Aircraft Stability and Control

Prof. A. K. Ghosh
Deptartment of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture-21
Directional Stability

Today, I will be discussing about directional stability, directional control and then later on lateral stability and lateral control. You know by now longitudinal stability and what was the understanding that if this airplane is flying this is the tail plane and if there is any angle of attack disturbance coming like this positive angle then it will generate a lift and this lift will give moment about center of gravity nose down, so you say for a positive Alpha the tail is producing the nose down moment which is negative.

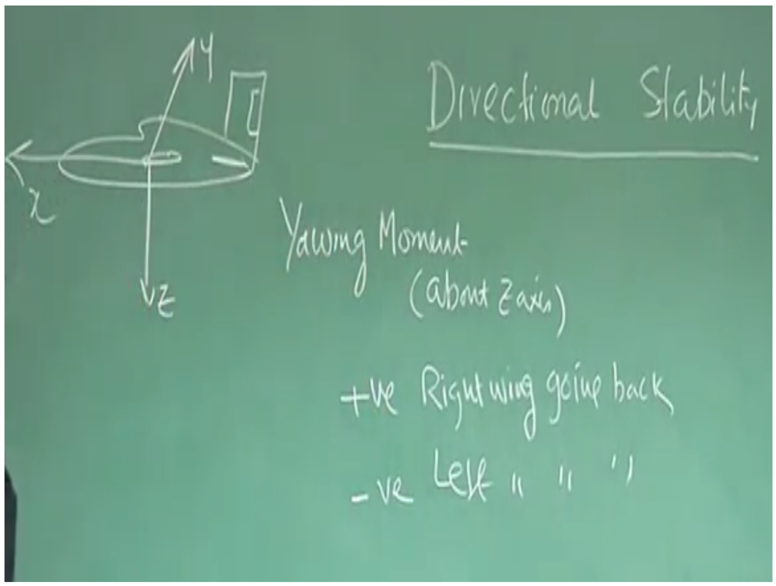
And you say CM Alpha is negative so the contribution of the tail is towards the making the aircraft statically stable in a longitudinal plane right. Today we will be talking about the directional stability and what is the directional motion if this is the airplane, if this is the airplane let say and this is the X axis, this is the Y axis and this is Z axis then directional motion is about Z axis like this and we will see that this horizontal stabilizer was helping for longitudinal stability.

Now this vertical stabilizer this whole vertical stabilizer will help to give directional stability and as in longitudinal case elevator was giving longitudinal control that is why when I was pulling the elevator up the airplane also will pitch up and when I was putting elevator down the aircraft also will pitch down. So we are discussing this in terms of longitudinal control and this is the elevator control similarly for directional case you could easily see if we deflect the rudder this way.

And airplane is moving in this direction so it will generate a force in this direction and that force will give a yawing moment taking the nose towards left and if I do it like this then the force will be towards me and this will give a yawing moment, which will take the right wing back right. As per as the convention is concerned will assume that which everybody as following in this convention that.

When I am flying like this rudder towards our left is a positive rudder deflection and rather towards right is it negative deflection so keep this back of your mind now we will go to the class room and we will discuss in detail. We have just now seen vertical tail or equal vertical stabilizer and also a rudder and the purpose of today's discussion will be on directional stability.

(Refer Slide Time: 03:00)



If I draw the diagram for axis system this is X, this is Y and this is Z and we know that when we were discussing the longitudinal stability, longitudinal static stability. We are talking about motion about Y axis that is pitch up pitch down but now once you talking about direction stability we will be talking about motion about Z axis. And the convention is if this, this are the two wings if right wing going back the yawing moment is positive that is if right wing going back that convention wise that will be yawing moment positive and left wing going back is yawing moment negative what is the yawing moment.

Yawing moment is the moment about Z axis right. So as per as yawing moment which is about Z axis and convention by positive if right wing going back while flying going back right. And this is negative when you see right wing going back yawing moment is positive and when left wing going back this left wing going back then yawing moment is negative right. Who causes this yawing moment you can understand that if some motion as to be there about Z axis then the force as to come like this right so for example if I take this case.

