Advanced Aircraft Control Systems With MATLAB / Simulink

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Lecture 15

Separation Principle

Hello everyone. In today's session, we will be discussing the principle of separation. This is a very important method in control algorithm design. The principle states that if the system is controllable and observable, the design of the state control and observer can be done separately without affecting the overall performance of the combined system. Once both the methods are combined,

the system stability can be guaranteed. This method actually simplifies the control design process in the control system implementation. In this direction, in this lecture, we will have the mathematical modeling of the separation principle, how it can be implemented for the observer and state-of-the-art control. We will come up with the system dynamic model. And in the next few lectures, we'll have examples of how to implement this method for our practical problems. Then, we'll conclude this part of state feedback control and observer design for a system, and then we'll come up with the new topic: optimal control algorithm. As I mentioned, the separation principle is basically combined system stability analysis.

Where we combine both the state feedback control and observer together to improve the system stability. It actually simplifies the control system algorithm. So, let us start the mathematical analysis of the separation principle. So, here we are going to assume the tracking problem, tracking problem. $X_d \neq 0$, and based on that, we can design the control algorithm. The control algorithm U will be

$$U = K[X_d - X_o] - K_d X_d \dots Eq(1)$$

So, where X is the desired state vector and X_o is the estimated state vector. K is the feedback gain matrix and K_d is the feedforward gain matrix in the system,

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

And the observer part, what we have done before, observer, if you remember, we already have done.

$$\dot{X}_o = (A - LC)X_o + (B - LD)U + LY \ldots Eq(2)$$

Now, if you substitute the control input here in the equation 2, our observer dynamics yields to be

$$\dot{X}_o = (A - LC - BK + LDK)X_o + (B - LD)(K - K_d)X_d + LY \ldots Eq(3)$$

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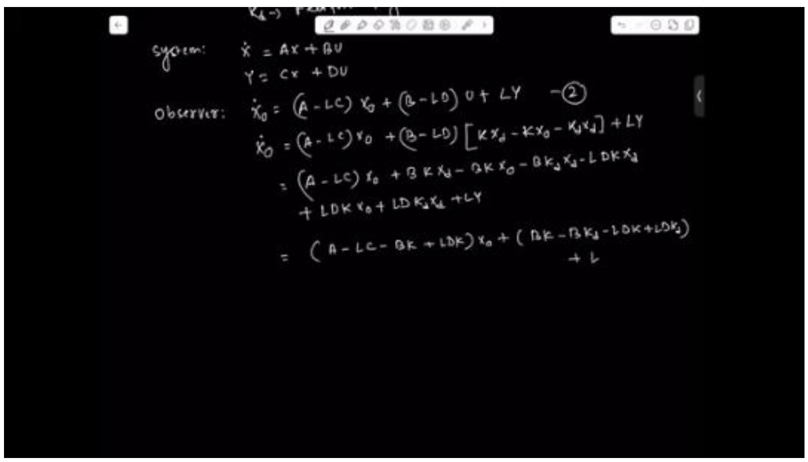

So, you can write this. Also, you know that. We know that the output equation from the original system is

$$Y = CX + DU$$

Now, if you substitute Y here y in equation three, we can write.

$$\dot{X}_o = (A - LC - BK + LDK)X_o + BKX_d - BK_dX_d - LDKX_d + LDK_dX_d$$

$$+LCX + LDKK_d - LDKX_o - LDK_dX_d$$

$$= (A - LC - BK)X_o + B(K - K_d)X_d + LCX \ldots Eq(4)$$

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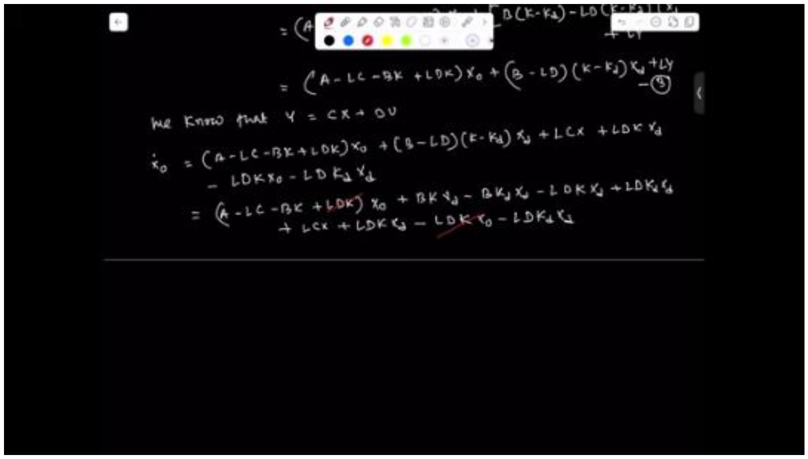

