Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 18

Example of Observer Design for aircraft system

Hello everyone, this is the lecture where we'll be studying how we can come up with the conclusion on the observability of the system before we proceed to design the observer. Here, we have the same system that we had in the last lecture.

$$\dot{p} = -15p - 15\beta + 2.5\delta_a + 3\delta_r$$

$$\dot{r} = -0.8r + 10\beta - 3.5\delta_r$$

$$\dot{\beta} = -r - 0.8\beta$$

$$\dot{\phi} = p$$

This is basically the lateral dynamics of the aircraft system. Based on the system, the question is: Is the aircraft observable with the output vector Y equal to p and r? In the output equation, we only have p and r as the state variables which can be measured from the output equation. If you notice here, r, β, and alpha, but in the output equation, we only have p and r. The other states are not present in the output equation. Based on this condition, we need to check whether the system is observable or not. Similarly, we have another question: Is the aircraft observable

with the output vector Y equal to p and ϕ? This is similar to the previous question number one. Here also, we have the output which is a function of the roll rate and the bank angle. And this is the second question. The third question we have is: Is the aircraft observable with the bank angle ϕ only? In this case, the bank angle is the only measured output in the output equation. The fourth question here is: Is the aircraft observable with only the β angle?

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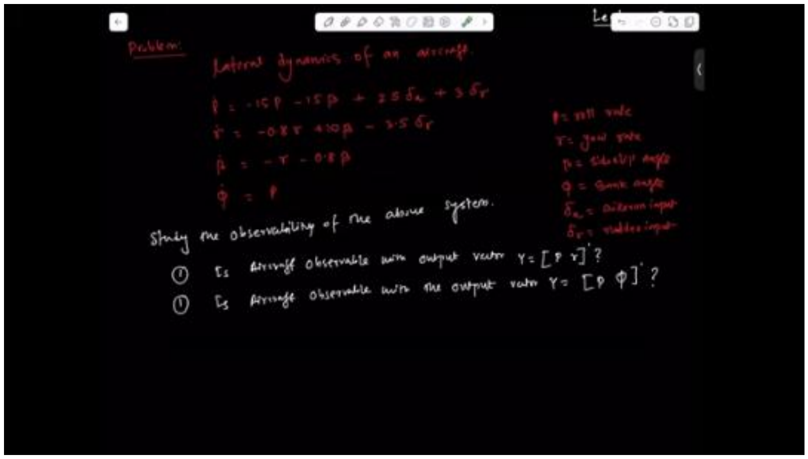

Is the aircraft observable with the side-slip angle β being the only measured output? The fourth question, sorry, the fifth question, is about the observer design part. So, you will be using the system observability of the different questions mentioned in 1, 2, 3, and 4. If you find the system to be observable for a particular condition, we can design the full state observer. So, here design

Full state observer for aircraft using only one of the state variables as output. Here, the desired poles of the observer are given. The desired poles for observer design are given to us as

$$s_{1,2} = -2 \pm 2i$$

$$s_3 = -16$$

$$s_4 = -2$$

So, these are the poles we'll be using for designing the observer where we'll be using

$$\dot{E}_0 = (A - LC)E_0$$

e dot equal to A minus LC. You know this equation already; you had it in the previous lecture, I think in lecture number 14. So, our main goal is to use these desired poles to find the observer gain L, and we'll try to simulate this equation. If all the eigenvalues of this matrix are negative If $eig(A - LC) < 0$, then we can say that if A - LC. Are all negative, then E_0 goes to 0 as t tends to infinity. So, this is the main motivation of designing the observer. Right now, we'll go to solve the problem. This solution will go step by step. So, the first question is the aircraft observer.

Output vector this. So, our output actually we can say X is here. X is actually we have p, R, β, ϕ. So, this state we are getting from solving. The state equation, which is basically we can write this is already you had, but let me do it again: $\dot{p}, \dot{r}, \dot{\beta}$, and $\dot{\phi}$, right? Equal to we have the matrix. We have

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -15 & 0 & -15 & 0 \\ 0 & -0.8 & 10 & 0 \\ 0 & -1 & -0.8 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} 2.5 & 3 \\ 0 & -3.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

So, this is our B matrix, and we have δ_r and δ_a. So, if you solve this equation, we are getting these states, right? And we are having the control input which is going to propagate the dynamics, right. So, now out of this state, we are taking few states. As if Y = CX. This x is coming here. And if you, based on the question given here, we can find C, right? So now, the first question is here.