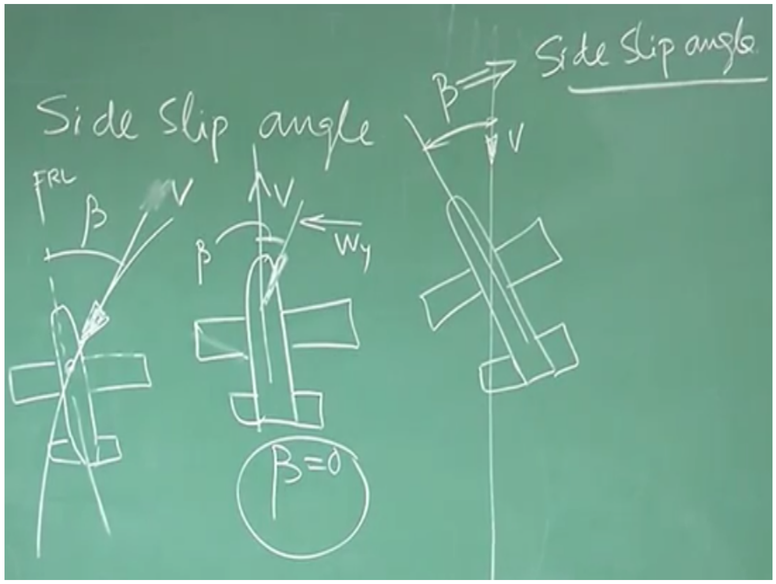
If I see vertical tail or vertical stabilizer and this is the airplane suppose like this Z is here so if it as to turn like this that means some force has to act here right. So vertical stabilizer will play that role will generates yawing moment which will try to help in giving static stability in the directional case but always remember one thing this directional case that is motion about Z axis and the motion about X axis they become coupled know how it happens you see you could see.

When I am yawing we are discussing about directional motion and try to see the directional motion and lateral motion gets coupled right. Think these are two wings if I am giving a positive yawing moment that is right wing going back see what is happening at the wing this wing is seen lesser velocity because I am moving forward but the wing is going back so relative air speed is less whereas relative air in the left view on the left wing is more.

So lift air will be more so naturally lift here is more compare to this, this will also bang that means what is happening as I am yawing I automatically get banged so I can say the directional motion and the lateral motion they get coupled but do we will now studying directional stability separately then lateral stability separately then we will discuss about their coupling etcetera okay.

This thing will not happen for longitudinal case for a small rate or small change in angle of attack it will not influence anything about other directional or lateral case at small angle of attack okay. Let us come back to directional stability what does it mean?

(Refer Slide Time: 06:46)



We have to define something called side slip angle for example suppose this is the path and the aircraft this is the velocity direction and this is the fuselage reference line. So now the relative here speed I can write it as if it is coming in this direction right. When I say relative line means it is relative to aircraft right. So now what is happening actually the airplane is moving like this okay not like this. For normal case if I am flying a machine I am going like this, but here actually if you see instead of this I am actually going like this.

So this angle Beta is called side slip angle Is it clear? I repeat again if this is the center line and if I am flying like this a relative air is coming like this. This is the axis, this angle is Beta which is called side slip angle and this will have an effect on the directional stability we will see how the airplane will gives a response to such Beta, such side slip angle to ensure that it is having some sort of an static stability in the directional case right.

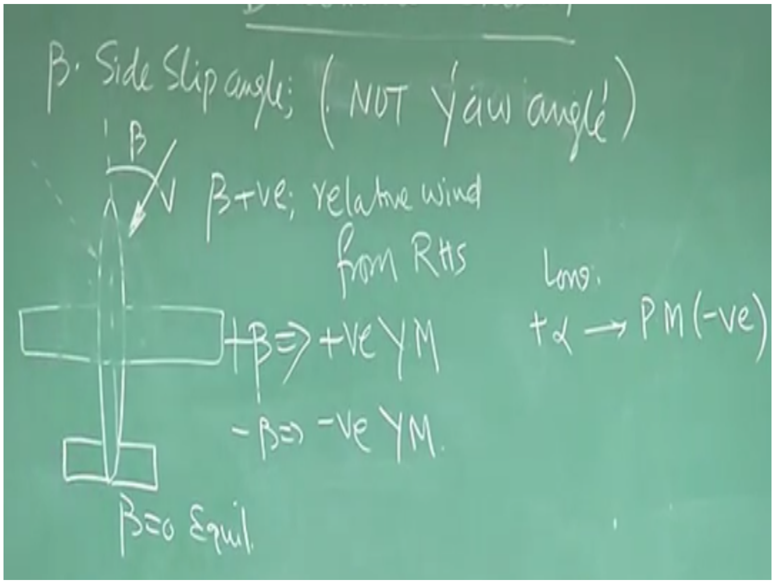
Now this Beta is let say I call side slip angle soon you will realize that how important is vertical tail to ensure that it does have static stability in directional case. So let us try to understand what is the meaning of static stability we have seen that static stability means if a is body distracts from its equilibrium state and if it as initial tendency to come back to the equilibrium then you say the body posses a static stability okay.

Now let say the body was flying like this Beta = 0 mostly the pilot will fly a machine at Beta = 0 so for all practical purpose Beta = 0 is the equilibrium state for an airplane there are some one or two occasion during especially while landing or when you try to reduce the speed increase the drag for some specific purpose that also will come to know the pilot may tend to fly at a particular yaw angle or particular side slip angle what accurately side slip angle it is not your yaw angle, yaw angle by convention is different than side slip angle which we will discuss as we go forward.

Let us talk only about Beta and that is side slip angle so if an pilot is flying at Beta = 0 and let say there are some wind coming from right hand side so this will make the relative air speed this way and so this will introduce a Beta but what is the equilibrium. Equilibrium is Beta = 0 so if the airplane as initial tendency to come back to Beta = 0.