Also, we have the plant equation. From the plant equation, if you substitute, in the plant equation, if you substitute the control equation. So, from the plant equation, we can also write plant state equation. So, we are designing this is the observer part. Now, we are working with the state feedback control.

$$\dot{X} = AX + BU$$

$$= AX - BKX_o + B(K - K_d)X_d \dots Eq(5)$$

Now, writing equation four and five in compact form or in matrix form, we can write equations four and five in matrix form. We can write

$$\begin{bmatrix} \dot{X} \\ \dot{X}_0 \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & A - LC - BK \end{bmatrix} \begin{bmatrix} X \\ X_0 \end{bmatrix} + \begin{bmatrix} B(K - K_d) \\ B(K - K_d) \end{bmatrix} X_d$$

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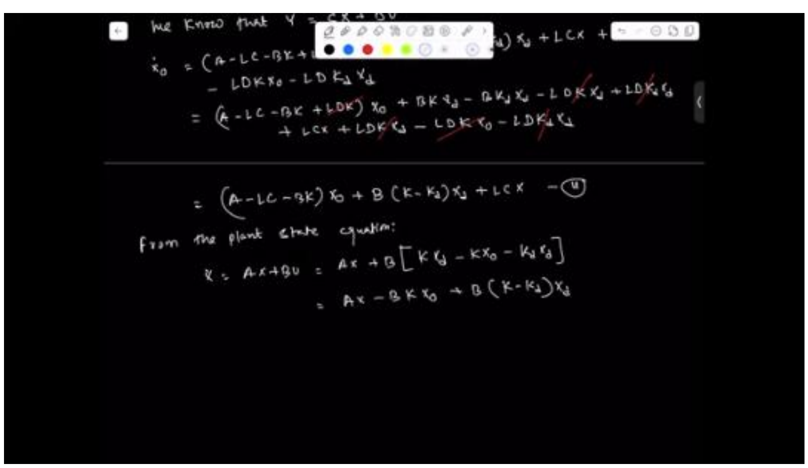

So, here this is the closed-loop control system. So, for this, we can denote that this is A_{CL} This is B_{CL}. Here, if you notice carefully, the order of this closed-loop system, the order of the above system we have 2n because for the first row for the $\dot{X}$ design and for $\dot{X}$ also, so 2n we can write. Also, if you notice here, the input is basically here X_d . $X_d$is the input. So, X_d can be considered as input to this combined system. Now, let us find the error dynamics. So, we have the combined state feedback equation on our observer. So now we will come up with the error equation for this system. So, let's write, assume, assume estimation error $E_o = X - X_o$.

And now the $\dot{X}$ equation we can write based on the zero dynamics. The equation we have, equation five, the equation five yields to be

$$\dot{X} = AX - BKX_o + B(K - K_d)X_d$$

So, here in this equation, we will take the inner part in this equation. For this, we are adding adding and subtracting B K X to the $\dot{X}$ equation. We have, we can write

$$\dot{X} = (A - BK)X + BKE_o + B(K - K_d)X_d \dots Eq(7)$$

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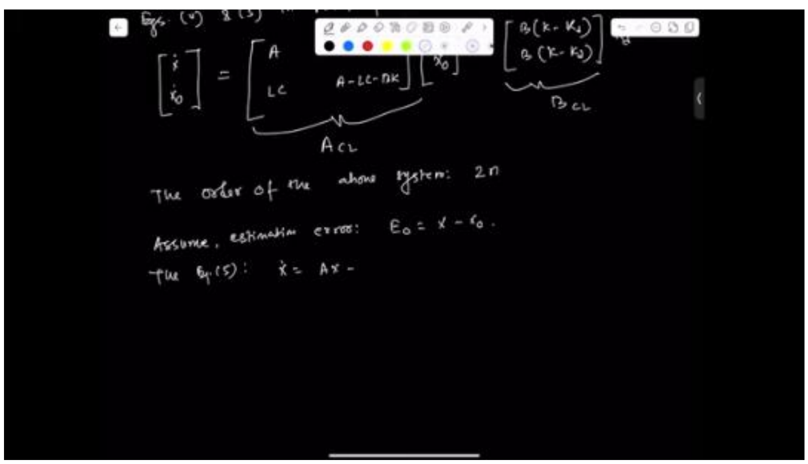

Now, if you subtract X_0 dynamics. So instead of dynamics we have here equation 4. From equation 7 we can write now subtracting equation 5.

