

# **Advanced Aircraft Control Systems With MATLAB / Simulink**

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**Lecture 01**

**Introduction**

This is the first lecture in the advanced aircraft control system. Here we are not going to talk much on the contents or the references what you are going to consider for this course. This is already discussed in the website. So, in today's lecture, we will be discussing why we need to design the control in the state space domain, what is the motivation, why it is important for aircraft system.

So, we will go through the details and also we have the motivational example, how the transfer function based control or state space based control are very important and which is better and which is not good for the multi input and multi output systems. so let's start with the course, so here we're going to study the control system as i mentioned, so let's assume we have some system which is some scalar system, let's assume some scalar system which is

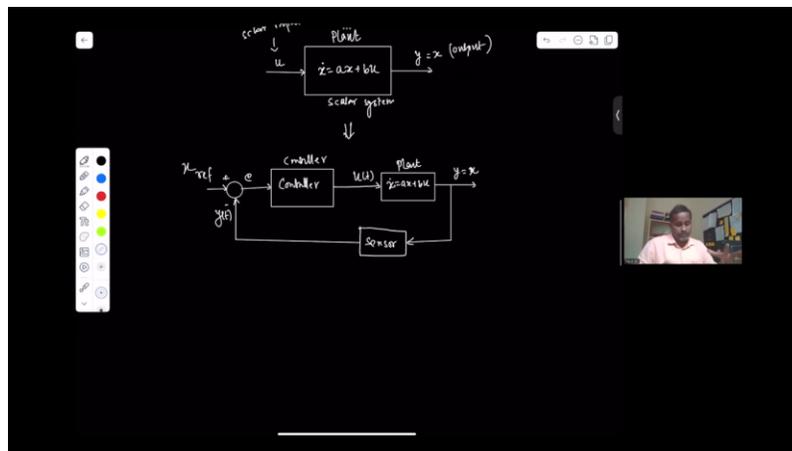
$$\dot{x} = ax + bu$$

and which is controlled by a control input which is  $bu$  and the output of the system is defined by  $y$ , so this is the plant we can say, this is the plant and this is we can say  $y$  equal to  $x$ , so this is the you can say scalar system scalar input and this is the output. And if you notice in the system, the system is controlled by the scalar input  $u$ , whatever the input is defined and based on the input the system will respond, will get the output.

So, there is no way to control the control input  $u$ . whatever based on the predefined signal, the system will respond. Now if you translate this system into a closed loop form, where we can make the system automated. So we need to design in this case,  $u$  should be predefined. but you can design some kind of closed loop control system, where  $u$  can be generated automatically, which helps the system output to track the desired reference signal, so let's assume we have the controller here and the controller is gives the control input which is  $u(t)$

and this control input equal to the plant, which is  $\dot{x}$  equal to  $x$  plus  $bu$  and the plant output which is given by  $y$  of equal to  $x$ , this is the plant, this is the controller and this controller takes the error signal which is coming from the summing point, in summing point we get the difference between the reference signal of  $x$  and also it gets the feedback signal from sensor, which let's denote it as  $y$  of  $t$  and this is the sensor, for example We have the sensor on board the aircraft or on board the plant is the sensor and the sensor senses the current value of the system output. So, here our main aim is basically, this is the closed loop control system, I can say closed loop control system. and this is we can say, this is the figure of the open loop control system.

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So, here from this block diagram we can write,  $u(t) = x_{ref} - y$  so here as we have to design  $u(t)$  in such a way that  $e$  of  $t$  goes to 0 as  $t$  tends to infinity, so in this condition if  $e$  of  $t$  goes to 0, we can say  $x_{ref} - y = 0$ , so in this case we can say,  $y$  is going to track  $x$  reference So, this is how we can design the controller  $U(t)$ , through which we can track the desired signal. So, this is the main purpose of the control system. This is the scalar system and how we can design the closed loop control system for a particular system, here is what we have defined, here we can do.