Then what the airplane should do, airplane should initially generate a yawing moment like this, right wing going back then only Beta will become 0 am I correct is it clear? So, it should for positive Beta you should generate a positive yawing moment. Why positive yawing moment? Because we know that as per the convention right wing going back is positive right. Let us understand this concept little more clearly before you go to next step.

(Refer Slide Time: 10:57)



We have just introduced something called Beta which is side slip angle we have introduced symbol Beta which is side slip angle and I have told you be careful this is not yaw angle. We will define yaw angle later at this point we only just stick through Beta side slip angle and what is the side slip angle this is the airplane.

The top V I am taking this is the central line and if the aircraft CG is moving in this direction so it is moving like this the airplane moving like this okay. This angle is Beta and Beta is positive convention wise when you are flying the machine and the relative wind is coming from the right end side okay. The relative wind from right hand side so what will be the Beta negative convention wise Beta negative will be other way it is like this that is Beta negative.

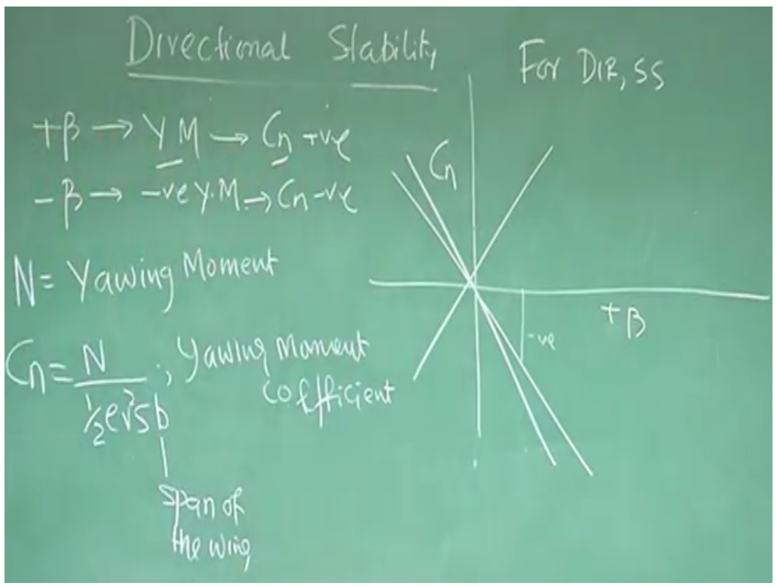
Now you know the most of the time Beta = 0 is the equilibrium so suppose I am going with Beta = 0 and suddenly there is a disturbance that has end with Beta so if it has initial tendency to make it to Beta = 0 we say statically stable that means it as to turn like this, turn like this means right wing going back that means it has to generate for a positive Beta it must generate positive yawing moment okay that is clear.

Similarly for negative Beta it should generate negative yawing moment right. Remember for longitudinal case it was for positive Alpha longitudinal case positive Alpha the pitching moment was negative to make it statically stable but for directional stability you are seeing for positive

Beta it should generate positive yawing moment right. So if I am going like this suddenly some wind disturbance has come which has given as Beta so I should able to time like this to make Beta = 0 and then only you will say this aircraft as directional stability.

And we will also discuss how many directional stability is required as we develop our lecture right? If this is clear so, now see how to design an airplane so that this phenomena can happen so.

(Refer Slide Time: 13:48)

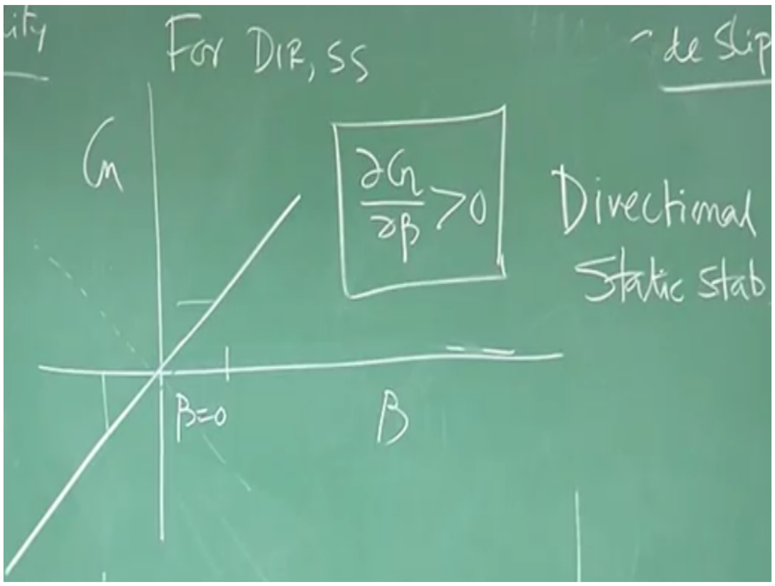


What do you have seen for static stability for positive Beta it should give positive yawing moment and for negative Beta it should give a negative yawing moment and generally by notation we use N for yawing moment okay. And similarly we will define CN yawing moment coefficient let N by half row V square S and B, see this is moment so this is the force and this is the length some for longitudinal case we are using mean aerodynamic chord as that length to be is for non-dimensional we are using C bar as a length to non-dimensionalized pitching moment.

