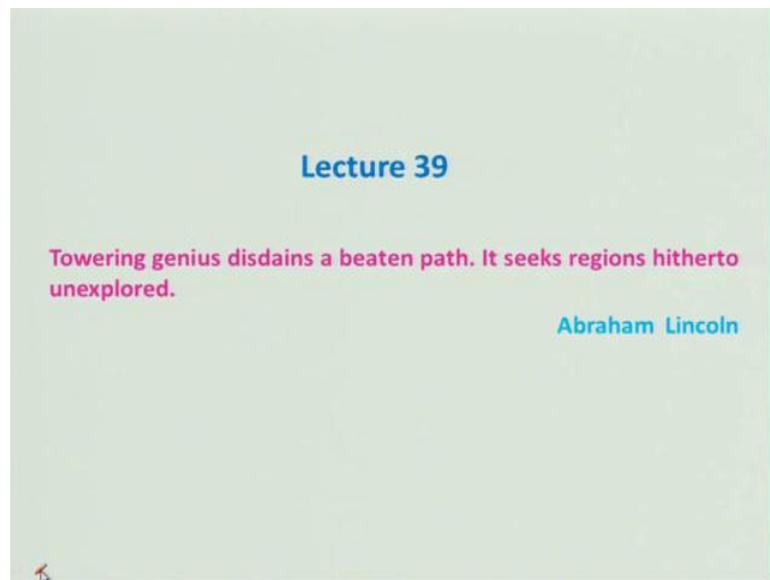


Fundamentals of Aerospace Propulsion
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Lecture - 39

Let us start this lecture thirty nine with thought process from Abraham Lincoln, the first president of America which says that towering genius disdains a beaten path, it seeks regions hitherto unexplored.

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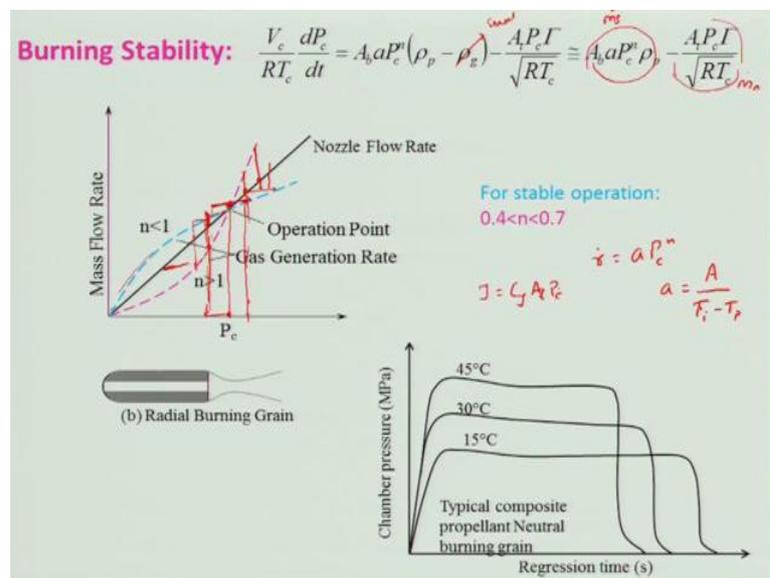
Of course, in our life we take easier path, unexplored path is difficult, but however that will give the meaning to the live and as usual let us recall what we learnt in the last lecture. In the beginning, we started with the various kinds of propellant right in the solid propellant and the liquid propellant; you know we have divided into various categories. Particularly, in liquid propellant I discuss in last lecture mono propellant bi propellant of course, it can be divided into hyperbolic non hyperbolic propellants.

Then, we moved into the solid propellant engines and then various components we looked at and what are the processes is involved during the combustion, which is quite complex in nature. It involves all kinds of heat transfer reaction with solid phase reaction and also the gas is phase reaction, when I talked about solid phase when a basically condense phase reaction not only the combustion, but also the fireflies is gasification. All those things takes place, this is quite complex in nature and then we moved into like

about how this pressure really vary with respect to you know surface area. Then, burning rate we derive some expression for the chamber pressure and the steady state and unsteady state as well.

So, and then from that steady state expression we learn that how we can design the various grain size for obtaining a particular kind of thrust law. For example, you know kind of thrust law neutral unprogressive kind of things and also even dual nature's kind of things. We have looked and what will do, we will carry out the analysis and look at some other respects which are quite important that is the burning stability.

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This burning stability if look at for that we need to look at expression what we had derived earlier that if look at this is we see divide by $R T_c$, $d P_c$ by $d t$. This basically changing chamber pressure with respect to time, which is nothing but the amount of the propellant being burnt. You know that is which will let it to the burning surface a v by chamber pressure P_c and ρ_p . The density of the propellant keep in mind that here have neglected this term ρ_g is almost 0 or very small you know very small as compare to ρ_p . So, we have neglected the amount of mass which is passing through the nozzle, so if we look at this is basically the mass passing through the nozzle this is mass being generated.

So, whenever there is imbalance between these two mass flow rates, then you will get a accumulation of the mass and resulting in increasing pressure and vice versa. If there is a decrease you know that also pressure will decrease because mass would not be accumulated, it will be depleted. Therefore, now what will be looking at we will be the looking at taking these mass flow rate being generated which is the function of chamber pressure right and the mass flow rate passing through the nozzle which is also a function of chamber pressure. So, if I look at that you know mass flow rate versus the chamber pressure if you look at it goes linearly right the mass flow rate with the nozzle, this is the flow rate.

It goes linearly, you know the pressure is 0 that will be 0 if pressure is and like higher then it will be go because it is linearly you know dependent on the chamber pressure, but if and this mass flow rate of course, it will be depend on an a b. For the time being, we can consider that surface area is remaining constant is not changing with respect to the time. So, if we assume this a b constant, then it is a function of P_c , P_c means chamber pressure and density of the propellant for the same propellant remains same. We do not really change, we are talking about appraisal thing.

