Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 3
Lecture No 16
Alpha-derivatives

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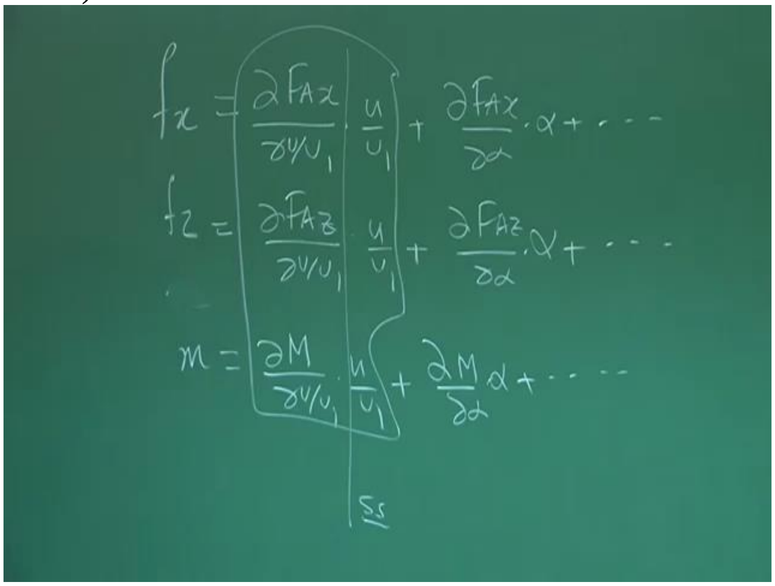

Good morning friends. If you recall, we were trying to find out the expression for partial derivatives which were required to model the perturbed aerodynamic forces and moments and if you recall, this was expanded as FAX by DU by U1 + DFAX + D alpha into alpha. Like this. Similarly, this was DFAZ by DU by U1 into U by U1 + DFAZ by D alpha into alpha and then Q terms, alpha dot terms.

And similarly for pitching moment, we expanded it like this, DM by DU by U1 into U by U1 + DM by D alpha into alpha + it comes with Q, alpha dot and delta E. We have so far completed these U derivatives. The expressions for U derivatives are known and we realise that if I know the aerodynamic characteristics, then I will be able to find out the values of these derivatives DFAX by DU by DU1, DFAZ by DU1, DM by DU by DU1 and we also realize these have to be evaluated that steady-state.

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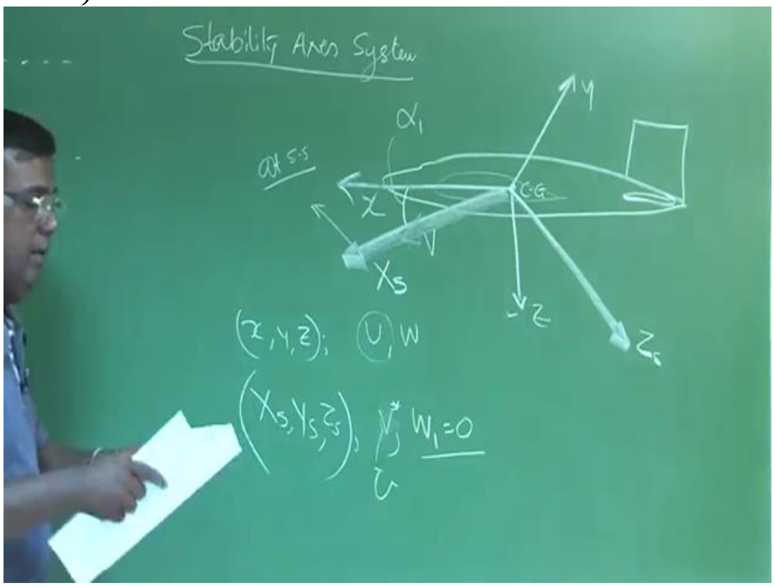

And also, very importantly, we should not forget, we are using stability axis system because the perturbed equations of motion were simplified using stability axis system. So it is important, we again try to understand what is this stability axis system? Let us say this is the airplane and this is body axis, X, Y, and Z. This is the centre of gravity and this is the wing. This X, Y, Z is the body fixed axis system.