But for non-dimensionalizing yawing moment or roll moment you will see we will be using B that is the span of the wing right okay. So this is CN which is also called yawing moment, yawing moment coefficient like for longitudinal we have pitching moment coefficient CN for directional case we have yawing moment coefficient CN.

And as we have understood for static stability for directional static stability, we have seen for positive Beta yawing moment should be positive so I can write CN should be positive so when here CN should be negative other anyway CN and yawing moment are related like this. So if I write CN here and Beta here and equilibrium is at Beta = 0 so if I draw it like this do you think this will have static stability check here for positive Beta yawing moment should be positive or CN is positive for positive Beta at is negative so this is wrong this is not correct.

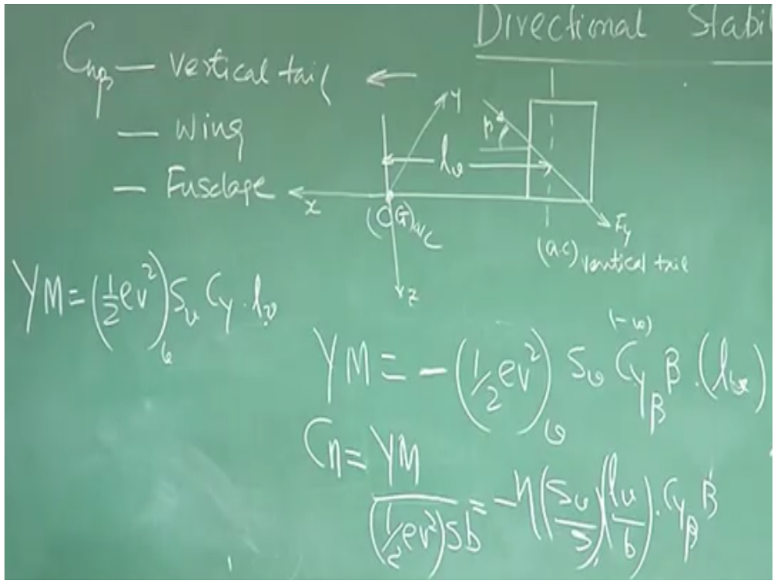
(Refer Slide Time: 16:03)



So what should be the correct thing if I draw this is as CN and this is as Beta then correct thing should be this equilibrium at Beta = 0. Now you will see for positive Beta it should give CN positive CN positive is coming for negative Beta CN negative is coming so what is the meaning of this, its meaning is DCN slope of CN versus Beta should be greater than 0 for directional static stability, clear. Please recall this sort of a slope negative was there for longitudinal case.

For which CM Alpha was supposed to be less than 0 but for directional case CN Beta has to be greater than 0 that keep back of your mind okay, So this is the condition for directional stability as per as static part is concerned. Now see who are going to contribute towards this.

(Refer Slide Time: 17:19)



As all the time we do CN Beta I can get from vertical tail primarily I can get from wing I can get from fuselage, let us see first the contribution due to vertical tail okay. I am again simplify diagram here center of gravity of the airplane and let say this is the vertical tail and this is the AC of the vertical tail and I let me need write this distance as LV that is distance between the AC of the vertical tail and CG of the airplane.

For longitudinal case if we recall we use some time called LT and what was the LT, LT was distance between AC of the horizontal tail and CG of the aircraft, but now I am using a define with LV for vertical tail because it is not necessary that horizontal tail AC and vertical tail AC will be a same point.

There are very configuration we have find horizontal tail like this and vertical tail is somewhere here. So they can be lead out differently so in general I am using a term a notation LV as length between CG of the aircraft and AC of the vertical tail right. And I also know as per the sign convention and axis convention this is the X, this will be the Y and this is the Z. This is the body fit axis what I want to see please come back here.

We want to check if there is a Beta disturbance given to this vertical tail whether it will generate a positive CN or not if it generates positive CN that means CN Beta because of the vertical tail is positive so I would say vertical tail is providing directional stability to the airplane right. So let us see let us give a positive Beta to the vertical tail and you know positive Beta means the wing if I am moving like this the relative wing should come from the right hand side.

So this is the positive Beta what will happen if there positive Beta let us also see. Let us see this is a vertical tail okay vertical tail I was moving like this now suddenly there is a crosswind coming like this because of the relative velocity, relative air velocity has become like this. So this is the positive Beta configuration, because relative towards my right. Now what this Beta will do? This air will give a force on the vertical tail in this direction ideally to perpendicular to the velocity vector but if this is small I can say it is something like this.

