Advanced Aircraft Control Systems With MATLAB / Simulink

Prof. Dipak K. Giri

Department of Aerospace Engineering

Indian Institute of Technology Kanpur

Lecture 12

Example of Observer Design

In today's lecture, we will be taking an example of how we can design an observer for a system where some state is not available for measurement. So, we will design this concept based on the theoretical foundation we had in the last lecture. So, let us set the problem. We have a system

$$\dot{X} = AX + BU$$

$$Y = CX$$

Where matrix A is given to us as

$$A = \begin{bmatrix} -3 & 8 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

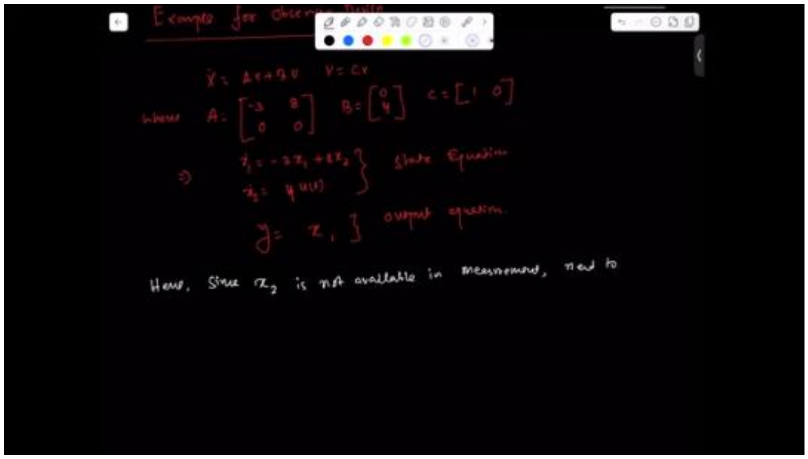
If you write the system in state equation format, we have

$$\dot{x}_1 = -3x_1 + 8x_2$$

$$\dot{x}_2 = 4u(t)$$

Only $y = x_1$, only a function of x1. So, we can write here, since x_2 is not available in the measurement, a state observer to observe x_2. To solve this, first, to solve this, first, we assume that all states are available. So, first, we are assuming that assume all states are available.

(Refer Slide Time: 03:21)



First assumption for feedback and design state feedback gains to meet the closed-loop performance. So, based on this assumption, design a state feedback control to meet the closed-loop performance. Here, we are assuming, for designing the state feedback, here, for designing the state feedback, we are assuming the closed-loop performance. Assume the desired closed-loop performance as following characteristics. So, where we are having the natural frequency and damping ratio as

$$\omega_n = 25\frac{rad}{sec} \qquad \xi = 0.707$$

So, using these parameters, using these given parameters, we can form a characteristic equation. So, using these, we can find the second-order standard characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 35.35s + 625 = 0 \ldots Eq(1)$$

This is the desired characteristic equation given to us. Now, this is the characteristic equation given to us for finding the gain of the state feedback control. So, let us define this equation. Based on these values, based on this, okay, now we will find the characteristic equation for the given system. So, this is the procedure we have followed before to find the gain of the system.

So, we have the desired characteristic equation and the system characteristic equation. So, there we can find

$$|SI - (A - BK)| = 0 \ldots Eq(2)$$

$$K = [K_1 \quad K_2]$$

If you compare this equation and that equation, we can find the control gain parameters, the controller gain matrix. We have the gain matrix.