So, then if a n is greater than one that is the index combustion index what we call if it is greater than 1, what will happen it will be like this it will goes on increasing. Now, whenever it will be if we look at this is the point right whenever it is cross each other that is a point where it is having a stable pressure. That means it would not change because both are matching that means these term will be 0, dP_c by dt will be 0 because mass flow rate of you know mass flow rate passing through the nozzle is same as that of the mass being generated due to the propellant burning. For example, if it is happening it is the rocket engine is stably being operative or it being operated in a stable manner, now due to some region if there is a changing the pressure.

Let us say pressure will decrease here right what will happen if I just pressure decrease, then this is having nozzle that means more amount of the mass is passing through the nozzle. It has compared to the amount of mass flow rate being generated due to changing the pressure earlier it was here, but now it has gone decrease right then what will happen to the chamber pressure. Naturally, it will decrease because more amount of mass flow rate is passing through the nozzle and then it being generated. Therefore, there will be

depletion of the mass instead of accumulation of the mass in the combustion chamber they will be decrease so as a result the pressure is being reduced.

So, if it is reduce what happen if there is a some change then again the point a operation will move toward back again it will be changing. So, it will be moving up point of operation see this is your operation point it is goes on moving towards the left side. That means after that what will happen certain critical pressure will be there beyond which the flame will extinguish and burning will stop. Then, you cannot really operate and this is dangerous, suppose sometimes it is happening due to some fluctuations and it has happen so you will be in trouble.

Let us take another case like where due to some reason there is a increase in pressure that means you are operating here you are going towards that this is my pressure right if it is operating here that means the less amount of mass is going through the nozzle as compared to the mass being generated at the more mass being generated as compared to the mass being going out through the nozzle that naturally what will happen mass will be accumulated if it is accumulated in the combustion chamber then what will happen to pressure.

Pressure will go on increasing that means my operating point is moved from toward that again it will I mean I am saying suppose they in again you know it is increasing. Then, again there will be a problem, again it will go on increase that means if there is a little part aviation in these kind of operation. Then, it will become unstable, then this is really dangerous thing because during operation you know there might be little part aviation here and take which is likely talker.

Therefore, this is not really very good, so let us look at another we call a situation, where a n is less than 1 if n is less than 1, and then the curve will be like this that is your dash line and the blue color.

Now, if it is operating here that is the stable operating point therefore, absolutely no problem, but due to some changes if it is occurring, what is happening. That means my pressure is change you know it has from here, earlier it was here, now it is here. So, then what happen in this case the amount of mass being generated will be higher as compared to the mass being going through the nozzle. Therefore, what will happen the pressure will increase that means this point of operation you know it will go towards that.

I think some of you are not getting, let me explain you again that is I am here right at this point and due to some reason. Now, instead of operating this point, I am the chamber pressure being reduce here so that means my point of operation will be here in this line. Now, if we look at the amount of mass which is passing through the nozzle is lower than the amount of mass being generated. So, if the more amounts of mass being generated and less mass is going through the nozzle.

Therefore, there will be accumulation of mass if there is a accumulation of mass in the combustion chamber then pressure will increase. Then, it will go back to the original of point of operation that means any part operation will lead to make the system to come back to the original point. So, this is a good point right and similarly, let us see what happens when there is a increasing pressure due some reason your operating here and now you are operating in this point. So, then what is happening here in this case, the nozzle amount of mass flow rate passing through the nozzle is higher as compared to the mass being generated. So, then what will happen the pressure will decrease because depletion of mass in the combustion chamber.

Then, the pressure whatever part aviation increasing pressure was there it will come back to the original one that means this is the very stable operation. Therefore, whenever you that designing a propellant they combustion index should be less than equal to one is a very important thing are you getting a point. Therefore, when you designing combustors or designing the propellant and you know then one has to take care of it has to be you know tested and found out you may say look I want you know more than one. So, my burning rate will be higher, but it will be leading problem therefore, for stable operation in is being use you know 0.4 to 0.7 not even 1, why it is not closer to 1 will see that in a moment.

So, now look at another important aspect so that is what we will see we are seen that r dot right is equal to $a P_c n$ and where a is 0 capital a t ignition minus T_P we have seen in the this is of course best and empirical result, but however it is good and or save. Now, if my T_P the propellant temperature is changing it is 15 degree Celsius and another case it is 30 degree Celsius another case 45 degree Celsius. So, what is happening I am getting a chamber pressure you know very high, over here if I take this integrated area you know that will tell bee the total impulse what I will be getting.

This area will be same as that of the 30 degree and 45, here only difference is there is a increase in the p chamber pressure what is the meaning of that if the chamber pressure increases we have seen earlier from the thrust coefficient in a relationship. You know thrust will be higher because we have seen that thrust will be thrust coefficient a t and P c or p t 2 that means if P c is increasing and for the same you know throughout area and same c t tau as you know what you call the thrust coefficient. You will get the increasing pressure thrust will be increasing, but there might be a problem if it is goes on increasing right then there might be explosive because if it is very high pressure.

That will depend on what depend one on this n index right and if it is a too small, then may be combustion may not occur then one has to choose properly. It will be dependent on the T i and how it is depend on T i that is self ignition temperature of the propellant that we will see. That means we need to find out the sensitivity of regression rate of the burning of the respective t p. T p is what propellant temperature because propellant will be attaining certain temperature.

So, for example, you are operating in the cold country the propellant will be having certain temperature right and it is summer we are having 45 degree even 50 degree Celsius some places and you are operating it will be 50 degree or 45 degree. Therefore, you know during summer and winter it will be causing lot of problem.