What the force will do? This force will CG somewhere here this force will give a moment which will take right wing back so indeed this will make yawing moment positive so vertical tail will give adequate directional stability. Let me repeat if there a positive Beta this will give force here and about CG it will give a yawing moment which will take right wing back so that CN is positive and if we design properly vertical tail we can get adequate directional stability because of vertical tail.

If that is the understanding let us formulate it okay. Let us see for a Beta what the force in the Y direction okay. We are assuming small Beta so force will be half row V square at the vertical tail dynamic pressure the vertical tail is vertical tail and CY because of vertical tail like we have CL and here we have CY I put V to ensure that it goes to your mind because of vertical tail. So you can see from CY is nothing but force.

FY divided by dynamic pressure into area reference area it will come if we are doing it for overall airplane but it for the tail so we are writing as V right. Now what happens this force will do what this force will give force in this direction I have just demonstrated this will give a yawing moment positive so yawing moment will be given this FY that is half row V square vertical tail is vertical tail CY into LV right. We have to ensure that in this the yawing moment whatever we are calculating it should have appropriate sign.

Now to ensure that sign we will have to do little more work because this sign convention etcetera we have to defined so we should be very very careful. So what is CY let us see if I want to ask what is CL we write CL Alpha into Alpha assuming CL0 is 0 so what is CY, CY I will write CY Beta into Beta correct okay. That is if I making more elaborative DCY by D Beta into Beta like this for DCL by D Alpha into Alpha for longitudinal case.

Now let us see what is the sign of DCY by D Beta that is important you could see that for a positive Beta this force CY will be in the opposite direction of Y if this is the Y the force in the opposite direction if I demonstrate here you see this is the X and this is the Y towards the you and this is Z positive Beta is like this. This will give force in this direction which is opposite in relation of Y so positive Beta will give you negative force.

As well as axis system is concerned because Y towards you and positive Beta the force in this direction so I have to take care of the sign here. So what I do I say I can understand from here the sign of CY Beta is less than 0 because for a positive Beta if FY is negative because why this is less than 0 we know for a positive Beta FY is negative and this negative and CY is FY by half row V square S and of course CY Beta means just take one derivative so naturally if this is from here if FY is negative then CY Beta automatically becomes negative correct.

Which physically also you understand I repeat again I will take this a register for you to get more clarified if this is the vertical tail and assume that this is the Y direction because this XY and Beta is coming like this positive that will give a force in this direction so it will give a negative in direction as far as force direction is concerned. So for the positive Beta the CY becomes negative I will say CY Beta is negative okay.

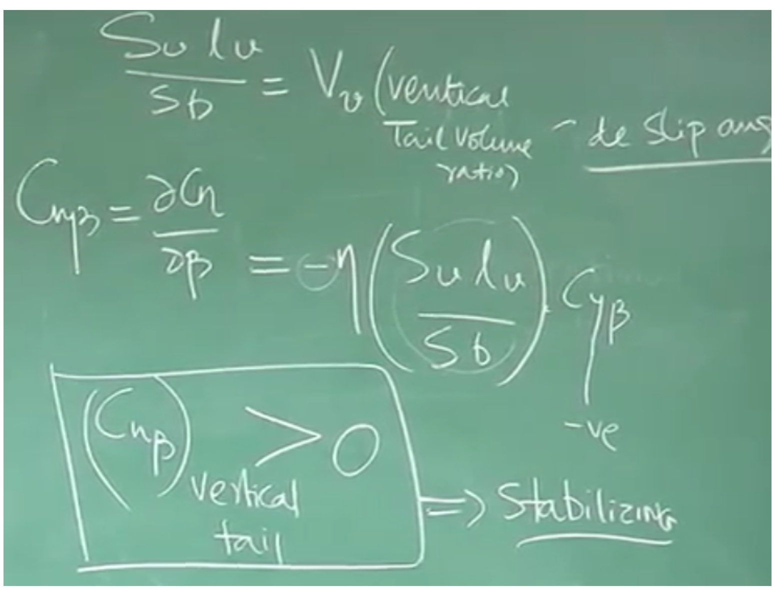
Please not get confused with CN Beta this is CY Beta. So now if I want to see this yawing moment here I further now go into little bit of detail to ensure the signs are consistent so then I write yawing moment is = half row V square dynamic pressure vertical tail S vertical tail and for CY I write CY Beta into Beta into LV which is the vertical tail arm. And if I write it like this since we know CY Beta is negative by definition now that we have defined so then this tells you that if Beta is positive the Yawing moment become negative but which is not correct.

For Beta positive although CY become negative but it is giving a positive yawing moment that is please understand again and again I am repeating this if this is the vertical tail Beta is positive CY is negative but this is give you a moment right wing going back yawing moment positive right? So to ensure that you are getting yawing moment positive you have to put here minus sign now see what happens if Beta positive.

