going to design u_1, which is going to control these states so if you look in state space, how the control has been designed, so this is our system, which we are getting from the linearization method and if you consider this is our control, this is the control let us assume we are going to control this longitudinal motion of the aircraft. So, for that we have to design u_1 in such a way that $\Delta u, \Delta w, \Delta q$ and $\Delta\theta$ is going to 0.

This is our main aim of this control having the control loop. So, here this is controller This is the controller, we can say that is the pilot and this controller gives that ideal control input U, which is going to detect the control surfaces and the engine and due to which we can get the force, and torque. and which is going to propagate the dynamics. So, this is our dynamic equation, this is going to propagate it and this is the sensor which is going to sense the current attitude and position of the aircraft, which is going to feed back to the summing point, where we can find the error and this error is going to the

controller which provides the desired, sorry, ideal control input, which is to be provided to the control surfaces. So, this is how the closed loop control system in state space can be developed. So, here main question is this K matrix. So, how we will design this K matrix, for the state space model? So, here

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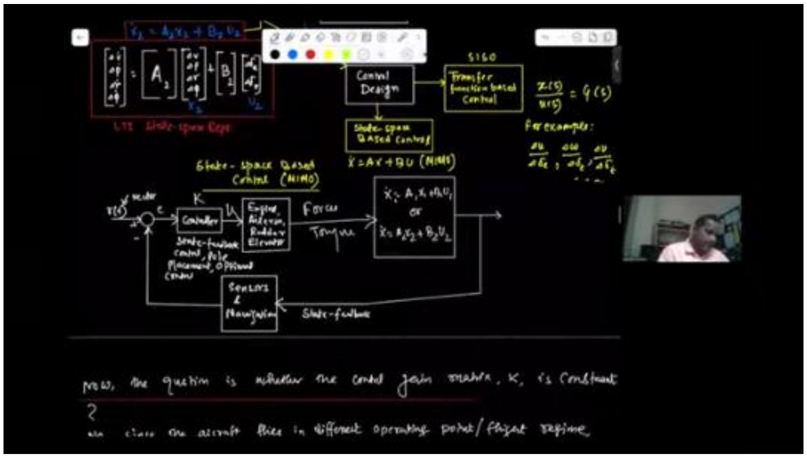

k is not scalar, here k is the vector or we can say in matrix. Why? We are going to control multiple states here at a time. if you look back our classical control approach, we had only one control input and one output, so there we can have one control signal gain, which is going to provide the control input and through which the system can be propagated, but here the control parameter is in matrix form because we are controlling multiple states at a time and Now, the question is how this K can be designed?

So, if you look in this particular case, the system is derived in this state space model, this state space model here, this state space model is being derived at a particular equilibrium point. So, at a particular equilibrium point, the state space is formulated through the linearization technique and at particular flight regime also during the level flight. So, this is the equilibrium condition, one equilibrium condition can be multiple parameters, but at particular level flight condition, this is our equilibrium condition of the flight and based on this equilibrium condition, we have arrived with this linearized model or state space model. So, if you consider multiple flight regime, we will have multiple equilibrium point and we will have multiple state space model.

So, in this case, if you are considering particular level flight condition, the K will be some constant matrix. So, if you have different equilibrium condition, we will have different K-gain matrix. So, we name this phenomena in flight mechanics as a gain scheduling concept, where we have list of gains for the different flight regime, which can be used for different transition of the flight. So, this is how the gain can be chosen here. So, this is I have written here in the notes.

So, if you notice here our the system what you are having in this case. So, here basically the system has been written in the state space form. So, we are having matrices, A is a matrix, B is a matrix, also we have vectors. So, the modern control depends too much on matrices. So, we should have some basic knowledge on linear algebra.

So, let us start with the vector space here. So a vector space V is a set of objects called vectors, for which the operation of vector addition and scalar multiplication are defined. And if you assume these are the vectors in the vector space V and these are the scalars, if they satisfy this type condition, we can say that these are the vectors in the vector space. The similar definition can be defined for the subspace. So, if we add two vectors in the subspace, the sum of these vectors remains in the subspace.