The alignment of X is along the fuse Lodge reference line or some symmetric line. However now once we talk about stability axis system, we have to understand one thing very clearly. What is that? Let us understand. This is the airplane. Let us say at steady-state, let us say this airplane is flying at a speed, V. It is a relative air speed. Airplane is going like this with angle of attack and the direction of V, relative airspeed, we have shown it like this.

This is the condition at steady-state. This is important. This is at steady-state. We define stability axis system in steady-state only. That is, what do we do? We said, now my X orientation will not be along any symmetric line, not my fuse Lodge reference line or a line where the product of inertia may vanish. But what we are saying? That X is now aligned with the direction of velocity in the vertical plane.

And this is XS. And this gentleman then becomes ZS. If we now try to see what exactly it means? It means this angle, earlier this was alpha, angle of attack at steady-state. That was velocity vector, angle between velocity vector and the X axis. That is how we define angle of attack. And when I am saying that, we are also assuming that X is along the chord line of the wing.

However, when I am defining XS like this, then I know that there is no angle between XS and V. That is very important. What is the implication of that? You see, if I am using X, Y, Z axis system, then there will be U, and there will be W. That is, the component of V along X direction is U, component of V along Z direction is W. They are nonzero if there is an alpha1. Right?

I am restricting my discussion to motion in vertical plane. However once I am taking XS, YS and ZS axis system which is stability access system, now you know that U, that is the velocity along X direction is U or the total velocity, V in the vertical plane. And there cannot be any component of this total velocity along a direction perpendicular to X axis.

So along ZS, there will not be any components. So we say, W1 equal to 0. And this viz a viz total which is nothing but U along the stability axis system. Correct? So the advantage we get is that W1 is 0 when I am using XS, YS, ZS. So what is common important thing is that I am defining stability axis system at steady-state. That is I find what is the angle at steady-state and I align this X axis along the velocity vector so that there is no component of total velocity along ZS axis.

And simply, if I am flying like this, and this was my X axis and this was the velocity vector, so I am just aligning this X axis along the velocity vector. So that is why, W1 becomes 0. And this orientation is done only referred to condition at steady-state. That is very important. Now we are talking about perturbations.

So what do we do? We will give some perturbation and see how this diagram is to be interpreted.

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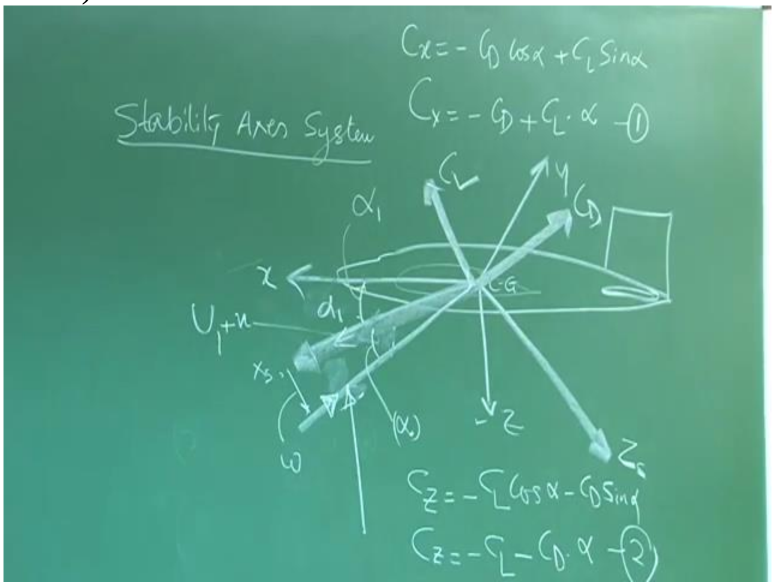

So we have no objection that this is alpha 1, this angle and we have, there is some disturbance that has happened. Because of that, total velocity has changed from the steady-state velocity. Let the component here is, earlier it was U1 at steady-state and + U is resolved along XS axis. This is a very small disturbance which is perturbed alpha. Okay? Clear? What is the message?