CY Beta is negative and this minus sign is there this gives yawing moment positive so from here if I write CN, CN will be Yawing moment divided by half row V square S referencing V, V is a span so this will become you are not expert Neeta the ratio of dynamic pressure at the vertical tail and the half V square free stream then SV by S you will get then LV by B you will get then CY Beta into Beta of course you have to put a minus sign here right.

And then what is CN Beta? It is very clear so using this CN expression CN Beta is linear if we just divide by Beta here or differentiate.

(Refer Slide Time: 27:08)



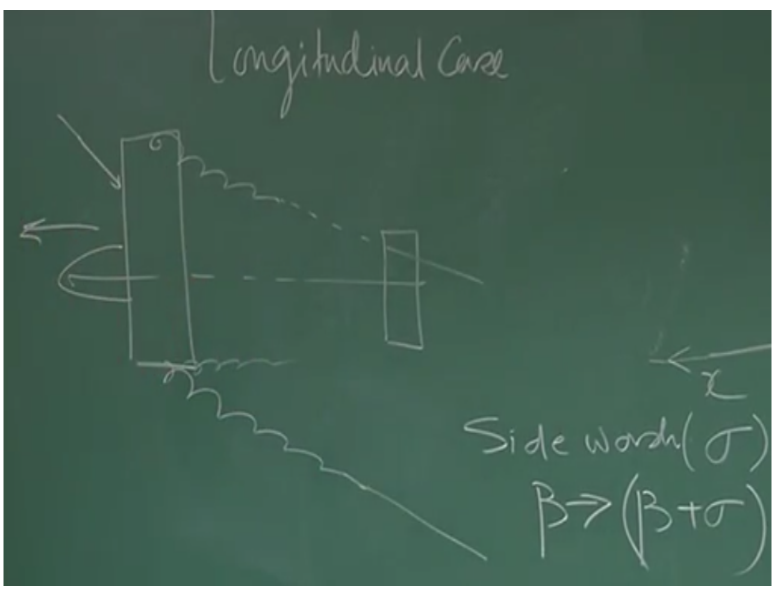
So CN Beta will be = DCN by D Beta which will be = - Neeta SV LV by SB into CY Beta am I correct. SV LV right so what is the sign of CN Beta? Here the CN Beta is because of what? It is because of vertical tail right.

So this is because of vertical tail or vertical stabilizer the sign will be what? Will be greater than 0 because I know this man is negative this is positive this is negative so this is positive so indeed vertical tail or vertical stabilizer they are stabilizing directionally correct and you could do now see what is this SV LV by SB let that help you in thinking something similar happen in longitudinal case that was ST LT by SC bar so it is that time that was called horizontal tail volume ratio and this is called VV that is vertical tail volume ratio.

You could see that this VV this whole expression is so important if I increase the value of VV I can increase the contribution of vertical tail towards directional static stability what is the meaning of increasing this expression or this value. Is that you have to either increase the vertical tail size or increase the length of its placement from the CG or doing both.

So this is another very important design parameter for designing an aircraft. So just now we finished the expression for CN Beta vertical tail and we see that it is having stabilizing effect on the airplane directional stability is it clear to everybody? Okay.

(Refer Slide Time: 29:30)

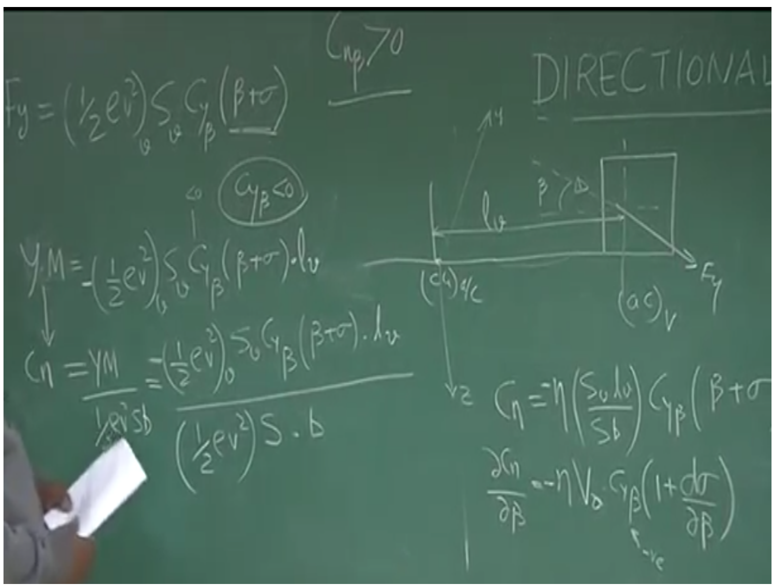


Another important thing for longitudinal case another important thing you must take a note for a longitudinal case if there is a wing here and here there is a tail horizontal tail because of downwash the angle was whatever angle it is being seen – Epsilon downwash angle right.