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The sensitivity of regression/burning rate with T_p at constant P_c

$$\sigma_{\dot{r}} = \left(\frac{\partial \ln \dot{r}}{\partial T_p} \right)_{P_c} = \frac{1}{\dot{r}} \left(\frac{\partial \dot{r}}{\partial T_p} \right)_{P_c}$$

But we know; $\dot{r} = a P_c^n$

$$\sigma_{\dot{r}} = \frac{1}{\dot{r}} \left(\frac{\partial \dot{r}}{\partial T_p} \right)_{P_c} = \frac{P_c^n}{a P_c^n} \frac{da}{dT_p} = \frac{(T_i - T_p)}{A} \times \frac{-A}{(T_i - T_p)^2} = -\frac{1}{T_i - T_p}$$

But, $a = \frac{A}{T_i - T_p}$
 $T_r < T_i$

For DB: $\sigma_{\dot{r}} = 5 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$; For typical composite propellant: $\sigma_{\dot{r}} = 2 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$.

How is the chamber pressure P_c sensitive to grain temperature?

$$\sigma_{P_c} = \left(\frac{\partial \ln P_c}{\partial T_p} \right)_{AR} = \frac{1}{P_c} \left(\frac{\partial P_c}{\partial T_p} \right)_{AR}$$

$$P_c = \left[\frac{A_b}{A} \frac{a(\rho_p - \rho_g)}{\Gamma \sqrt{RT_c}} \right]^{1/(1-n)}$$

$$P_c = K a^{1/(1-n)}$$

$$\sigma_{P_c} = \frac{1}{K a^{1/(1-n)}} \left(\frac{d(K a^{1/(1-n)})}{dT_p} \right)_{AR} = \frac{1}{1-n} \sigma_{\dot{r}}$$

Hence the sensitivity of P_c to grain temperature is greater as compared to the sensitivity of regression rate to grain temperature.

So, we need to understand how good it is so for sensitivity of regression rate is concern and how bad it is for that we need to define a term. That is σ_r which is nothing but change of regression rate with respect to the propellant temperature per unit regression rate, while the chamber pressure remaining constant. So, what we will do we are already derive an expression or we are already got an expression empirical relation see that r dot is equal to a P_c^n . We know that this we can use this r dot and put into this definition of the sensitivity of the regression rate right with T_P I will just put it here. If we look at this one over r dot is nothing but a P_c^n and if I keep this P_c is constant.

That is nothing but $P_c^n \frac{d}{dT_P}$ keep in mind that I can cancel it out very easily and in place of a I can use this a is nothing but you are a by $T_i - T_P$. So, I will just put it $k (T_i - T_P)^a$ and when I difference it this a I will take minus a divided by $T_i - T_P$ whole square. So, this will cancel it out and this will cancel it out what it says that σ_r dot is equal to minus 1 above $T_i - T_P$ what it indicates if T_P is very closer to t ignition that is self ignition temperature of the propellant. Then, what happen this is the very big term right and that means sensitivity will be very high it is not really called.

So, therefore let us look at a typical you know value for the double base propellant σ_r dot is equal to 5×10^{-3} degree Celsius that means 1 over Celsius. You know if you look at this temperature, of course you can use any temperature because this temperature difference you know I can use Kelvin. I can use any thing that you should keep in mind where as for the composite propellant typical composite propellant σ_r dot is 2×10^{-3} . You know inverse 1 over degree Celsius what it indicates. It indicates that the double base propellant is more sensitive as compare to the composite propellant. Keep in mind these numbers, you should not take it is sacrosanct that means it can be hiring from hiring from propellant to propellant, but I have taken some example.

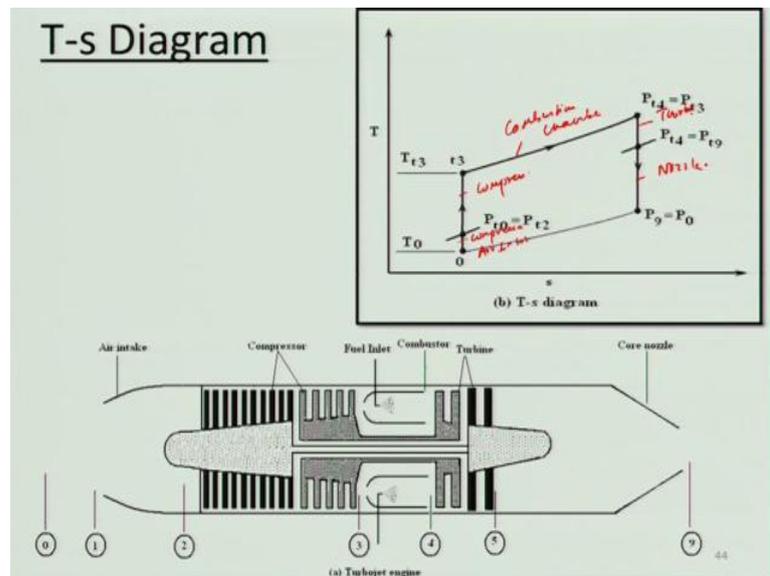
The value of T_P will less than T_i I always know, it is σ_r has to be negative because there is minus 1, which one T_P , T_P should be less than T_i minus sign minus 1 upon t . That really does not matter is that like you know what is the value how it is changing, you know that means you know it will be T_P always will be less than you know T_i . Otherwise, it will lead to the problem because if it is you know very small quantities, you

know, then it will be very higher values because if it is 0, it will be infinity. If it is T_i equal to T_P , therefore one has to worry about it.

Now, how is chamber pressure sensitive to the grain temperature because that is a very important aspect one has to look at it because what we have seen just now that how these you know regression rate is sensitive to the T_P . Now, how this chamber pressure will be you know sensitive to the grain temperature that is T_P . So, we know that P_c is basically for a stable state, of course this is valid for stepper is a v divide by a t into the other terms like a ρ_p minus ρ_g divide by γ know divided by root over $R T_c$ power to the 1 over 1 minus n .