$$K = [16.5 \quad 8.09]$$

We are not doing this, so this is already done. Please follow the procedure you have done to find the controller gain matrix. You have to just compare the equation. Equation 1 and equation 2, you have to compare the coefficients of S for both equations, and you can find K_1 and K_2 . So, this is basically the state feedback control algorithm. So, here our state feedback control algorithm is

$$U = -K_1 x_1 - K_2 x_2 = -16.5 x_1 - 8.09 x_2$$

So if you notice here in the control algorithm, we have x_2 , but as per the system given to us in the output equation, we only have x_1 . So we need to design the observer here. Before proceeding to the observer part, first we have to test the observability matrix to see whether it is full rank or not. If it is full rank, then we can say that we can design the observer for this particular system. Since this is a second-order system, we can find the observability matrix. The observability matrix N equals,

$$N = [C^T \quad A^T C^T]$$

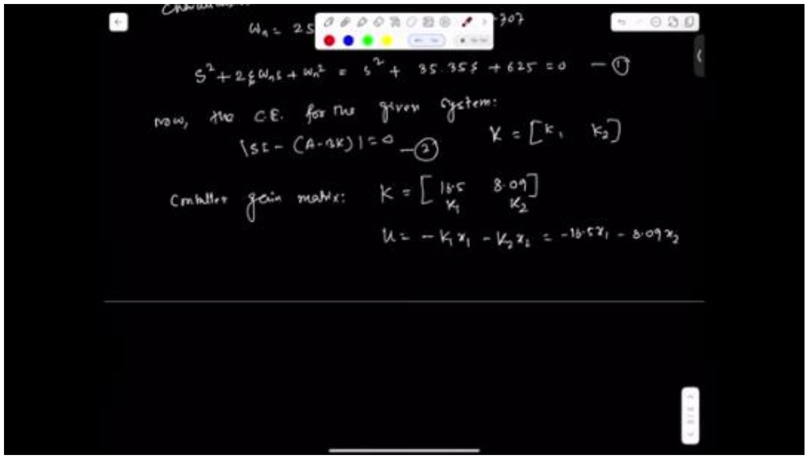
Let's find the second term:

$$A^T C^T = \begin{bmatrix} -3 \\ 8 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -3 \\ 0 & 8 \end{bmatrix}$$

Now if you find that, so there are two ways you can find whether the system is observed or not. One is we can find the determinant. And it is found to be 8 and not equal to 0. It is full rank. Another is rank. If you find the rank of N, it is considered to be 2. So both conditions are satisfied. So the system is observable. Hence, the system is observable. Okay, so now we will find.

(Refer Slide Time: 10:02)



The characteristic equation of the system error system. So, we know our observability observer dynamics from the observability mathematical formulation of the observer design. We had the error dynamics

$$\dot{E}_0 = A_0 E_0$$

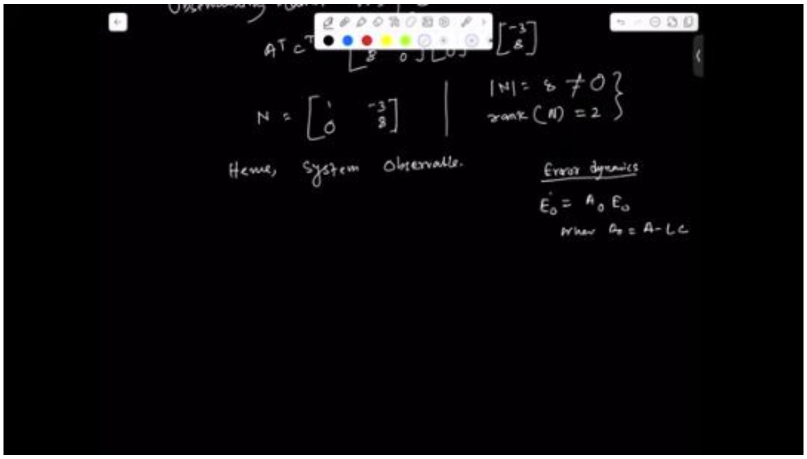
So, where $A_0 = A - LC$, right. So, this is the error dynamics we found in the last lecture. So, if you can make E to 0, then we can say that all states are available for designing the controller. If you can make E_0 zero as time tends to infinity, so for this, you have to find the characteristic equation of the aerodynamics, sorry, yeah, aerodynamics. So, here we can find the characteristic equation of this is the equation let us assume.