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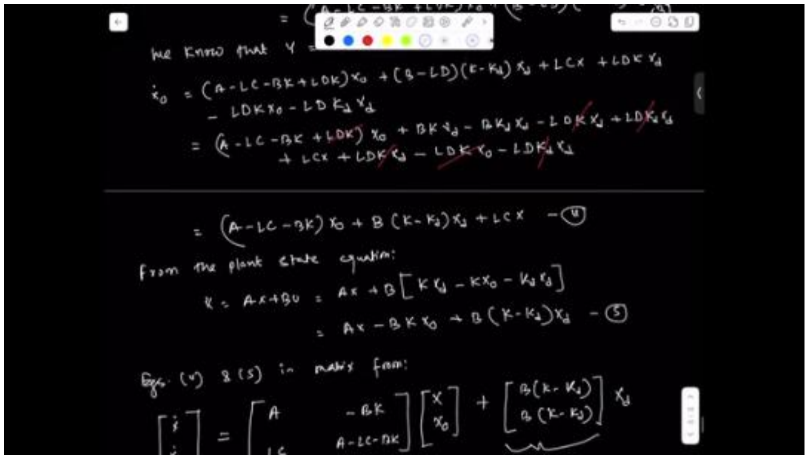

$$\dot{X} - \dot{X}_o = (A - LC)E_0 \dots Eq(8)$$

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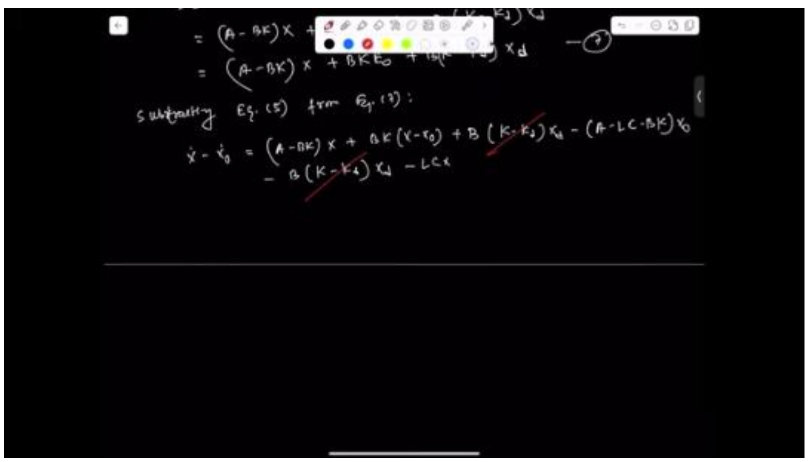

Okay, now, let's find the tracking error dynamics. This is the In order to observe our dynamics now, if you find that tracking error dynamics, you can easily find. So here, tracking error. So now, we are going to find E and E_0.

$$E = X_d - X$$

$$\dot{E} = \dot{X}_d - \dot{X}$$

$$= A_d X_d - AX + BKX - BKE_0 - BKX_d + BK_d X_d \dots Eq(9)$$

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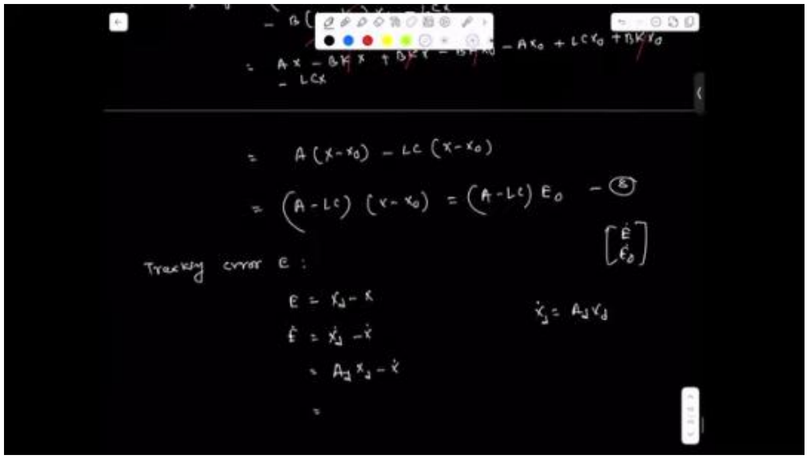

So, now we are adding and subtracting AX_d to the equation on the right-hand side of equation 9.

$$\dot{E} = (A - BK)E + (A_d - A + BK_d)X_d - BKE_o \dots Eq(10)$$

Now, if you combine equation 8 and 10. So, in matrix form of equation. 8 and 10, we can write,

$$\begin{bmatrix} \dot{E} \\ \dot{E}_0 \end{bmatrix} = \begin{bmatrix} A - BK & -BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} E \\ E_0 \end{bmatrix} + \begin{bmatrix} A_d - A + BK_d \\ 0 \end{bmatrix} X_d \dots Eq(11)$$

So in the next step, we can write and let's write this as closed-loop matrix, augmented system A_{CL} and this is B_{CL} let's denote, okay so you can easily see from equation this equation 11 that for the estimation error here if you would like to estimation error E_0 to zero.