So, what will see that P_c will basically function of this term a by a , so what will assume that a is remain constant. Then, we can think about because a already we know that is you know a is basically what we call a by T_i by minus T_P . So, if we look at that, then we can define the ρ_p as change in chamber pressure with respect to the change in T_P per unit chamber pressure when a r this is a r area ratio remaining constant. So, what we will do is will just take these as a P_c because all other things are remaining constant γ and then $R T_c$ everything remaining constant only it is varying with a .

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So, if you look at this process is basically what you call compressions compression intake air intake and this is your compression and this is your combustion chamber constant pressure heat addition. This portion is your expansion in nozzle, sorry

expansion in turbine and this is in your nozzle grate expansion. It is similar to what that Ramjet only think you are having a com contribution from compression and a turbine that is all, but it is similar. If you look at PV diagram, it will be similar to that only the division will be there in the compression and expansion.

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The thrust produced by an ideal turbojet by assuming complete expansion in the exhaust nozzle ($P_9 = P_0$) and ($\dot{m}_9 = \dot{m}_0$) is

$$T = (\dot{m}_9 V_9 - \dot{m}_0 V_0) = \dot{m}_0 a_0 \left(\frac{V_9}{a_0} - M_0 \right)$$

where a_0 is the speed of sound and $a_0 = \sqrt{\gamma R T_0}$ is the flight Machnumber at station number (0)

The specific thrust T_s can be derived as,

$$T_s = \frac{T}{\dot{m}_0} = a_0 \left(\frac{V_9}{a_0} - M_0 \right)$$

But, we know that

$$\left(\frac{V_9}{a_0} \right)^2 = \frac{a_0^2 M_9^2}{\gamma_9 T_9} = \frac{T_9}{T_0} \cdot M_9^2$$

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So, now what we will do we will do the similar you know methodology to derive the expression for the thrust produce when ideal turbojet engine will be same that what we have derived M naught 9 v minus M naught V naught. If I take this a naught out, you know and M naught 9 out, I will get V 9 by a naught minus M naught M naught is a flight Mach number. So, a naught is speed of sound and we have already know these things and the specific thrust can be you know defined as I told you earlier the thrust divided mass flow rate of air is nothing but a naught V 9 by a naught minus M naught.

This same thing what we have done, so we know that V 9 by a not square is nothing but a 9 square M 9 square and a naught 9 we know that it is gamma 9 or 9 T 9 and a naught square, you know that has gamma naught or naught T naught. So, this is cancel it out gamma is same as that, so we will land in getting T 9 by T naught into M 9 square, so what we will be doing, we will be doing the similar thrust of relating the various you know pressure and temperature parameters in each component. It is a just extension what we are done for the ramjet engine, so p T 9 by p you know 9, we know that it can be express for isentropic flow in terms of Mach number and exit Mach number.

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The exit Mach number M_9 can be expressed in terms of pressure ratio by

$$\left(\frac{P_{t9}}{P_9}\right) = \left[1 + \frac{(\gamma-1)}{2} M_9^2\right]^{\frac{\gamma}{\gamma-1}} \quad M_9^2 = \frac{2}{(\gamma-1)} \left[\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

The pressure ratio P_{t9}/P_9 can be expressed in terms of pressure ratio across individual components as

$$\frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_{t5}} \cdot \frac{P_{t5}}{P_{t4}} \cdot \frac{P_{t4}}{P_{t3}} \cdot \frac{P_{t3}}{P_{t2}} \cdot \frac{P_{t2}}{P_{t1}} \cdot \frac{P_{t1}}{P_9} \cdot \frac{P_0}{P_9} \quad (1)$$

Assuming $P_9 = P_0$, Eq. (1) can be expressed in terms of pressure ratio across each component as

$$\frac{P_{t9}}{P_0} = \pi_n \cdot \pi_t \cdot \pi_b \cdot \pi_c \cdot \pi_d \cdot \pi_r \quad (2)$$

If I just rewrite back, that it will be M_9^2 is equal to $\frac{2}{\gamma-1} \left[\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$. So, we will be looking at this $\frac{P_{t9}}{P_9}$ and look at a various pressure ratios, so you look at a similar way and I have just written down it here and if you look at this is a fully expanded. So, this will be 1 and $\frac{P_{t9}}{P_9}$ what it would be this will be your π_r and $\frac{P_{t2}}{P_{t1}}$, $\frac{P_{t3}}{P_{t2}}$ by π_c . This is nothing but your π_d and this is $\frac{P_{t3}}{P_{t2}}$ is π_c and $\frac{P_{t4}}{P_{t3}}$ by π_b and $\frac{P_{t5}}{P_{t4}}$ is your π_t and this is your nozzle you can say.

We know that this what you call and we can express in this similar in terms of pressure ratios and under ideal condition, what we can say we can say that π_b is equal to 1. What happens to your turbine π_t can I say 1 π_c can say one no and π_d can I say 1, total pressure ratio what happens in the π_d no compressor no turbine. You are saying because we have you know getting sometimes doing the work on it and extracting the work in the turbine, so what about π_d . What we did in the ramjet engine total pressure, what is happening ram pressure will be what total pressure will be remaining same or it will be different this is isentropic process.

So, what about nozzle this also one because we are just converting you know kinetic energy into the where into the static pressure or a dynamic air to the static air in case of the air intake and this is other way around then nozzle is just opposite. The static way

you know like static pressure, we are converting into dynamic, but the total pressure is remaining constant. Therefore, you will have to be you know understand this concept, so $\pi_n \pi_b$ and π_d is equal to 1.