This was my total velocity, U because there were no object components. Now because of perturbation, there is a perturbed angle of attack has come and this direction is the total velocity after perturbation. So this is perturbed angle of attack and this small U is the perturbed speed along XS direction. So it becomes, U1 + U.

And the lift for CL and CD will be along and perpendicular to the current direction of V, which is after the perturbation. So this has to be understood and of course, along this direction, this will be small W which is the component along ZS axis. Please understand, this W is a perturbed quantity. At steady-state, there was no W component along ZS.

But because of perturbation, the velocity vector has changed. So there will be a W along ZS direction which is a perturbed quantity. Once I do that, I can always use this advantage. I can write, CX is equal to - CD Cos alpha + CL sin alpha. And we know that we are talking about small perturbation. Because this alpha is perturbed angle of attack, so for small angle I can write, CX is equal to - CD + CL into alpha.

Remember, this is small perturbation. Similarly, I can write CZ is equal to - CL Cos alpha - CD sin alpha. You can refer this diagram. And for a small angle, I can write this as - CL - CD alpha. So there are 2 equations which are very handy. We will be using this. So I thought, before we use these equations, we again reiterate what is our understanding about stability axis and what do we mean by perturbation about steady-state. Right? This I presume is clear with you.

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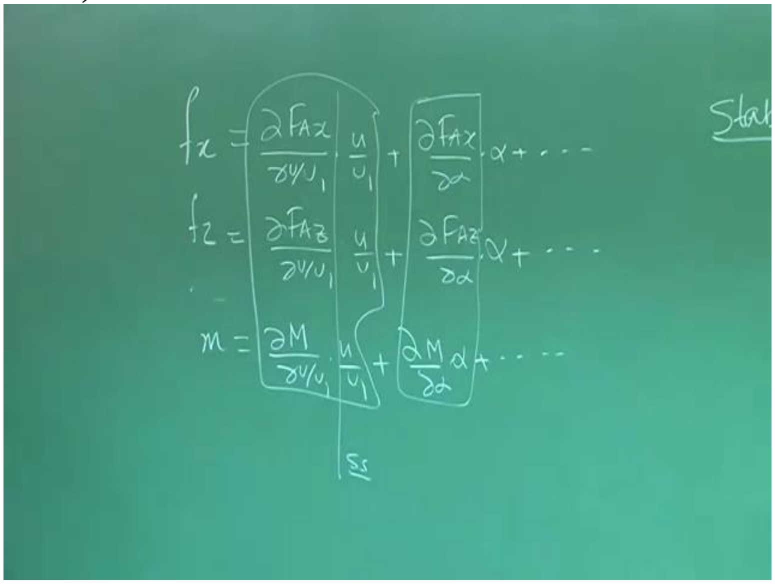

Now after these U derivatives are over, we will now focus on the alpha derivative. What we are going to do? We want to derive expression for alpha derivative.

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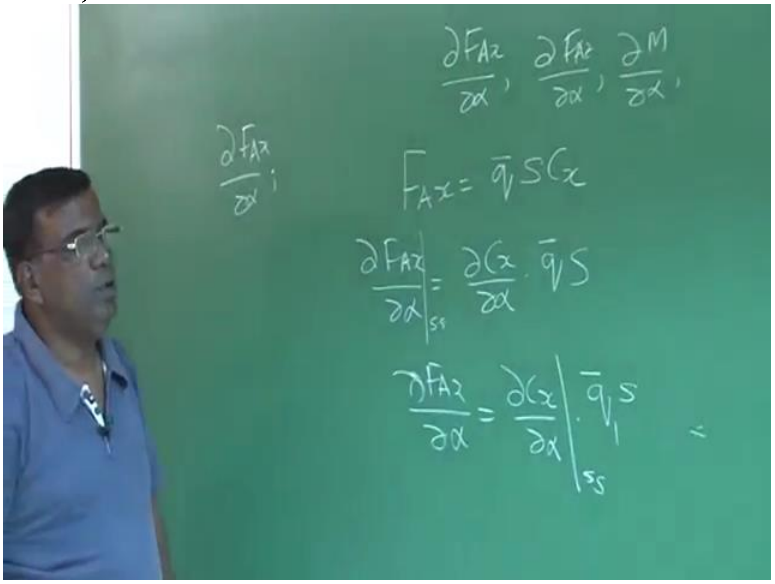

Let me start with DFAX by D alpha. Although we will be doing all the things, let me write that also and DM by D alpha. Primarily, these 3 partial derivatives will be derived in terms of aerodynamic coefficients. So let me start with DFAX by D alpha. So let us take DFAX by D alpha and the procedure is similar. What do we write?