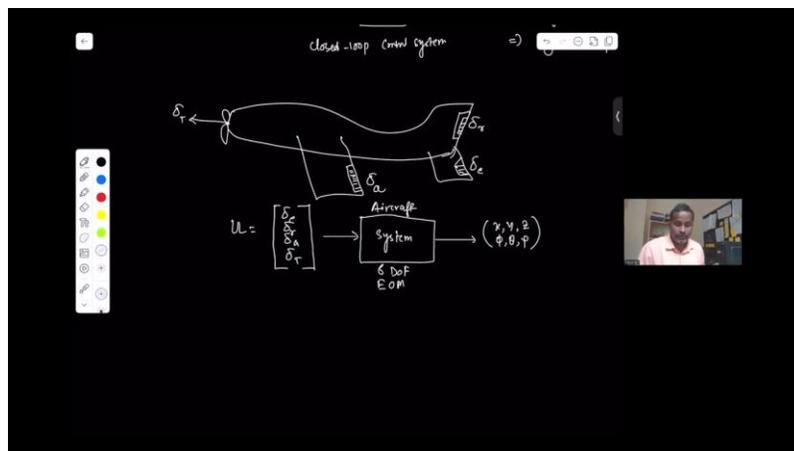
now if you would like to explain this concept for the aircraft application, so let's assume this is our aircraft and these are the control surfaces, so here we can say we can deflect to the desired angle and show that, the control signal can be provided to control the motion of the aircraft, and also we have the engine which provides the desired thrust to the system, so let us assume that, of course we are getting  $\Delta t$  and this is the control  $\Delta A$  and this is the control, which is going to control the surface  $\Delta E$  elevator to  $\Delta R$ . So, these are the control signal going to deflect the control surfaces, so that we can get

the desired signal to propagate by the aircraft dynamics. So, if you notice here to maintain the orientation and position of this aircraft We need to apply different controls to the different surfaces and also to the engine.

So, for this let us assume the control input for this aircraft system. Let us write in vector form  $\delta E$  elevator,  $\delta R$  rudder,  $\delta A$  aileron. So, this is the control input that can be given by the controller. And this controller input is going to the system, our system is, this is the system. This is nothing but for this purpose, it is aircraft.

so this is basically we can say six DOF equation of motion, so this is already we have done in our first course and from this, we can get this here, output from this aircraft system which can be your x y z position of the aircraft and also the angle of the aircraft which is phi theta and psi, so our aim is we are going to provide the control inputs which is the u signal, which is going to the servo system and servo system is going to provide desired force and torque to the dynamics of the aircraft and the dynamics of the aircraft can be propagated so that we can get the desired position and angle, we can maintain the desired rotation and angle of the aircraft, another question is how we gonna measure this signal because these are the current position and angle of the aircraft, how we are going to measure this signal, so here we should have some navigation system, this is you can say navigation also, you can say sensor navigation system which senses the two states of these values, this can be installed on the onboard the aircraft or maybe in the ground, which senses the current position and altitude of the aircraft and through some estimation technique, we can use some kind of navigation filter and we can get the estimated values of the

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states, we can say this is estimated states. So, if you notice in this flow, in this flow diagram, we have going to have multiple control input at a time, which is going to control multiple states of the system and all the states can be sensed by sensors, onboard sensors. So in this case it can be accelerometer, gyro or any other sensors or GPS system for our sensors installed in the aircraft. Those sensors will sense the current attitude and position of the aircraft.

Right. So now if you put them all together in closed loop fashion, what you have done for this particular scenario, for this case. So here also you can do the same thing for this particular case as well. So let's now draw a figure, where we can see how the different subsystems in the aircraft are connected in closed loop fashion. So we have the

let's start with the reference signal, so reference signal here, we are going to control different state position as well as the angle, so x reference nothing but a vector, so here x y z, phi theta and psi, these are my reference signal, down a little bit this is my reference signal and this reference signal again is different, so this reference signal i'm going to the summing point, where we can do the reference signal, we're going to our next part which is controller. This is the controller and controller gives the desired signal which is going to the servo system on board the aircraft. We can say this is the servo system. and this servo system provide the desired forces, i can say torque and forces which is going to the aircraft dynamics

