Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 17

Example of Control Design for aircraft system

In this lecture we will be discussing very important part how we can study whether the particular control input going to control the states or if there are multiple control inputs how they are going to control the states of the system. So here we are going to consider the aircraft lateral motion dynamics which is given by this equation.

$$\dot{p} = -15p - 15\beta + 2.5\delta_a + 3\delta_r$$

$$\dot{r} = -0.8r + 10\beta - 3.5\delta_r$$

$$\dot{\beta} = -r - 0.8\beta$$

$$\dot{\phi} = p$$

Here, p is the roll rate, r is the yaw-rate, β is the side-slip angle, and phi is the bank angle. And here we are having two control inputs. One is aileron, another is the rudder.

And there are three questions are set. Is the aircraft controllable using both inputs with all states? are affected by this two control input, is the aircraft controllable using only δ_a that is aileron control input or is the aircraft controllable using δ_r so while designing the classical controls over this modern controls this is very important point whether the control inputs what we are considering in the system are able to control all the states or not So let's start the problem. So from the given system, from the given system, we can form the state-space equation.

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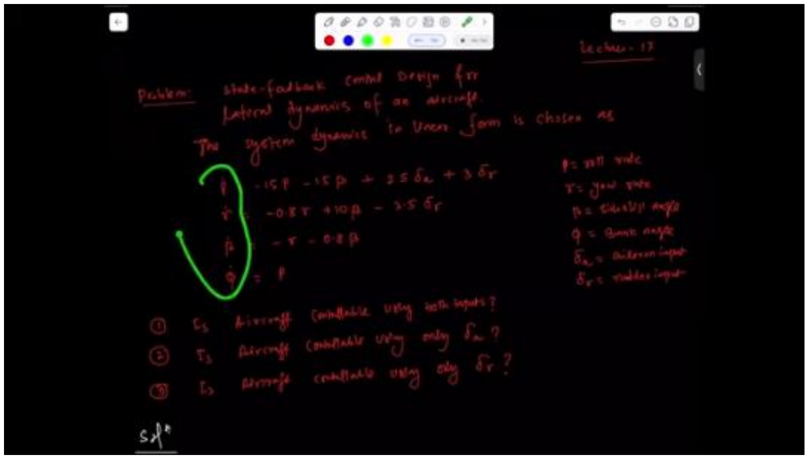

So we can write

$$\dot{X} = AX + BU$$

If you notice, there are two control inputs acting in the system, but as of now we handle only one control input in the state space one is the aileron and another is rudder right. So, here we can write in this form we can write in this form as well. So, here we can write

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -15 & 0 & -15 & 0 \\ 0 & -0.8 & 10 & 0 \\ 0 & -1 & -0.8 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} 2.5 & 3 \\ 0 & -3.5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

so since we are in the problem we are dealing with the controllability test so only you can go with the state equation output equation not required right once you are talking about the observability output equation is required so now for we need to first question is we need to use the two inputs so we need to consider the whole b matrix right so for that we can form the controllability test matrix so controllability test matrix we can form as

$$P = [B \quad AB \quad A^2B \quad A^3B]$$

$$P = ctrb(A, B);$$

$$r = rank(P);$$

$$r = 4$$

we can find rank of matrix and if you enter rank is found to be 4 hence we can say hence we can say system is controllable since n equal to equal to the number of states, right.

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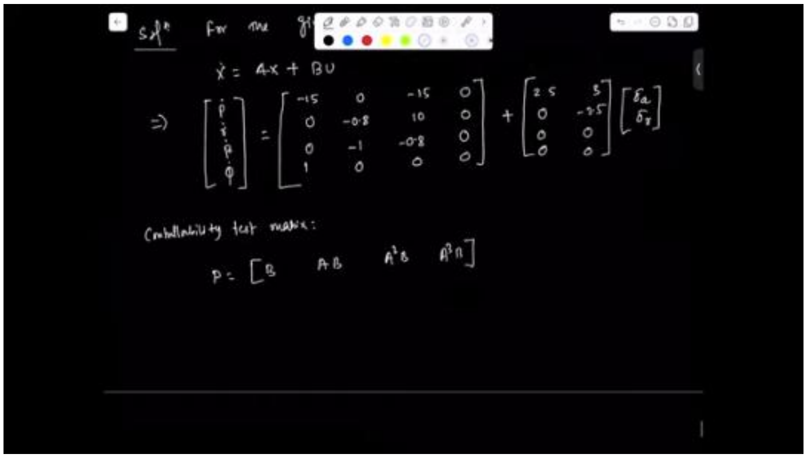

Now, second question, so first question, yes, we can control the system using both the inputs δ_a and δ_r. Second question we have, is the aircraft controllable using only δ_a? If you are having only δ_a, 11 input in the system, if we assume delta to be 0. For example, there is no deflection in the other input. So, in that case, how we can check?