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But, for an ideal engine $\pi_n = \pi_b = \pi_d = 1$. Now, Eq. (2) will be

$$\frac{P_{t9}}{P_0} = \pi_n \cdot \pi_t \cdot \pi_b \cdot \pi_c \cdot \pi_d \cdot \pi_r \quad (2) \quad \frac{P_{t9}}{P_9} = \pi_t \cdot \pi_c \cdot \pi_r \quad (3)$$

Using Eq. (3) the exit Mach number M_9^2 can be expressed as

$$M_9^2 = \frac{2}{(\gamma-1)} \left[\left(\frac{P_{t9}}{P_9} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] = \frac{2}{(\gamma-1)} \left[(\pi_t \pi_c \pi_r)^{\frac{\gamma-1}{\gamma}} - 1 \right] \quad (4)$$

Since, $\pi_r = \tau_r^{\frac{\gamma}{\gamma-1}}$ and for an ideal turbojet $\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}}$ and $\pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}$, Eq. (4) becomes,

$$M_9^2 = \frac{2}{(\gamma-1)} \left[\tau_t \cdot \tau_c \cdot \tau_r - 1 \right] \quad (5)$$

But, we can express T_9/T_0 in terms of temperature ratios as

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{T_{t9}/T_9}$$

So, this equation becomes you know like p_t by p_9 $\pi_t \pi_c \pi_r$ in case of ramjet engine π_t is equal to 1 π_c because there is no compression, no turbine. So, you can directly get this expression that means if I know this equation, you know expressions I will get ramjet turbojet just making some parameter one. You know that why that is a beauty of this analysis, I need not to do anything if I say $\pi_d \pi_c$ is equal to 1. That means it is ramjet that is the beauty of this relationship where computer you can do very easily. I can put a condition and do that ramjet same equation you know so that you should must appreciate of this relationship and using equation 3 exit Mach number, I can get because I have already derived that is nothing but same as that and except this.

You know π_t and π_c being coming to as compare to that ramjet engine, so if you look at this similar to that what we have done for the ramjet only two terms are come for the turbojet π_d and π_c .

So, π_r you know is we can relate to the τ_r gamma power to the gamma minus 1 and similarly, for π_c and π_t these are isentropic relationship, π means it is the pressure ratio τ means it is the temperature ratio. So, we know that pressure ratio can be related to the temperature ratio with the help of this gamma, you know of the index. So, M_9 you

know square, we can write down here itself if I put these values over in this place, you know and also for pi r i can get instead of pi i can get in terms of tau that is tau t tau c tau r.

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Let us express T_{t9}/T_0 in terms of τ across the various components as

$$\frac{T_{t9}}{T_0} = \frac{T_{t9}}{T_{t5}} \cdot \frac{T_{t5}}{T_{t4}} \cdot \frac{T_{t4}}{T_{t3}} \cdot \frac{T_{t3}}{T_{t2}} \cdot \frac{T_{t2}}{T_{t0}} \cdot \frac{T_{t0}}{T_0} = \tau_n \cdot \tau_t \cdot \tau_b \cdot \tau_c \cdot \tau_d \cdot \tau_r \quad (6)$$

We know that, for an isentropic process $\tau_d = \tau_n = 1$, so Eq. (6) becomes,

$$\frac{T_{t9}}{T_0} = \tau_t \cdot \tau_b \cdot \tau_c \cdot \tau_r$$

The expression for T_9/T_0 can be rewritten as

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{T_{t9}/T_9} = \frac{\tau_t \cdot \tau_b \cdot \tau_c \cdot \tau_r}{\tau_t \cdot \tau_c \cdot \tau_r} = \tau_b \quad (7)$$

An expression for T_9 can be obtained by combining Eqs. (1), (2), (5)

$$T_9 = \frac{T}{\dot{m}_0} = a_0 \left(\frac{V_9}{a_0} - M_0 \right) = a_0 \left[\sqrt{\left(\frac{2}{\gamma - 1} \right) \frac{\tau_\lambda}{\tau_c \tau_r} (\tau_t \cdot \tau_c \cdot \tau_r - 1)} - M_0 \right] \quad (8)$$

So, we will do the similar way of getting this T 9 by T naught you can write down t T 9 by T naught if your t T 9 by T 9 and we will go on doing that all those things if you look at it will you will be get this tau n tau c tau t tau b tau c tau d and tau r. If you look at tau d is what is it one because the adiabatic process in there air intake there is no heat. Therefore, that will be one, similarly in the nozzle is it you are adding some heat, you cannot you are not doing anything, therefore it will be 1. So, what about tau c and tau t, can I make it i 1 and tau b is certainly no, because I am adding some amount of heat can I make it.

I cannot make the tau t and tau c as 1 that means the total temperature across the compressor and across the turbine are changing. So, it cannot be one because in one case compress you are giving you know amount of work and in that the turbine you are extracting the work from the fuel it. So, there will be change in total temperature otherwise if you will get other thing. Therefore, you cannot really make it one, so t T 9 by T naught it will be tau t tau b tau c tau r and again you can say this here you can appreciate this point the tau c and tau t will be one in case of ramjet this same thing I am

repeating. So, that it will enter into your mind so the expression T 9 by T naught can be really if look at is a interesting.

If I put this t T 9 by T naught all those things this is can and also t T 9 by T 9 you will see that it is can be cancel it out. This, can be cancel it out and the tau c, you can cancel it out it happens to be tau b and what we have seen in case of your ramjet engine yes or no. We are doing the same thing, but make it little complex, but this is cancel it out which is obvious because the turbo in the turbojet engine. The, work you know harness by the turbine is being utilized by the compressor therefore, it must be T 9 by T naught will be tau b it will be how much heat added into the combustors that is the thing we were saying and it is true also Ramjet.

So, the T s you can just substitute these values V 9 by V naught and you will get a expression which looks to be little frightening, but however it is quite simple. I am not expecting that you should remember this expression, but however you must know how to go about how to derive it that is expected.