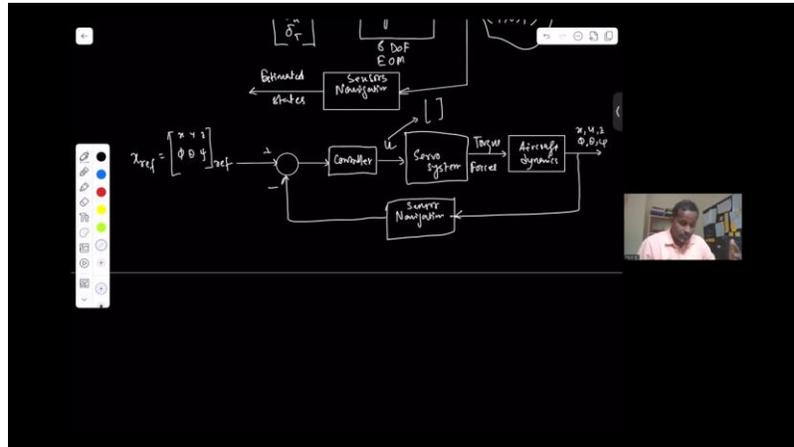
and this provide the actual values of the x, y, z of the aircraft position as well phi theta and psi. So, we can have a navigation system on the aircraft and we have the feedback. This is plus. So, I can see this is the navigation block. So, it is trying to estimate values of the state.

So, this is the sensor also. So, this is how the aircraft control system works. So, here if you notice here carefully, we can control multiple states at a time. There actually you can see this is basically in vector form.

So, we can send multiple control input, so that multiple states of the aircraft can be controlled. But if you look back in the previous course, in the transfer function based control, where we have designed the control algorithm, which can control one state at a time, basically the controller handles the system single input and single output, so here actually we can control multiple states at a time So, that's why state-space-based control design is quite important for the aircraft application. the aircraft system is complicated

with the non-linear terms and coupling nature, in that case, the state-space-based control system is very important for designing multiple states to be controlled.

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So, now This is how the closed loop control system works for the aircraft. Now, let us look at some concept in control. What are the different or how the control system is connected in real application? What are the different subsystems involved?

So, this is only for control, but here also we have the navigation part as well. But in this course, we are not going to discuss much on navigation. Here mostly we will be handling control, how to design the control algorithm, so that the system can follow the desired signal over time. So, in the control system, we are having different subsystems. So, let's go step by step.

So, here as we have done in the previous phase that In the control system, we should have the navigation block navigation, block which provides the estimated values of the system and these estimated values goes to the guidance system, and guidance system finds the error between the desired and actual values, so desired means the reference signal, so guidance system basically finds the error between the desired values and actual values and this command goes to the controller, so guidance system provides the error signal, which is going to the controller and controller tries to correct it, this is going to the controller and this controller  $u$  goes to the actuators So here actuator is nothing but the aileron, rudder and elevator.

We can write aileron, rudder, elevator and engine we can say. So this provides the force and torque. and this goes to the aircraft dynamics and this aircraft dynamics provides the true signal, basically this is the actual position and orientation of the aircraft and this is

supposed to go to the sensor part and these sensor values goes to the navigation, so that some estimation can be done in the navigation system.