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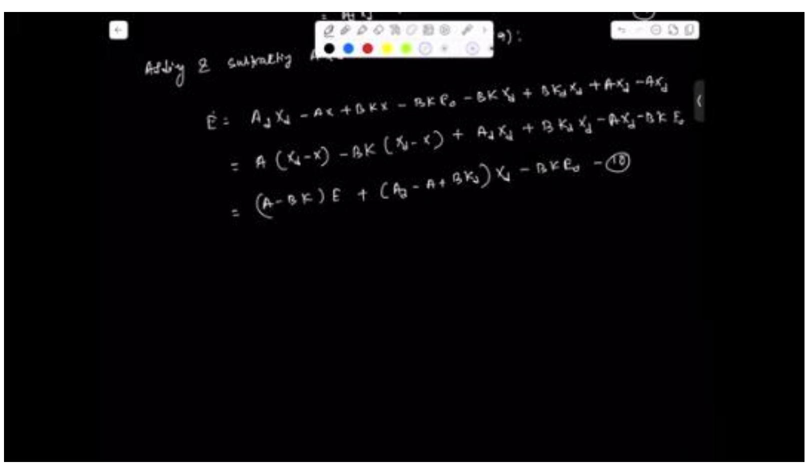

All eigenvalues of $A - LC$ must be negative, must be in the left-hand plane, and for tracking error, this is the tracking error, and this is the estimation error. This is the estimation error, and this is the tracking error. And for tracking error to go to zero, all eigenvalues of $A - BK$ must be in the left-hand plane. And so, this is the condition that should be. Along with this, there is another condition that should also be satisfied. This part should also be zero, so you can write that if E is going to zero, the eigenvalues of $A - BK$ should be negative, and for $\dot{E}$ to zero, the eigenvalues of $A - LC$ should be negative. The final values of this should lie in the left-hand plane, and along with this condition, this part should also be zero, so we can write along with the above condition,

$$A_d - A + BK_d = 0$$

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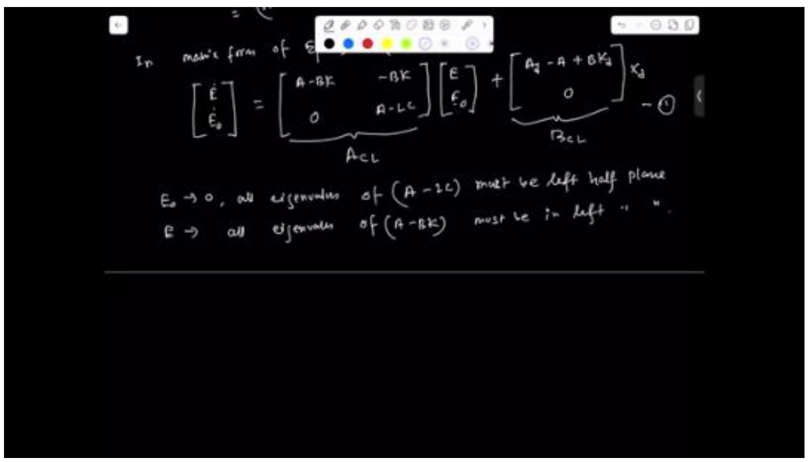

So, how can it be zero? So, how can it be zero? So, we have to make K_d in such a way that $A_d - A + BK_d$ is zero. So, I can write here that K_d should be selected. In such a way that all elements of $A_d - A + BK_d$ equal zero.

So, this is how we can do the combined analysis of the estimator and observer together. And this is how the separation principle is designed for a system. Here, we have also taken the critical cases like X_d not equal to zero, and we have used feed-forward control, but if X_d is not equal to zero, if it is a regulated case, the problem will be more simplified. And even if there is no feed-forward control, it will be simpler. The problem can be defined in a more simplified form. So, we have taken the critical cases here in this design.

So, we will validate this analysis example in the next lecture. Thank you.