You know about this by now but for the airplane having a Beta that means a relative wind is coming like this okay that is why a Beta it is suppose this is a fuselage we're moving in this direction so that vertices will now be attained direction along the velocity vector relative velocity vector. So this will give some sort of a side wash it is analogous to downwash we call side wash and denoted by sigma and the angle of attack at the vertical tail which was Beta now it is changes to Beta + sigma okay.

What is sigma? Sigma is the side wash okay. There are empirical ways of finding sigma value okay. So now what is happening so what is the force?

(Refer Slide Time: 30:52)



So the FY will be = half row V square at the vertical tail S vertical tail into CY Beta into Beta + sigma right. Because of the side wash this angle C needs Beta + sigma so the force will be half of the V square SB CY Beta into Beta + sigma but note down we have agreed that for the positive. Beta the FY should be negative but I do not see any negative sign here, but you know that CY Beta is less than 0. So that is taken care from the value of CY Beta so if I put CY Beta values to - point 3 - point 6 the FY will be negative okay.

Now from here for your clarity yawing moment about CG will be half row V square vertical tail S vertical CY Beta into Beta + sigma so from yawing moment how do I come to CN? You know that I have to divide the yawing moment by half row V square SB.

So I divide it by CN is = yawing moment divided by half row V square SB it is expand so this becomes half row V square at vertical tail. This is the dynamic pressure at vertical tail into SV into CY Beta into Beta + sigma before I write the yawing moment expression do not know if there is mistake? Half row V square S CY Beta into Beta is the force but I am talking about the yawing moment, so moment would be what? This is the force here FY into LV this distance should come LV that so this term was wrong a little so we correct it we write it LV. LV is the vertical surface tail moment arm okay.

Now am I correct I will write LV and this is divided by half row V square free stream you know by now S and B is non-dimensional length has to come because its moment is the force into distance moment and now if I write the expression I get CN = Neeta SV LV by SB into CY Beta into Beta + sigma right. So, have taken SV LV is the SNB so by this is by this is Neeta so its SV LV is by SB this is here CY here Beta + sigma here what is our aim?

We want to find an expression for CN Beta because we want to ensure that CN Beta is meeting up the zero and we are talking about only vertical tail contribution. So what will be CN Beta? So DCN by D Beta will be Neeta let me write this CY Beta into 1 + D sigma by D Beta what is the sign of CN Beta?

CN Beta is positive, right. And now what is happening? If I go like this I know CY Beta sign is negative but this shows that I have done something wrong in the expression what is that wrong? Let us go back and see so, this is extremely important for keeping the sign into mind. Let's see here this Beta is giving FY so FY is here half row V square SV CY Beta + sigma we have seen for a positive Beta FY should be negative so and we know that CY Beta is negative so taken care FY is negative.

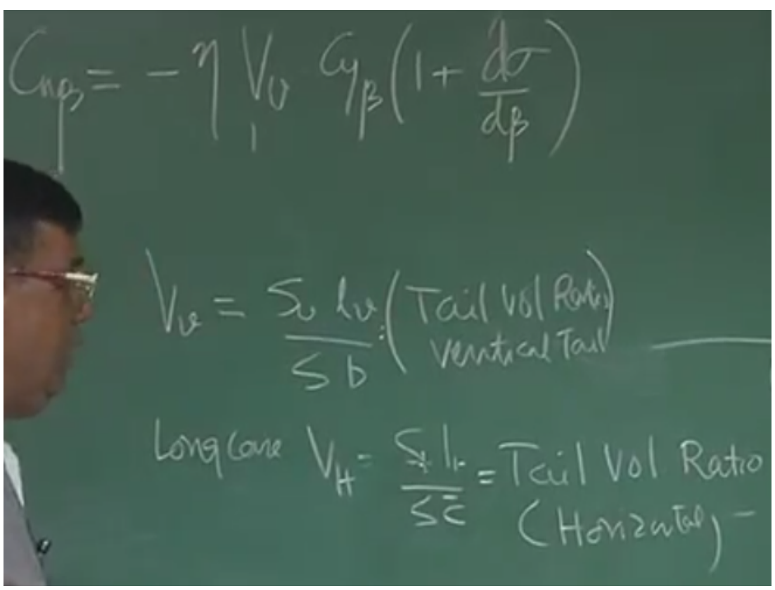
Now from here when I coming to moment please see for FY for whatever FY we are getting it should give me a positive yawing moment right. If you see this is the vertical tail and CG somewhere there if there is a Beta force is here that will give a moment so the right wing going back now I have to see whether I have taken that into care or not. If you see why M is the

Yawing moment and I know CY Beta is less than 0 so the whole expression is becoming negative which is not correct.

Because I know from here if CG is ahead of vertical tail which is true this Beta will give me a moment yawing moment where right wing going back so I have to take care of that so I have to artificially put a - sign here please note down this is extremely important right. So, that - sign will also continue here and this - sign is also continue here and - sign will continue here is it clear to you or not? Please let's go again we know that for a positive Beta this FY will generate a positive yawing moment.