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By applying one dimensional steady state energy equation to a CV containing the turbine, the work output \dot{W}_t is given by, $\dot{m}_f \ll \dot{m}_c \ll \dot{m}_a + \dot{m}_f$

$$\dot{W}_t = \dot{m}_5 (h_{t4} - h_{t5}) = \dot{m}_0 C_p (T_{t4} - T_{t5}) \quad (9)$$

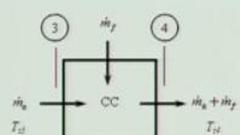
Similarly, the work input to the compressor \dot{W}_c is given by

$$\dot{W}_c = \dot{m}_3 (h_{t3} - h_{t2}) = \dot{m}_0 C_p (T_{t3} - T_{t2}) \quad (10)$$

But, $\dot{W}_t = \dot{W}_c$. Therefore, by equating Eq. (9) and (10), we can have

$$\tau_r = 1 - \frac{\tau_r}{\tau_\lambda} (\tau_c - 1); \quad \tau_\lambda = \frac{C_p T_{t4}}{C_{p,0} T_0} = \frac{T_{t4}}{T_{t3}} \cdot \frac{T_{t3}}{T_{t2}} \cdot \frac{T_{t2}}{T_{t0}} \cdot \frac{T_{t0}}{T_0} = \tau_b \cdot \tau_c \cdot \tau_d \cdot \tau_r = \tau_b \cdot \tau_c \cdot \tau_r$$

The fuel/air ratio f can be expressed in terms of known variables as



$$f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_p T_0}{\Delta H_C} (\tau_b \tau_r \tau_c - \tau_r \tau_c)$$

$$TSFC = \frac{\dot{m}_f}{T} = \frac{f}{T_3}$$

So, which will be quite simple to derive these expressions and that is very important; we need to now relate this turbine work into the compressor work, because we will have to get the relationship between the compressor and turbine. So, we will assume the one dimensional steady state energetic way, you know these things. We apply this to a energy equation control volume and the turbine work will be m dot 5 is equal to s t 5

minus $s t_5$, because the in the turbine. You keep in mind that here we are assuming, you know like the mass is continuity is maintain because one dimensional flow we are talking about $M_{naught 5}$ is equal to $M_{naught 4}$.

You are not adding anything except you know, so same fuel it is going therefore, which is nothing but that M_{naught} not keeping in mind that here I am saying it is very very less than $M_{naught 5}$ or is very less than M_{naught} . Therefore, I am saying this is M_{naught} is equal to $M_{naught 5}$ basically $M_{naught 5}$ is equal to M_{naught} plus $M_{naught f}$, but I am assuming this.

Therefore, $M_{naught f} C_p T_{t4} - T_{t5}$, similarly I can have for the compressor, which is same as that $M_{naught} C_p T_{t3} - T_{t2}$ and when we quit this work done by the turbine is same that as work you know taken by the compressor. Then, we will get an expression you know τ_t is equal to $1 - \tau_r$ by τ_λ in bracket τ_c minus 1.

Keep in mind that we can express in terms of τ_λ because that is a thing how much heat you know being added or the total enthalpy at the exit of the combustor divided at the amount of enthalpy entering into the engine.

So, if you look at it is this, you know it can be rewrite this because this is cancel it out T_{t4} by t_{t3} into t_{t3} by T_{t2} into t_{t2} by $t_{T_{naught}}$ and $t_{T_{naught}}$ by T_{naught} . So, you can write down in terms of all these τ_b τ_c this is compressor and τ_d is for air intake and τ_r . You know that τ_d is equal to 1 and because of it is adiabatic process we are saying no heat is going out, but in real situation it cannot be, but in ideal situation it is ok. So, the fuel air ratio can be express in terms of known variables as you know we will have to consider these combustors, which we have done a similar thing in case of ramjet engine it will same.

So, we can write down $m \dot{f}$ divide by $m \dot{a}$ nothing but $C_p T_{naught} \Delta h_c$ which you will see that $\tau_b \tau_r \tau_c - \tau_r \tau_c$. You see some step I have omitted here, but you can do very easily and in case you will find some problem. Let me know and TSFC you can get very easily that is f divided specific thrust and you will put this values. You know now we are having several terminologies are coming like several ratio temperature ratio of pressure ratio. Keep in mind that we can you know change, this τ_c in terms of pressure ratio compressors. Similarly, τ_p we can you know some

places wherever it required we can change into the pressure ratio across the turbine I use in the isentropic relationship.

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The relation for propulsive efficiency η_p from the basic definition can be expressed as

$$\eta_p = \frac{2TV_0}{\dot{m}_a(V_9^2 - V_0^2)} = \frac{2\dot{m}_a(V_9 - V_0)V_0/a_0}{\dot{m}_a(V_9^2 - V_0^2)} = \frac{2M_0}{V_9/a_0 + M_0}$$

The expression for thermal efficiency η_{th} can be written as

$$\eta_{th} = \frac{\dot{m}_a(V_9^2 - V_0^2)}{2\dot{m}_f\Delta H_c} = 1 - \frac{1}{\tau_r\tau_c}$$

The overall efficiency η_0 can be expressed as

$$\eta_0 = \eta_p \cdot \eta_{th}$$

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So, the relationship for propulsive efficiency we can write down $2TV_0$, basically the thrust power divide by kinetic energy change in the engine that is $\dot{m} V_9^2 - \dot{m} V_0^2$ divide by 2. If I just you know write in in place of thrust in this expression $\dot{m}(V_9 - V_0)$, you will see that I cancel it out and this will cancel it out and I will get $V_9 + V_0$. If I divided by a_0 here, similarly a_0 square. Here, I will get two Mach number divide by V_9 by V_0 plus M_0 . So, this is an easier way of saying that because I know expression V_9 by a_0 I can do very easily.