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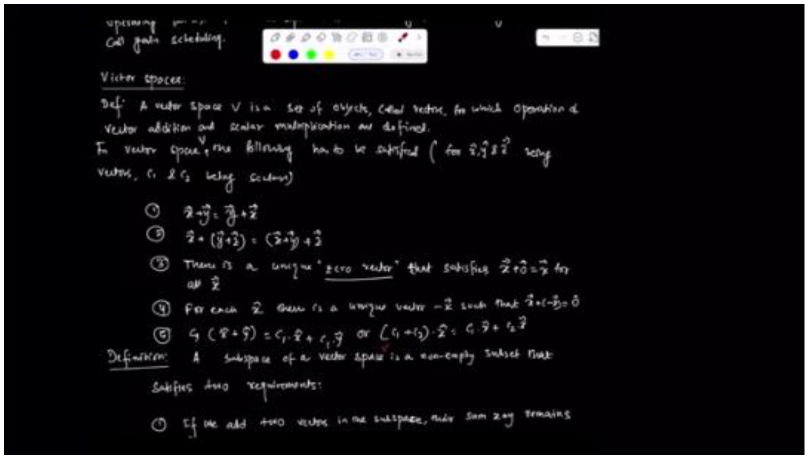

And if you multiply any vector by x by a scalar c, the multiplication c x also still in the subspace. So, this is how we can define vector space and subspace. Now, let us come to the linear dependency of vectors. So, let us consider there are v_1, v_2 and $v_n$vectors in the vector space V and they will be linearly dependent, if there is a set of scalars, this is the scalars with at least one non-zero scalar, this condition if they are satisfied.

So, if any of the vector can be written by the linear combination of the rest of the vectors, we can say that the vector is linearly dependent with the other vectors. For example, in this case, v_3, if you notice here, we can write the rest of other vectors for this condition. So, we can say v_3 is linearly dependent with other vectors. and if you again, if you look at this equation, if α_1 to α_n are 0 and if this condition satisfy in that case, the vectors will be linearly independent only the case when α_1 α_2 till α_1 are 0 other than that, if they will be linearly dependent to each other now Let's come to the basis vector.

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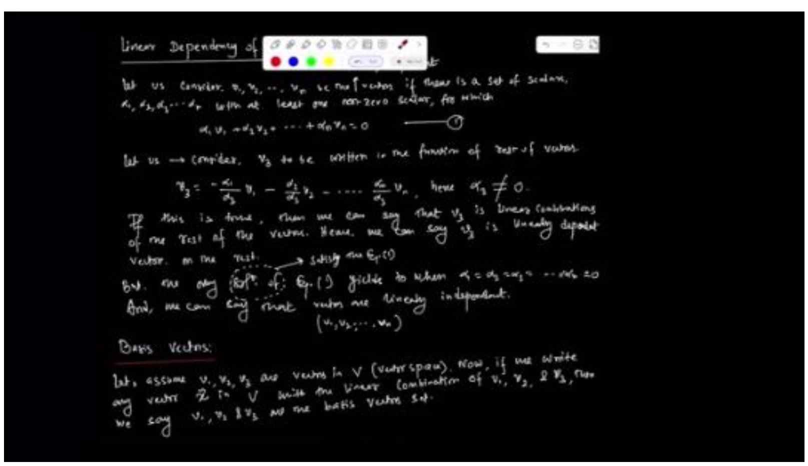

So, basis vector, let's assume there are three vectors v_1, v_2, v_3 and in vector space V. Now, if we can write any vector x in the vector space V with the linear combination of these vectors v_1, v_2, v_3, then we can say these are the vectors defined in the vector space. So, if we look in 2D space here, full space xy is the space and if you can define any vector for example x_1 with the linear combination of i and j and for example here this is the for example, this is a and this is b, so here you can write $x_1 = ai + bj$. So, it means we can define any vector with the help of this basis vector in the 2D space. So, since here we are having i j 2 vector, we define the basis vector.

So, we can say the dimension of the basis vector is 2. So, now let us come to the matrix and linear system. So, here a matrix A is called invertible, when A inverse exists. So, this is you can call invertible matrix, if A inverse exists. And if A and B are invertible matrices, then also they should satisfy this condition.

Then we can say these matrices A is invertible, also B is invertible. Now there is symmetric. If you can write any matrix in this, suppose A is a matrix and if you write the transpose of that matrix and if it comes out to be equal as A matrix, then we can say that matrix is symmetric matrix. And a matrix is skew symmetric, if A transpose equal to minus A. So here I can write here, suppose a matrix A equal to

$$A = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -1 \\ -3 & 1 & 0 \end{bmatrix}$$

so if you write if you can write, this is the skew symmetric matrix and if you write a transpose, it will get minus a so this is skew symmetry matrix and if this follows this condition, then you can say the matrix is symmetric and now let's come to the rank let's assume this is the matrix, one matrix is there and if it is n cross n matrix, square matrix, if all columns of this matrix are linearly independent to each other, then we can say that this is a full matrix, this rank is n. Similarly, if all rows are linearly independent to each other, then we can say the rank is n. If any of the columns is a linear combination of the other columns, then we can say, for example, this is the column, if you can write this column vector, the linear combination of the any of rest of the columns, then you can say the rank is $n - 1$.