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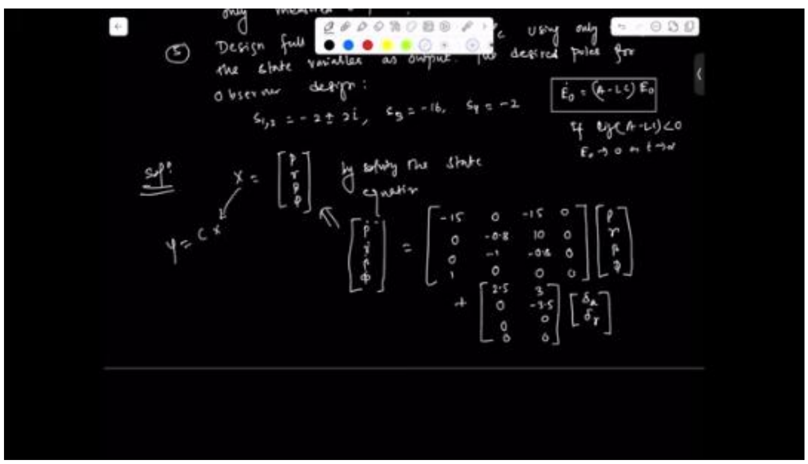

Question number 1 now, we can for the matrix, matrix for the given question

$$C = [1 \quad 1 \quad 0 \quad 0]$$

If you notice where p and r are the first two L states, that's why it is 1 1. Now, also, you can form the observability test matrix, which is

$$N = [C' \quad A'C' \quad C'^2C' \quad ...]$$

Now, we can find this matrix, this matrix N matrix in MATLAB. So, N you can write

$$N = obsv(A, C)$$

$$r = rank(N)$$

$$r = 3$$

r is found to be 3, so only P state we can measure from the observer. So, one state we can't measure. That's why we can say the system is not observable. For the condition given in question 1. Now, let us go to the second question. The second question is, the second question is, is the aircraft observed with the output equation p and ϕ. So, according to p and ϕ, P and 5, so it will be 1 0 0 1, right? So,

$$C = [1 \quad 0 \quad 0 \quad 1]$$

$$N = obsv(A, C)$$

$$r = rank(N)$$

$$r = 4$$

So, it is, you know, it is found to be 4. So, in this case, we can say since rank is 4, means the number of states equal to the rank of the system 4. So, it is. So, we can say the system is, but the system is observable, OK. So, this is very important. Now, we put the system is observable for the second question. Now, let's go to the third part. The third part, ah, in the third part, only in the output equation, we can measure ϕ. Φ is our bank angle. So, we can form C matrix,

$$C = [0 \quad 0 \quad 1 \quad 0]$$

$$N = obsv(A, C)$$

$$r = rank(N)$$

$$r = 2$$

so only two states we can estimate from the observer. So, the system is not fully observable. Now, let us go to question number four. In question number four, we are having only β available in the output equation.

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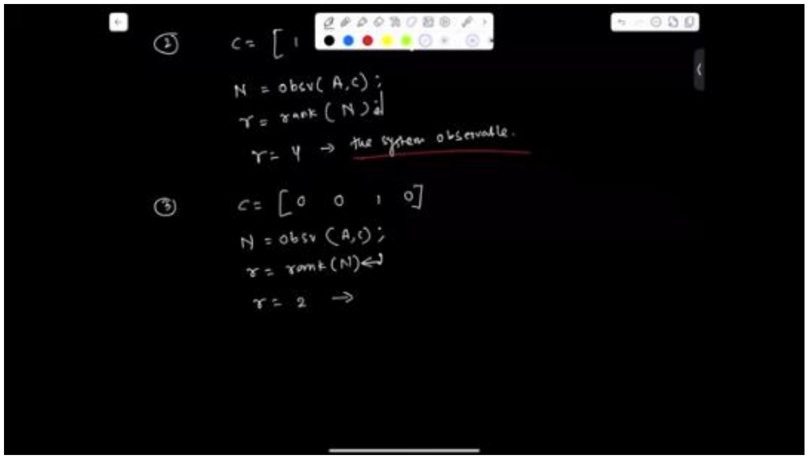

$$C = [0 \quad 0 \quad 0 \quad 1]$$

$$N = obsv(A, C)$$

$$r = rank(N)$$

$$r = 4$$

So, you can say the system is fully observable. You can observe all the states from the observer. Now we'll go to the last question we need to design the observer so here let's take since this condition is observable so let's take question number three the c matrix like this because this is anyway this observable for