FAX is Q bar SCX. So DFAX by D alpha will be equal to DCX by D alpha into Q bar S. But we know that we have to derive this expression, DFAX by D alpha at steady-state. So at steady-state, DFAX by D alpha will be DCX by D alpha to be evaluated at steady-state and for Q bar, we write Q1 S. What is this? Initially, our Q bar is what?

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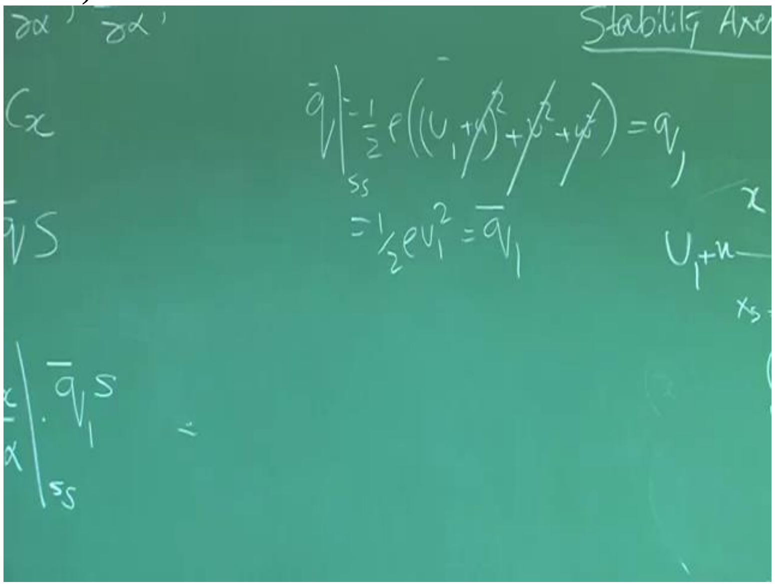

Q bar was half Rho U1 + U whole Square + V Square + W Square, all the perturbed quantities are included here. But when I write Q1, I am evaluating this at steady-state which becomes Q1 and steady-state, this is 0, this is 0, this is 0. So it is only half Rho U1 square and which is Q1 bar. 1 is for steady-state condition. So we have written this. As simple as that.

So we need to find out what is DCX by D alpha? What we will do? We will use this expression. What do we know?

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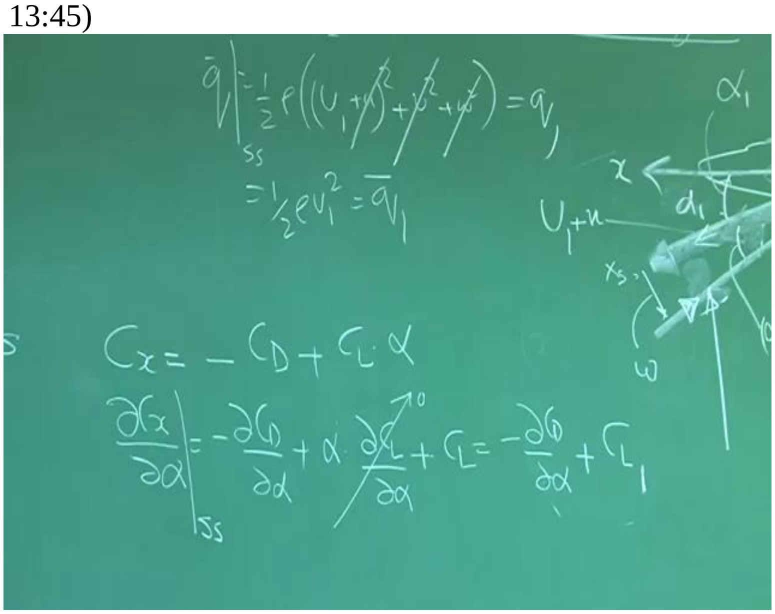