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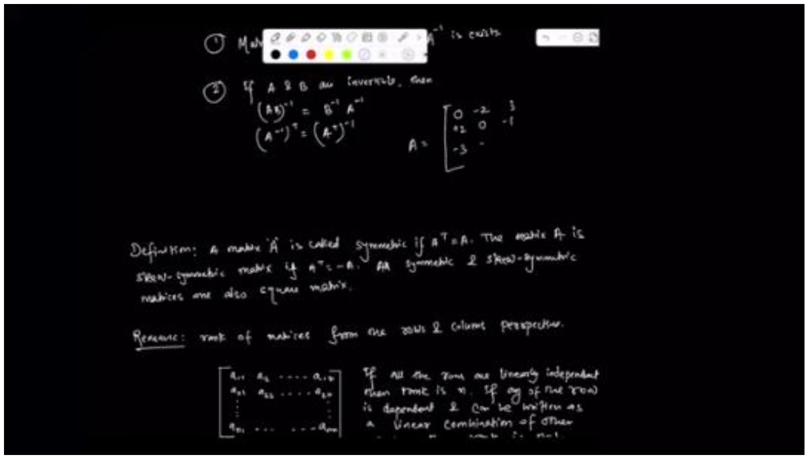

This is not full rank. Similar, the same property is also valid for the rows. Now, this is how we can define the rank. Now, a matrix is defined as square matrix. If $AX = B$ has the unique solution for any x, and for any b, which are in the vector space b $x = 0$ has the unique solution for $x = 0$, it's clear and also a inverse exists determinant of A not equal to 0 and rank of A equal to N. So, if this condition is valid, then we can say, of course, the matrix is invertible and we can find the unique solution of the system. So, now let us take an example and here also if determinant equal to 0, then the rank is not full rank. So, it is quite sure that any row or any column is linearly dependent with other rows or other columns. If it is columns, then it will be linear combination of other columns, if it is rows, linear combination of the other rows.

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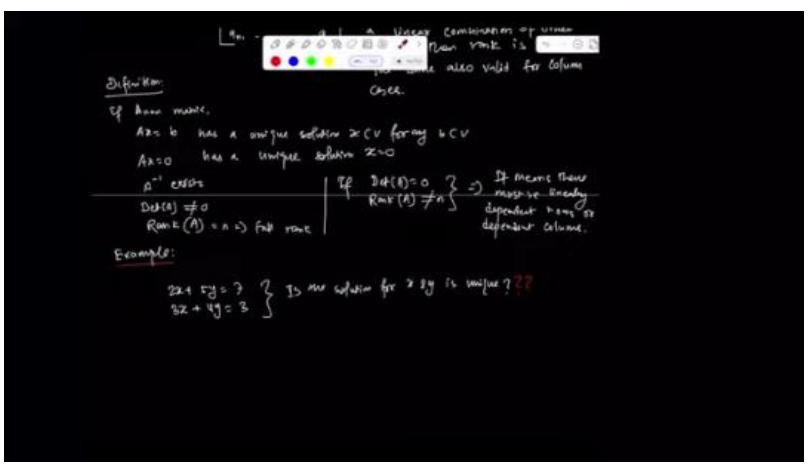

so let's take an example, this is the example where we can find the unique solution for x and y, so here in this case we can write, we can find the solution of this, so you can write, the question is, this is the solution that x and y is unique, so let's check it, we can write this system in a matrix form We can write

$$\begin{bmatrix} 2 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

So, here it is clear the rank of A equal to 2. So, and also determinant of A not equal to 0. So, it means for A matrix, A inverse exist, and is the full time matrix and also you can say that the solution for x is unique. This is how, we can find the solution of a linear equations. Now, let us come to the determinant. So, let us assume that the matrix, n cross n matrix, which is defined like this. so, a_{ij} are the elements of the matrix and the determinant of this matrix can be written in this form, so here these are the element of a particular row or column, in this case, we can say this is i, this is particular row of a matrix and these are the cofactors of that particular element, so now

if you take, this is the matrix and let's find the determinant, so determinant of A, we can find, if you take the third row, so the first element is a_{31} we can write, this is the cofactor, so this is a_{31} should be the cofactor, this is the cofactor and for a_{32}, this will be the cofactor, for a_{33}, this will form the factor, so based on that, we can find the final solution, so determinant is five, we can easily find and coming to important properties, if two rows are equal, determinants are zero because these are linearly dependent on each other And if determinant is 0, then A is singular matrix. This is quite sure. And it is singular matrix, if any of the row or column are linearly dependent on others, then we can also call it singular matrix because A inverse will not exist.