And when I writing yawing moment expression like this without - sign you know CY Beta is already negative we have seen that without - sign putting here this will give me a yawing moment in negative which is not correct for a positive Beta I should get positive yawing moment that is right wing going back. So I have to put a - sign here and that - sign should carry and finally I get an expression DCN by D Beta at this and let us have closer look on this expression what expression you have got?

(Refer Slide Time: 37:01)



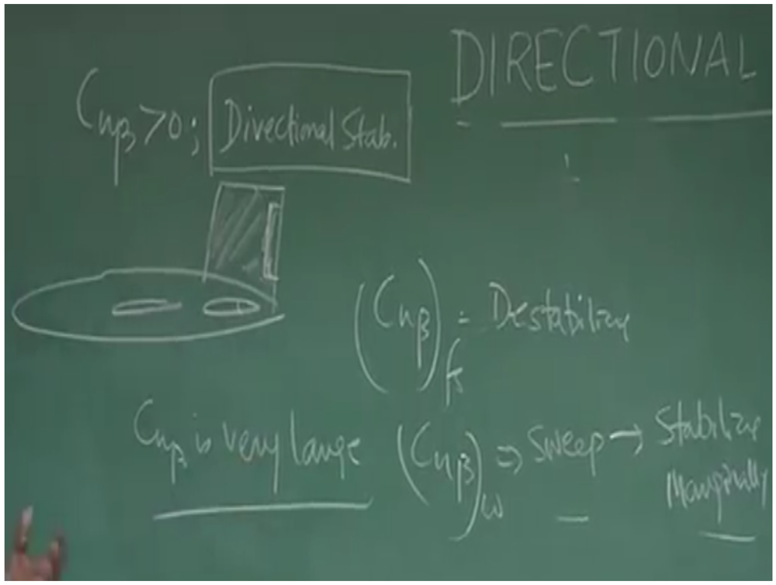
Is CN Beta is - Neeta V V into CY Beta into 1 + D sigma by D Beta okay what is VV? What is this? Could you recall similarity of this expression with longitudinal case? This is S vertical tail L vertical tail by S wing and B span. This is if I try to find a comfortable similarity for longitudinal case we had VH which was ST LT by SC bar and this used to refer to as horizontal tail volume ratio right. Horizontal so what is this then?

This is also instead of ST horizontal tail this is S of vertical tail instead of tail moment of the horizontal tail it is vertical tail and S is the wing area and B is for convention everybody follow we have taken span is instead of C bar. So, this becomes what this is again tail volume ratio that for vertical tail or vertical tail very clear no doubt about it? Now what was the role of VH? VH was we know that this was primarily giving the longitudinal stability to the horizontal tail if tail volume ratio is higher than we will have more and more static stability.

Similarly here you see we wanted CN Beta to be greater than 0 and we have seen that although - sign is here as per our convention we know that CY Beta is negative so this DCN by D Beta is actually greater than zero. So, for the positive Beta it is CN Beta is positive that means it is this vertical tail is giving directional stability okay. And who decides that? Not only CY Beta it also decided for a designer it is the vertical tail volume ratio so you can appropriately select vertical tail area or vertical tail moment arm.

I can put the vertical tail forward and aft or I can in a combination I can increase VV and as I increase VV that means it will add towards more and more directional stability okay.

(Refer Slide Time: 39:39)



So, let us go deep into this CN Beta is greater than 0 and shows directional stability we are talking about static stability we are talking about static stability. Now what is the issue we see next question comes to our mind how much directional stability we want okay, we know that if we make it very high in terms of static stability then if you want to change from an equilibrium to another equilibrium there is a problem there is a lot of effort is required.

We have clearly seen in the longitudinal case but see here what happens if CN Beta is very very large if CN Beta is very large so far let's say not we have only computed CN Beta because of vertical tail and this because we know that primary contributor to directional stability is vertical tail you will soon see that there will be contribution from wing and fuselage mostly fuselage will be destabilizing CN Beta you will see for fuselage will mostly destabilizing we will do that CN Beta from wing you will see if there is a sweep it is stabilizing marginally.

We will see these things we just to for continuity keep this back on mind if CN Beta is neutralized what will happen? If CN Beta is very large physically you see if I am flying a machine and CM Beta is very large is what the moment there is a Beta disturbance that means if there is a crosswind coming it will have an initial tendency to nullify these crosswind effect okay.

So, that means if CN Beta is very very large the airplane will become highly sensitive to cross wind if wind is coming like this it will turn like this if it is coming from right turn like this so the airplane will become highly sensitive to crosswind that is not a good design right. So there will be a debate on how much CN Beta or how much directional stability I need to have that will be very important question before we try to understand.

What is the directional control how a rudder will control we saw this (()) (refer time: 42:17) the directional stability of the airplane that is if CN Beta is very, very large then you have to put lot of effort on rudder to control this airplane at a particular direction okay or particular equilibrium right. Thank you very much.