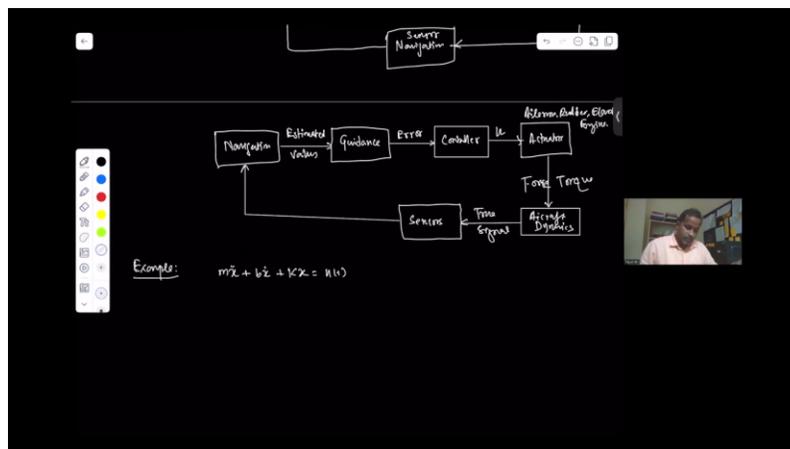
So this is how the typical aircraft control system works practically. So here we are having different subsystem, which has own merits to design the control objective fulfilled. So, this is how the modern control is very important practically and where we can control multiple states at a time, but in the classical control, it is fully transfer function based control. So, we can control one state at a time, but in the modern control, we can control multiple states at a time. So, which is very desirable for such complicated aircraft system.

So, now, how we can find for particular system, a state-space model and transfer function model, because this is also important, let us take some example. So, how we can convert a system, where we can convert into state-space model and also transfer function. So, let us assume we have simple mass spring damper system,

$$m\ddot{x} + b\dot{x} + kx = u(t)$$

So, I am not going to detail of it, this is simple second order system which can be derived from the simple mass-spring damper system. So, here from this system, we can write two different form of equation, one form of equation in transformation form and one form also you can write in state-space form.

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So, if you would like to write in state-space form, state-space form basically defines the system into first-order ODE. So, if you notice this equation is the second-order system. So, we have to convert into first-order differential equations. So, in that case, we need to apply the change of variables. So, here we can write  $x_1 = x$ ,  $\dot{x} = x_2$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{b}{m}x_2 - \frac{k}{m}x_1 + u(t)$$

So these two equations, we can find from the given equation. This equation we can be find from this. So in the transfer function form,

Okay, let me work on this mode. So, if you notice, this is basically two first order ODE, two equations. So, if you write in state space form, we can write. So, there are two states here, one is position and one is velocity. So, we can write in matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

but anyways in the system only one control is there, single control input, so here, if you notice in this equation, so two states can be controlled which is multiple states but if you write in transfer function form, one thing as you have done in the previous course, that to find the transfer function of the system we need to consider the initial condition to be zero. Because if you consider non-zero initial condition, the superposition principle cannot be validated. So, our system would be non-linear.

So, to find the transfer function, we need to consider the initial condition to be zero. So, here, if we consider zero initial conditions, if we apply the Laplace transform for this equation, we can get

$$ms^2X(S) + bsX(S) + kX(S) = U(S)$$

$$\frac{X(S)}{U(S)} = \frac{1}{ms^2 + bs + k}$$

So, this is the transfer function for this mass-spring-damper system and this is the state-space form of the same system, mass-spring-damper system. So, this is how we can study the classical control techniques and state-space-based control techniques.

In state-space-method, we have to convert the system of defined form so that we can design the controller in such a way that multiple states can be controlled. But in the transfer function base, basically if you notice here, it is actually SISO form, single input

and single output, but here we can say multiple input, multiple output. But in this case, it is a single input. we have more examples where we can have multiple input as well.

So, this is how we can study in modern control, how we can control multiple states of the system using the state-space-based control synthesis. Let's stop it here. We'll continue in the next lecture, how we can come up with the how we can design the different control parameters in the system and how we can look in more detail, in this course. Thank you.

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Example:  $m\ddot{x} + b\dot{x} + kx = u(t)$

$x_1 = x, \dot{x}_1 = \dot{x} = x_2$

$\dot{x}_2 = -\frac{b}{m}x_2 - \frac{k}{m}x_1 + \frac{1}{m}u(t)$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$

MIMO

$m s^2 x(s) + b s x(s) + k x(s) = u(s)$

$\Rightarrow \frac{x(s)}{u(s)} = \frac{1}{m s^2 + b s + k}$

SISO