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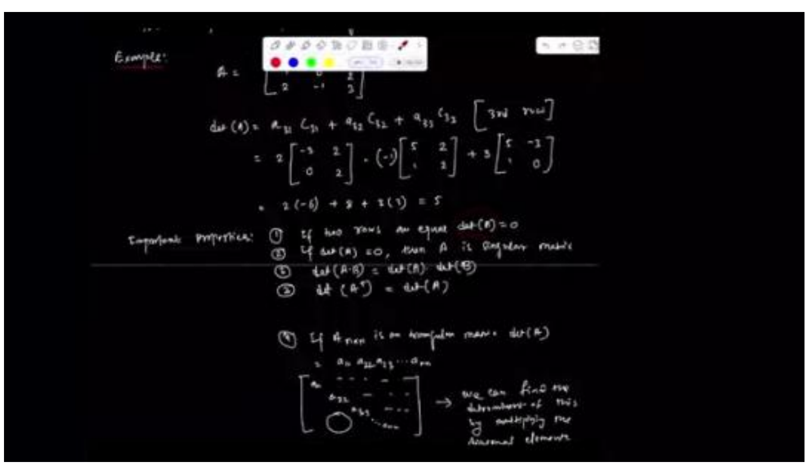

This is also valid. This is also for A and B. This is also you can write. And this is also the same value. determinant of A transpose and determinant of A, will be same. Now, this is the triangular matrix.

Triangular matrix, if you want to find the determinant of triangular matrix, it will multiply all the diagonal elements. Triangular matrix means it has only one side of the diagonal elements. So, in this case there are elements, but there are no elements. And the next part is eigenvalues and eigenvectors. Let A be the n cross n matrix.

let us call lambda be the eigenvalue of the matrix, one of the eigenvalue. And if there is a non-zero vector x, such that this condition will satisfy. This is very very important part. So, eigenvalue and eigenvector should satisfy this condition. If they satisfy this condition, we can say x is the eigenvector for the corresponding eigenvalue, lambda.

So, here let us take an example, so that x is a vector which is $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is eigenvector of matrix and find the corresponding eigenvalue. So, let us solve this problem. So, in that case, first let us find solution So, here solution we can write

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

so we have to validate this condition, we have to validate this property, okay then you can see, these are eigen values and eigenvectors, so here we can find

$$Ax = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$Ax = \lambda x$$

so here it is quite obvious that 4 is the eigenvalue, so now let us check another vector 1, so let us do now let's consider $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, so let's say whether this is eigenvector or not, in a similar way you can proceed,

$$Ax = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$4 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

So, these are not equal.

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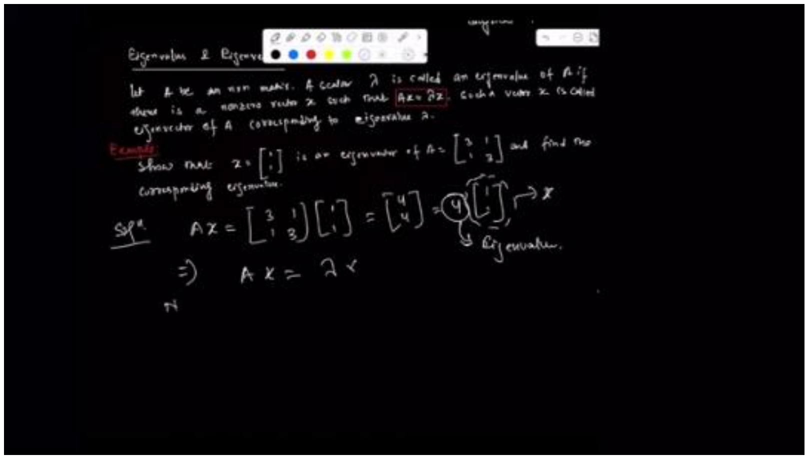

So, in this case, 1, 2 is not the eigenvector. So, this is how we can check, how can find the eigenvalue. Eigenvalue basically you have to find, and how to find eigenvalue? Lambda I minus A equal to 0. We will discuss all this stuff later.

This is how we can find the eigenvalue. And eigenvector, if you know the eigenvalue, you can find the eigenvector. So, this is how we can find the eigenvalue and also you can check whether the eigenvalue or whether the particular vector is the eigenvector or not of the particular matrix, so let's stop it here, these are the basics on linear algebra and we will be using very frequently these things in our subsequent complex system design and thank you, we will continue from the next lecture.

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