So, in this case, B problem, this is A. So, in B, since we are considering only the δ_a, since this is very important since we are considering only control input is del a, b can be modified as

$$B = \begin{bmatrix} 25 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P = ctrb(A, B);$$

$$r = rank(P);$$

$$r = 2$$

So, system hence system is not controllable. So here, let's look how it is happening. So if I take the dynamic equation here, let's look how this is happening.

So for this present case, we are assuming this part to be the delta part to be 0. So, this part we are not assuming here, right. This is we are assuming to be 0. So, only control is δ_a, right.

First, see δ_a. So, in this case, if you notice, of course, it is rank we found to be 2. So, system is controllable, not controllable. So, it means aileron cannot influence all the states. So, if you notice here, we can only influence P. This is very important. Only using δ_a, we can influence P. And there is no other states except pi dot. These two states, either they are not function with δ_a or not function with P. So, using δ_a, we can control P and P can control P. So, that's why the rank is 2. Only 2 states we can control out of 4 states. I hope it is clear. Now, let's go to the third case, third problem. So, third problem is Is the aircraft controllable using δ_r? So, here we are assuming δ_a to be 0. So, let us look.

Let us see. Now, the based on the controlling point. So, here we are assuming δ_r. B matrix is yields to be B equal to, we can write

$$B = \begin{bmatrix} 3 \\ -3.5 \\ 0 \\ 0 \end{bmatrix}$$

3 minus 3.5, 0, 0, right. Since there is no delta is acting. So, now let us look again in this case. Let me take the equation here. So, here if you notice, we are assuming this to be 0, this to be 0 here in this case, there is no element is at any, right. So, here if you notice, δ_r going to control P and also δ_r going to control, right, both P and R control. And also β is function of r. This is also β can be controlled.

And phi also function of p. So this is controlled. So it means all the states supposed to be controlled. Right. So now, I hope it is clear. Now let us go to the MATLAB code.

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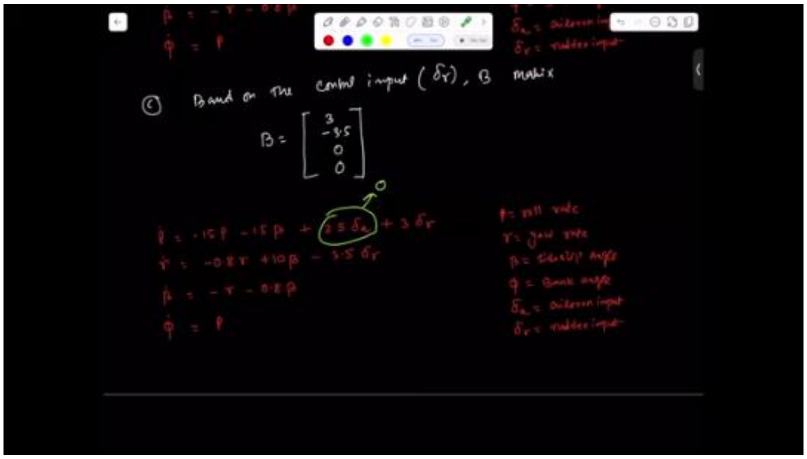

$$P = ctrb(A, B);$$

$$r = rank(P);$$

$$r = 4$$

We can say, here, system is controlled, right. So, now, since system is controlled by using only δ_r, δ_r, now, if you see the poles of the plant, they get So, we can see the, we can check whether the system, how the system is going to control. We can design the controller as well using only δ_r, using δ_r. So, we are doing some more stuff apart from the question being asked in the problem. So, we can come up another question maybe here, how to determine the compensator for this, permitting regulator for this problem. Okay. So, here we are going to do some MATLAB as well. So, we will see the DAMP command to find the pole location of the system.

If you are having only δ_r is acting in the system. Okay. So, DAMP. Okay. And if you enter, you can see we are getting the

$$0 \quad -15 \quad -0.8 + 3.16i \quad -0.8 - 3.16i$$

So these are the poles we are getting from a matrix. So if you notice carefully, we are having one pole at origin. So for this particular case, so system is not global stable or asymptotically stable because one pole at origin. So here we are trying to shift these poles to the left hand side of S plane using some state feedback control.