CX is equal to - CD + CL into alpha. So DCX by D alpha will be - DCD by D alpha into alpha into DCL by the alpha + CL. This is clear? Now tell me, what do we want to find out? We want to find out DCX by D alpha at steady-state. So how this right-hand side will get modified? Before doing that, let us ask ourselves, what is this alpha? Alpha is the perturbed angle of attack.

So when I am evaluating at steady-state, perturbed angle of attack is 0. So this gentleman goes off. So I get, at steady-state, this is - DCD by D alpha + CL but CL what? CL at steady-state. So CL1. So what do we get? The expression of DCX by D alpha as - DCD by D alpha + CL1.

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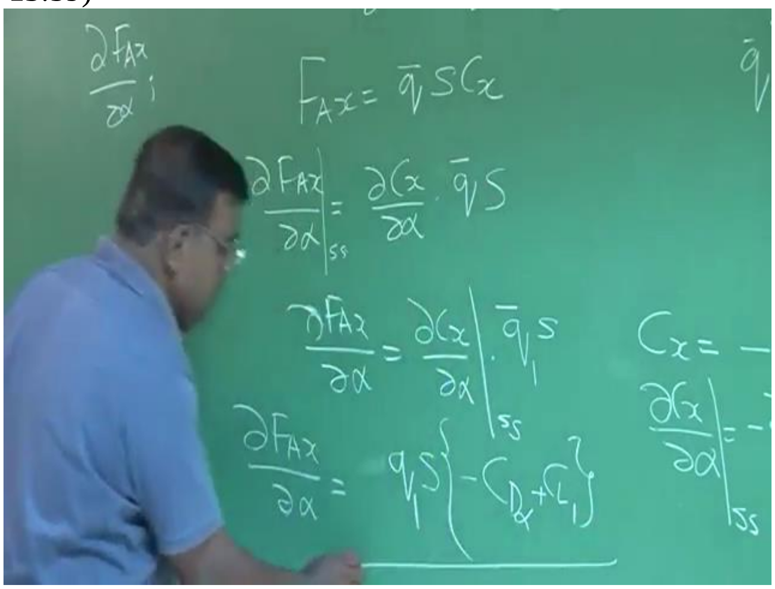

So DFAX by D alpha becomes - Q1S. I take - here. So Q1 S. So it is - CD alpha + CL1. This is the expression.

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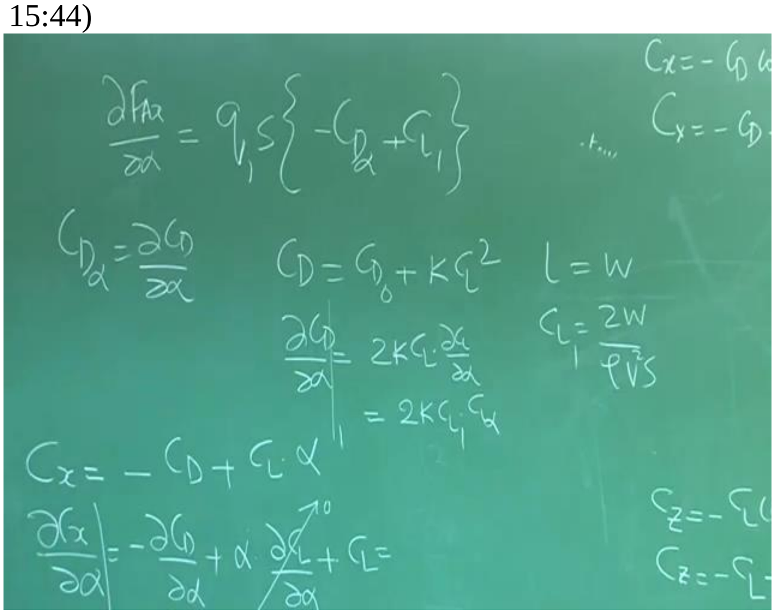