$$C = [0 \quad 0 \quad 0 \quad 1]$$

so in this case we need to design we need to design the observer gain matrix gain vector we can say l right so to find l we need to have the desired observer poles so v is given to us in the problem v is here so we can form the v vector using this desired pole location right

$$V = [-2 - 2i \quad -2 + 2i \quad -16 \quad -2]'$$

$$L_{acker} = acker(A', C', V)$$

$$L_{place} = place(A', C', V)$$

$$L_{acker} = [-12.28 \quad -28.745 \quad 3.6672 \quad 5.4]$$

$$L_{place} = [-12.28 \quad -28.745 \quad 3.6672 \quad 5.4]$$

So, the same observer gain vector we are getting for these two different commands. Now, if you use this observer gain vector, we can find $A - LC$, right? A is given to us; we already have it. We just found L, which will come here, and C we already have, right?

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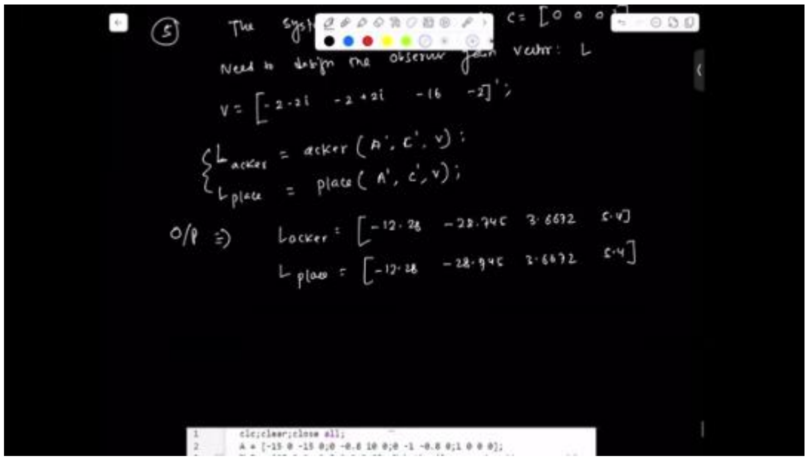

So here, we can use the MATLAB command:

$$A_{CL} = A - L_{acker} * C;$$

$$eigen = eig(A_{CL})$$

$$eigen = [-16 \quad -2 \quad -2 - 2i \quad -2 + 2i]$$

So, if you notice, all the eigenvalues are negative. Since all the eigenvalues are negative, we can say that

$$\dot{E}_0 = (A - LC)E_0$$

So, this dynamic is stable. So, it means we can say since $A - LC$ eigenvalues are negative, E_0 goes to zero as t tends to infinity; the system is stable, and we can estimate all the states. So, this is the MATLAB code for this example. This you can use, and you can try MATLAB; the same thing we have done here. Whatever things we have done in this example, the same thing is given in this MATLAB code. I am not going through this MATLAB code; you can have a look and try the same thing for other systems as well, and you can design the observer. So, let's stop it here. In this part, the state feedback control observer, from the next lecture onwards, we will be starting a new topic on optimal control. How we can design the mathematical model, then how also how we can implement the optimal control methods for aircraft systems.

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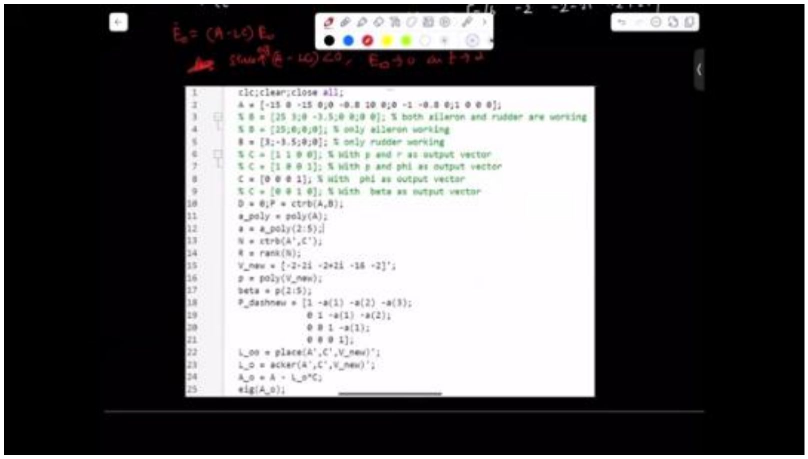

Thank you.