So for that we are assuming we need to design a regulator So here, design a regulator which places the closed-loop poles of the aircraft

$$s_{1,2} = -1 \pm i$$

$$s_3 = -15$$

$$s_4 = -0.8$$

so if you notice carefully all poles are in the left hand side there is no poles in the origin okay or right hand side so here also we are assuming only design the control design the control using only on sorry only the rudder input. So, here we are going to look the MATLAB code for this. So, let us write. So, V is

$$V = [-1 + i \quad -1 - i \quad -15 \quad -0.8]$$

$$K_{acker} = acker(A, B, V)$$

$$K_{place} = acker(A, B, V)$$

$$K_{acker} = [-0.0777 \quad -0.4095 \quad -2.2324 \quad -1.1662]$$

$$K_{place} = [-0.0777 \quad -0.4095 \quad -2.2324 \quad -1.1662]$$

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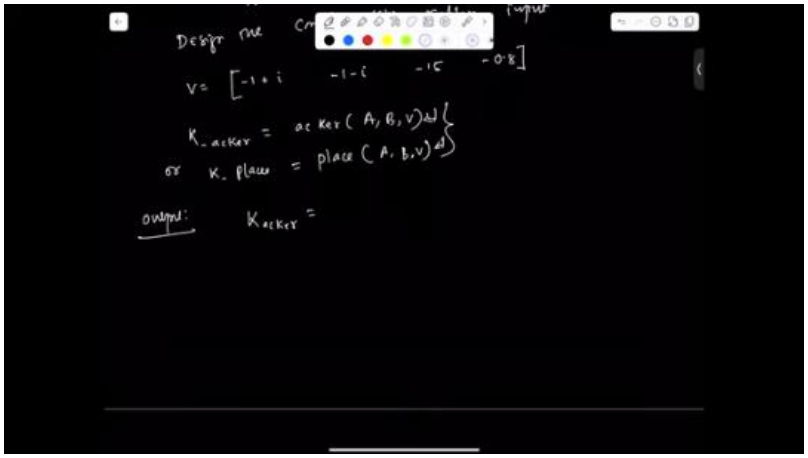

So in both the cases, we are getting the same control gains. So now also you can find whether the closed loop system, the closed loop system or augmented matrix, you know,

$$A_{CL} = A - BK$$

This is our augmented matrix. Arguments of the augmented matrix are in the left-hand side or not. So for that,

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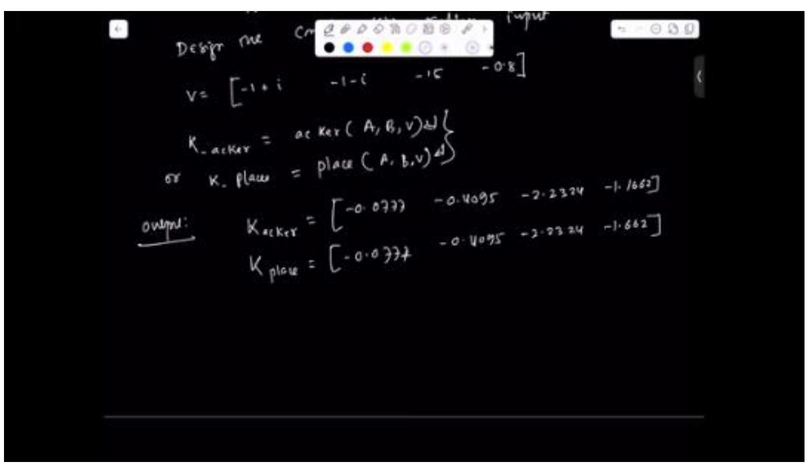

you can use

$$eigen = eig(A)$$

$$eigen = [-15 \quad -0.8 \quad -1+i \quad -1+i]$$

If you notice, whatever the desired pole you choose, it is almost same. So, now, so these are the poles we choose, right, for the desired poles and we are getting the same result almost here. So, here that is why the conclusion of this part is using rudder control input, using rudder control input, We can design the state feedback control for all the states.

P, R, β, phi, we can control all the states using only rudder deflection. This is very, very important problem. So, I request, please see the response because we have to find the closed loop system. Then you can use the lsim command, you can find the response of the system. this is the homework for you but whatever you have done till now we have clear code for you i will show you no this is not been done but you can try and this is easy to do because we have done a lot of MATLAB code for the other complicated system so this is this is also we have done the same thing for other system also so i hope you can do easily

But in the next example, we will take the same problem and how we can come up with some different control design part. Thank you.