Now we see, DFAX by D alpha is Q1 is - CD alpha + CL1. So try to understand what is the meaning of CD alpha? What is CD alpha? It is DCD by D alpha. What is the meaning of that? How do I find that? You recall, drag puller is given by CD equal to CD not + K CL square. So I can easily find out DCD by D alpha as CD not if I assume constant with the angle of attack for small angles, this will be 2K CL into DCL by D alpha. So this will be 2KCL into CL alpha.

So you could see easily that I can find out the value of DCD by D alpha if I know what is the aspect ratio of the wing because K is nothing but 1 by Pi aspect ratio E, if I know what is C, this gentleman is cruising. Because these are evaluated at steady-state, CL1 will be given by CL equal to what? Lift equal to weight. So CL equal to 2W by Rho V Square S. So CL1 we can easily find out from this expression corresponding to the steady-state, corresponding to the cruise which is our steady-state condition. So simple to understand this. So once I now DFAX by D alpha, will not try to find out DFAZ by D alpha.

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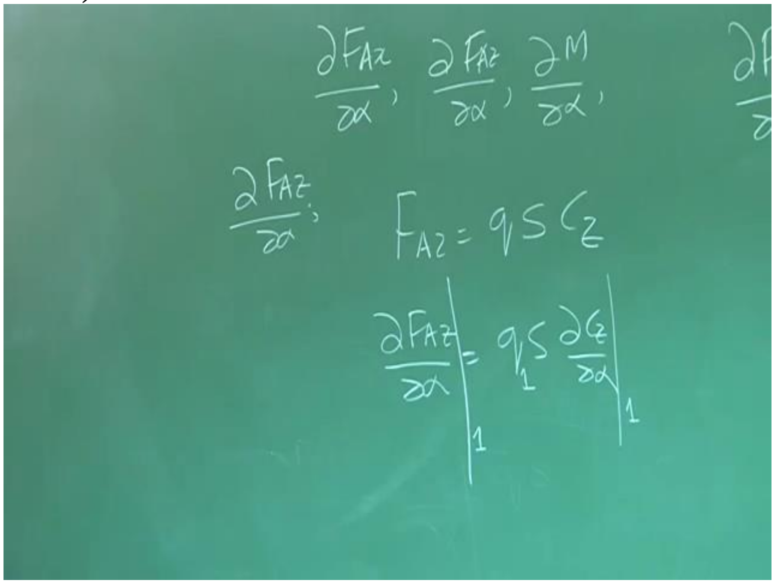

So let us find out DFAZ by D alpha. Again mechanically I can follow. FAZ equal to QSCZ. So DFAZ by D alpha will be Q1S DCZ by D alpha. But we have to evaluate this at steady-state. So Q will become Q1 and this has to be evaluated at steady-state and for steady-state, I am using the notation 1. A similar thing we did for DFAX by D alpha also. Nothing new. There, I wanted to have an expression of DCX by D alpha. Now here, DCZ by D alpha. So again we go here.

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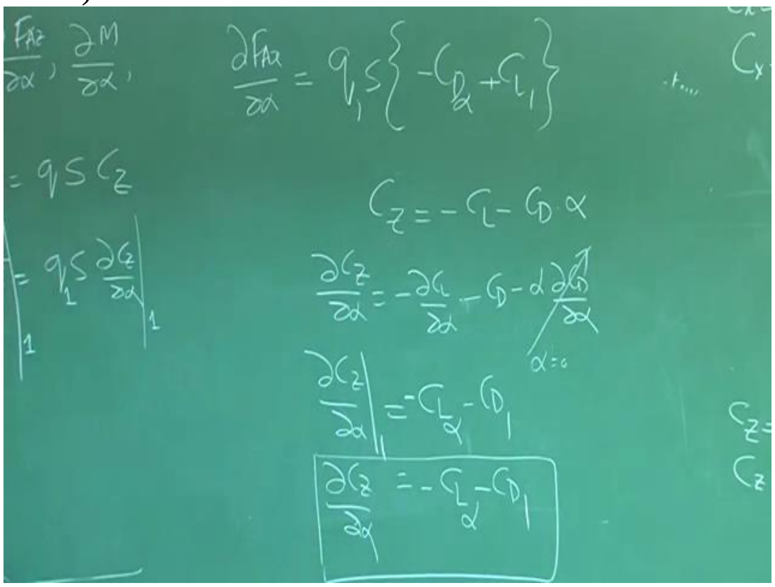