$$|\lambda I - (A - LC)| = 0$$

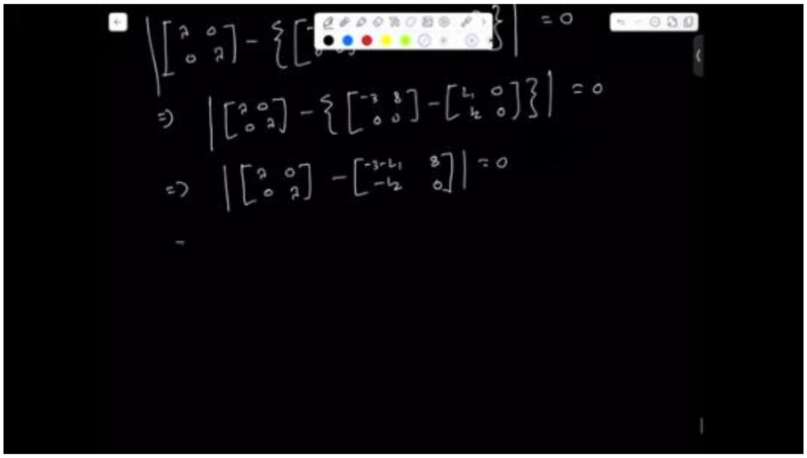
$$\begin{vmatrix} \lambda + 3 + L_1 & -8 \\ L_2 & \lambda \end{vmatrix} = 0$$

$$\lambda^2 + (3 + L_1)\lambda + 8L_2 = 0 \ldots Eq(3)$$

(Refer Slide Time: 12:42)



(Refer Slide Time: 14:29)



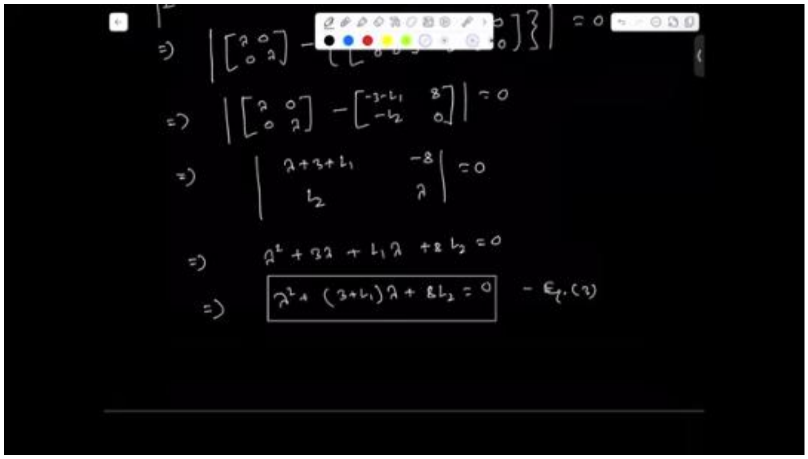
So, now as we have mentioned in the earlier lecture that the eigenvalues of the observer should be faster than the eigenvalues of the desired system dynamics. So before that, we need to find the eigenvalues of our desired polynomial for feedback control design. So we have the desired polynomial for the feedback design. This is the equation we are having, right? This is the desired dynamics we are having for state feedback control design. So let's work on this and find the eigenvalues or the poles of the characteristic equation. So given desired dynamics, given desired dynamics for state feedback control, we are having

$$s^2 + 35.35s + 625 = 0$$

$$s_{1,2} = -17.68 \pm 17.68i$$

From the system, we see there are two poles

(Refer Slide Time: 15:56)



So, from the poles' location, it is quite clear that the system is stable, right? So there is oscillation initially, but over time, it will be negative zero because this is the stable system. All poles are on the left-hand side. But as per the observer design, what you have learned is that the observer must be faster than the system being controlled. Here, we assume observer roots are four times as large as the desired closed-loop performance, right? This is already something we had in the last lecture. So now, four times of these roots, right? So we can find the observer roots.