Also, you can see that how these Mach number is really affecting your propulsive efficiency and thermal efficiency you can get you know do all those thing algebra. You will get $1 - \frac{1}{\tau_r\tau_c}$ keep in mind that here τ_c is coming to the picture in case of the turbojet. Earlier, it was only $1 - \frac{1}{\tau_r}$ in case of Ramjet τ_c is equal to 1 in case of Ramjet. So, propulsive efficiency overall efficiency will be nothing but propulsive efficiency multiplied by thermal efficiency will give nozzle that means by this we have derived all the expressions. We are armed with all the expression to the carry out parametric studies, and just to summarize this is what we have derived.

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Summary: Turbojet Engine Analysis

$$T_s = \frac{T}{\dot{m}_0} = a_0 \left(\frac{V_9}{a_0} - M_0 \right) = a_0 \left[\sqrt{\left(\frac{2}{\gamma - 1} \right) \frac{\tau_\lambda}{\tau_c \tau_r} (\tau_r \cdot \tau_c \cdot \tau_r - 1)} - M_0 \right]$$

$$TSFC = \frac{f}{T/\dot{m}_0} \quad f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{C_p T_0}{\Delta H_c} (\tau_b \tau_r \tau_c - \tau_r \tau_c)$$

$$\eta_p = \frac{2M_0}{V_9/a_0 + M_0}$$

$$\eta_{th} = 1 - \frac{1}{\tau_r}$$

$$\eta_0 = \eta_p \eta_{th}$$

Parametric Analysis

Alt = sea level, 12 km, 15 km

M₀ = 0, 0.85, 1.5

T₀₄ = 1600 K

$\pi_c = 1 \text{ to } 40$

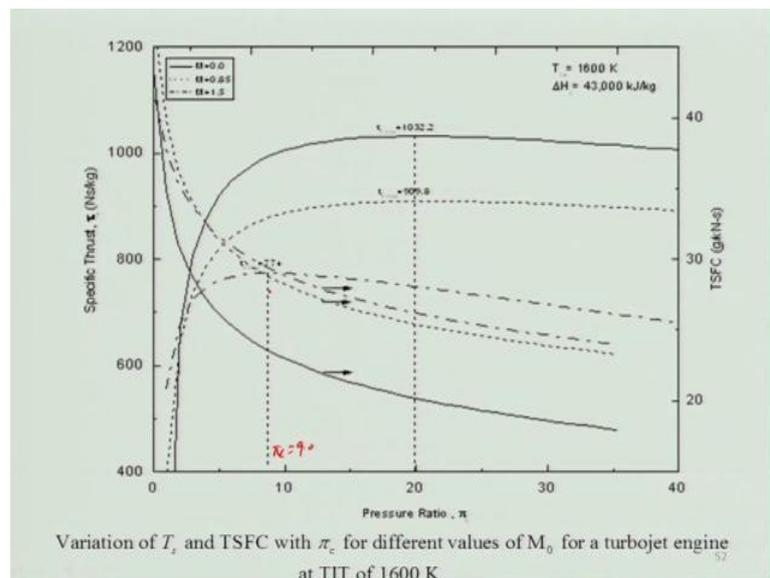
That is the specific thrust which we have done and these are the expression which in terms of various tau b tau c tau r. We are using also tau lambda and in place of tau lambda may be you know this term tau lambda divide by tau c tau r is nothing but tau b some places you can use tau b as well. So, what is helping this specific thrust expression is basically separating each parameter. It is arming with us or helping with us say that we can do parametric analysis that you must appreciate. Otherwise, I need not to go for this you know unless I am not interested in parametric studies and which is essential when I am trying to understand you know how it is the performance is affected by the various parametric efficiency.

That is f c and we know that f is you can be express in terms of tau b tau r tau c minus tau r tau c multiplied by C p T naught divide by delta H c. and these are expressions for propulsive efficiency thermal efficiency which we have discussed just now and what we will do now we will take three cases. One is the Mach number of je w that is the sea level conditions. Another we will take a long range you know vehicle or other aircraft that we use passenger rate at 0.85 which can operate at 12 kilometers altitude. Then, the fighter aircraft which is corresponding to 15 kilometers altitude value and we are keeping this the turbine, you know like inlet temperature or combustor exit temperature is 1600 Kelvin.

So, and what we will do now what are the variables we will be using we will be basically using this, you know we can vary this pressure ratio across the compression unit. You can say that why not vary the pressure ratio across the turbine. You can do as well both are you know can be related, basically with the work you know because a work what about you are getting from the turbine is being utilized by the compressors, so generally compressor is very important one.

Therefore, we use the compressor as parameter that you will be varying from 1 to 40 and see that how we can choose a compressors. How we can choose a, you know pressure ratio, because the compressor size and you know cost will be dependent on the pressure ratio you want to have for each, so that is a very important one.

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So, what we are looking at this specific thrust being plotted on the y axis and the pressure ratio plotted on the x axis for a turbojet engine which is having turbine inlet temperature of 1600 Kelvin. We are also varying this Mach number as I told you three cases we have taken representative, case one is sea level conditions or you can say static conditions other is 0.8. Mach number flight, Mach number and other is 1.5 or you can note here that the when this flight Mach number is 0 this solid line the specific thrust increases with the increase in pressure ratio across the compressor. Then, it reaches the peak values here and then after that it decreases fully.