What is CZ? CZ is nothing but - CL - CD alpha for small angle approximation. So you can find out DCZ by D alpha equal to - DCL by D alpha - CD - Alpha into DCD by D alpha. Simple derivation. I am just taking a derivative on both sides with respect to alpha. So DCZ by D alpha at steady-state, this becomes CL alpha, - sign at steady-state - CD1 and this, because at steady-state, alpha equal to 0, so this goes. So this is - CL alpha - CD 1. This is DCZ by D alpha. I can remove this alpha 1. CL alpha is CL alpha. Lift of slope of the whole airplane.

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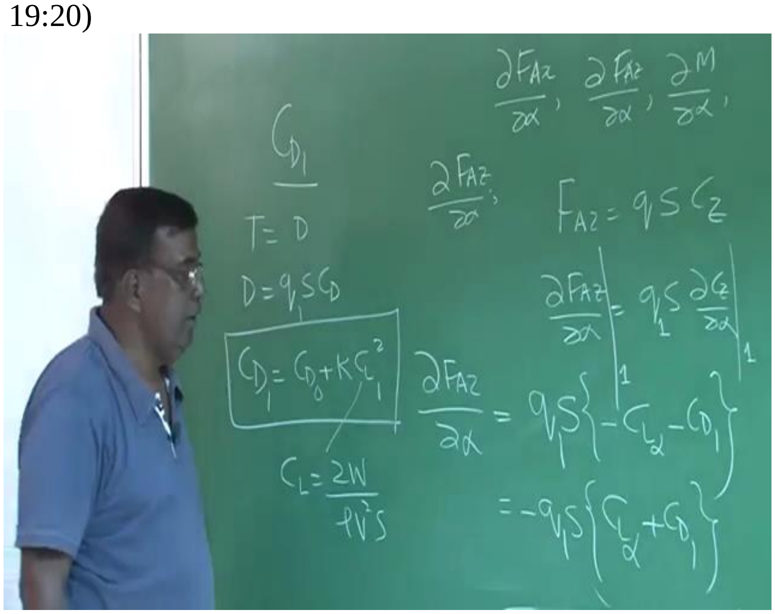

So once I know this, then DFAZ by D alpha will look like this, Q1S into - CL alpha - CD1 and in textbook you might have seen, - is taken outside and this is written as CL alpha + CD 1. Now the question comes, what is CD1? CD1 means drag coefficient at steady-state and for our case, steady-state is the cruise.

And I know that thrust equal to drag and you know, drag and can be written as Q1 SCD and CD 1 will be CD not + K CL1 square. So I can easily find out CD 1 if I know drag puller of the airplane and if I know at what CL the airplane is flying which I always know from the fact that lift equal to weight gives me CL equal to 2W by Rho V Square S. So simple. So you can find out DFAZ by D alpha.

What do you need to know? What is the lift slope of the air plane? What is the CD at steady-state? That is what is the drag puller and I know that from drag puller, what is the dynamic pressure at steady-state I am flying, the altitude and the speed and area of the wing. As simple as that. So this also we know how to do. Now the last part we will do DM by D alpha. Now we will be trying to find out derivative DM by D alpha which is very very straightforward.