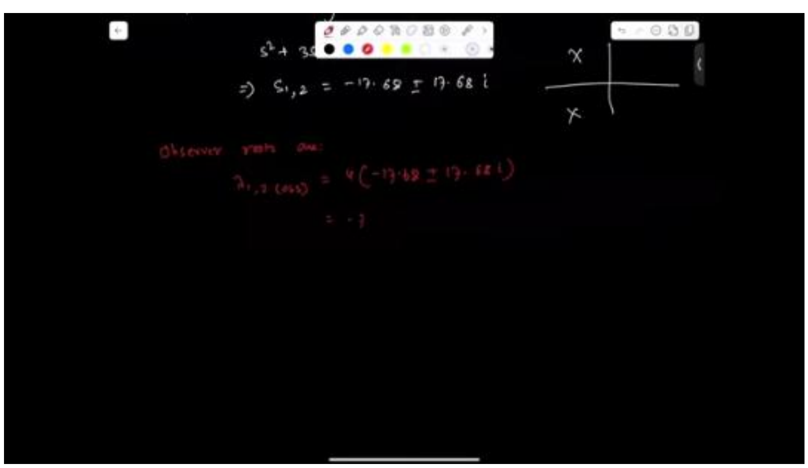
$$\lambda_{1,2} = 4(-17.68 \pm 17.68i)$$

$$= -70.72 \pm 70.72i$$

And the characteristic equation, the characteristic equation we can find for the observer. For the observer, we can find

$$\lambda^2 + 141.44\lambda + 10003 = 0 \ldots Eq(4)$$

(Refer Slide Time: 18:35)



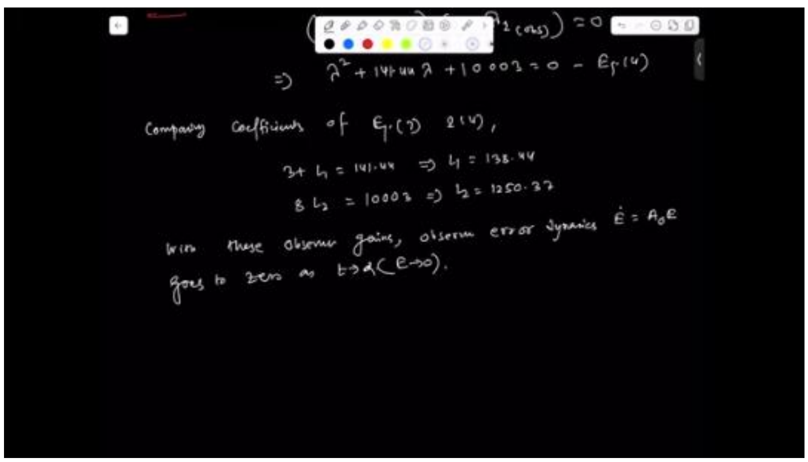
Now we will compare equation 4 and equation 3 to get the values of

$$L_1 = 138.44$$

$$L_2 = 1250.37$$

So, this is the observer gain matrix. With this observer gain, observer error dynamics goes to zero. Now, if you do the MATLAB simulation with this observer and gains, observer error dynamics dynamics, which is $\dot{E} = A_0 E$, goes to 0 as t tends to infinity. So, you can say E goes to 0 as t goes to infinity. So, this is how we can validate that All states are available, even if they are not measurable in the output, but states are available to get the controller for the system.

(Refer Slide Time: 22:35)



So, this is a very, very important part: how we can find the state which is not available in the output equation, but we can find the estimation of the state which is not available in the output equation for designing controls. So, let's stop it here. We will continue from the next lecture how we can find the gain methods because, as of now, if you look, whatever the controller methods we have designed as of now, we are assuming some desired dynamics and also we are finding the characteristic equation of the system, and we are comparing both the equations, and we are finding the controller gains. But there are other ways also we can find the controller gain parameters without assuming the desired dynamics. From the next lectures onwards, we will have some controller concept, another way to find the controller, and we will go step by step.

Thank you.