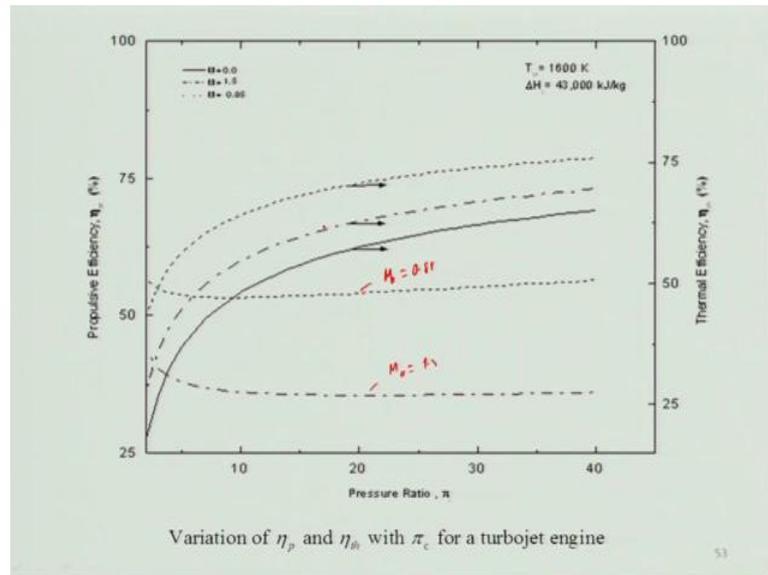
So, what it indicates that means there is an optimum value for the what you call pressure ratio across the compressor for which you will get the maximum specific thrust and the however if you look at TSFC which I have shown here. It is a very high value at the low the pressure ratio of one, you know because we cannot have zero pressure ratios across the compressor. So, one is you know that that means no compressor factors at that value it is going towards infinity very large. It has no meaning and you cannot have any meaning turbojet engine having the pressure ratio of 1, but however it decreases.

You know it decreases continuously as you increases in pressure ratio, you will see that this is having not a minimum values. Unlike in a ramjet engine, there is a minimum values and of course, you will have to choose that, but there is another interesting thing you can observe. When I will go for the flight what you call Mach number of 0.8. You will see that this is having, you know it is also having similar features goes on increasing and then of course, it is having a certain values of t and looking where you get the maximum specific thrust, but if you go for the Mach number of 1.5, you will have similar features of the curve.

It is decreasing you know after reaching a value of certain maximum value at a pressure ratio of 9, t is equal to 9, you are having and then it decreases little at a higher rate as compared to the both the sea level conditions and other static condition and the flight Mach number. What it indicates, it indicates that when the flight or the engine is moving at a higher speed, it can used that pressure and you need not to go for a high pressure ratio compression because the ram pressure can utilize two kind of for increasing its pressure.

So, therefore if you look at the fighter aircraft, we will be having this smaller compressor because you would need a low pressure altitude, whereas you go for a long range passenger aircraft, you need to have to go for a twenty kind of engines. At the same diameter, what thrust you need to because at a static conditions with static process is also important. So, you will have to play around and see that what really you need and what is the level blight flight. So, these are the things you can get by just doing you know specific parametric analysis that is the beauty of this method. You can learnt a lot by just playing around and see what is happening, why it is happening and what are the reason whether you can have any scope to improve it further.

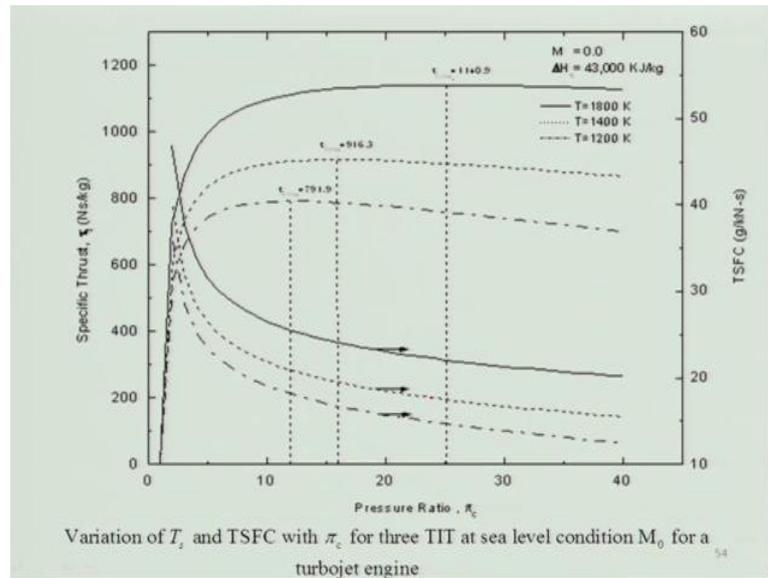
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So, let us look at a propulsive efficiency and the thermal efficiency if you look at these are the curves which is you know basically the propulsive efficiency. These two curves and it is a static condition there will be no meaning of having propulsive efficiency even Mach number type number is equal to 0. So, what you call this one is basically Mach number of 0.85, it is having higher propulsive efficiency as compare to 1.5.

It is having similar feature features in the when it is pressure ratio increases, it is very higher over here and decreases may be at 7 pressure it will minimum which is not very obvious in this diagram. When you look at number, it indicate and thermal efficiency, it is goes on increasing, you know from the lower pressure ratio to higher pressure ratio and you will get a lower thermal efficiency. In this case it indicates you know zero flight Mach number and when you increase this 0.85, you know you will get thermal higher efficiency when it is the 1.5 kind of thing, then it decreases because of you know what your energy utilization wants.

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So, what we will do now, I will just look at you know what happens the effect of the temperature ratios and which is having similar views you can see that this is having highest temperature you know like 1,800 Kelvin. So, if you look at you will have a higher pressure ratio, you need to get to get a higher specific thrust and as a temperature decreases, you will get a lower specific thrust. You know like a maximum you know specific thrust at lower pressure ratios, because you know you need to have a higher what you call the temperature to be achieved. Therefore, you need to give you know more amount of pressure and you can get a higher kind of value and after that you peak reaches.

So, the higher temperature when you want to get and you want to get also higher specific thrust. So, naturally you will pay penalty for having a higher TSFC and all having similar view you know a value what you call trend like as you goes on decreasing the turbine inlet temperature, you will go on decreasing the TSFC. So, you will have to make a these things where you want to go and with this I will stop over in the next class, we will take an example to see how we can solve this problem.

Thank you.