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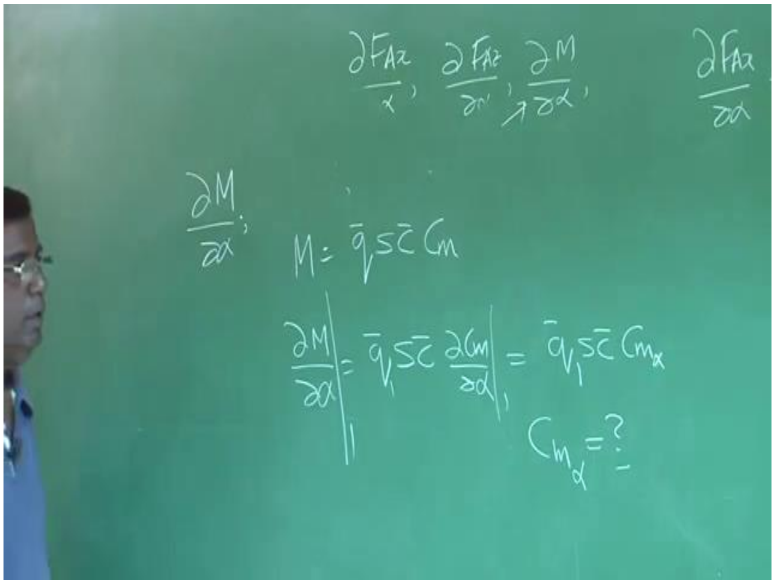

Even now, M I can write as Q bar SC bar CM. So DM by D alpha which is partial derivative will give me DCM by D alpha and when I evaluate this at steady-state, this has to be evaluated at steady-state, this becomes 1. So this will become Q1 bar SC bar CM alpha. And what is CM alpha of the air plane? What is CM alpha? What rings to your mind when you see CM alpha?

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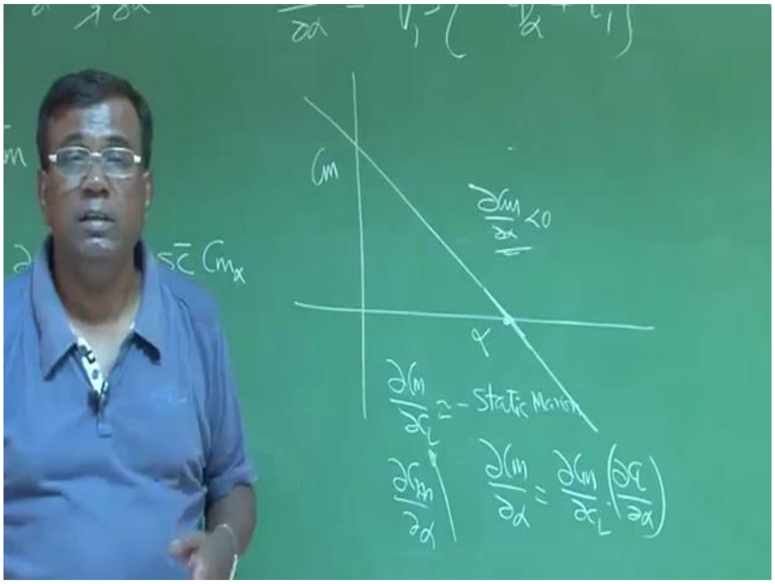

Because of static stability, when you plot it, CM vs alpha, we said there is a requirement of these airplane to be statically stable about the trim. Then the slope should be negative. So DCM by D alpha should be less than 0. So this is the condition for static stability. So typically, these values will be negative. And you know that also, you cannot go on making this more and more negative because more stable means difficult to control.

So there are typical values, typical range of values that the designer will select for an airplane to have adequate, comfortable static stability which are known. Absolutely from the configuration you can find it out. So again, I know how to calculate DM by D alpha if I know dynamic pressure, area, (())(21:56) and the derivative CM alpha which is also linked to static margin. Remember that.

Roughly, we remember, DCM by DCL is approximately equal to - static margin. So you can easily find out DCM by D alpha using this because DCM by D alpha I can write this as DCM by DCL into DCL by D alpha which is nothing but CL alpha. And DCM by DCL is nothing but static margin. 10% static margin, 5% static margin, 15% static margin. So all these things are known to a designer before he starts the design and it is a typical number.

So we are comfortable in finding expression for alpha derivative. In the next class, we will go for alpha dot, Q and Delta derivatives. Okay, thank you very much.