Lecture – 34

Let us start this lecture thirty-four, and we will be today discussing about turbo fan engine and we will carry out a cycle a real cycle in this. But before that we will take an example in turbo jet engine, if you recall we really looked at in the last lecture, how to incorporate the losses while carrying out a real cycle right; that means, we have already discussed how to daily look at incorporate those losses in the ideal cycle of turbo jet engine. And then we also had carried out a parametric analysis for a very limited case, I had taken only the one magnum, but 0.8, and then looked at that how this specific thrust will be different for a real cycle.

And if you recall that we are getting the what do you call higher specific thrust for case of real engine for the same pressure ratio and same turbine pressure same magnum everything remain the same right, but whereas, the t s f C we are having higher values in case of real cycle as compared to the ideal. And we looked at other efficiencies we found that it is basically decreasing particularly the overall efficiency whereas, the propulsive efficiency you know will have an higher efficiency as compared to ideal one.

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Example 1:

An turbojet is flying at flight Mach number $M_0 = 0.8$ at an altitude of 15 km. The pressure ratio across the compressor with polytropic efficiency of 85% is 12. The turbine has a polytropic efficiency of 88%. At the combustion chamber exit, the temperature is 1600 K. The burner has an efficiency of 98% and a total pressure ratio of 0.95. A convergent nozzle with isentropic efficiency of 95% is used to produce thrust. The mechanical efficiency of this engine is assumed to be 98%. If the total pressure recovery factor in the air intake is 0.91, estimate the net thrust produced by the engine if it consumes 50 kg/s of air. Determine T_s, TSFC, η_p and η_{th} if the heat of combustion is 43,000 kJ/kg.

Given:

$Al = 15$ km	$T_0 = 216.7$ K	$P_0 = 11.2$ kPa
$T_{t4} = 1600$ K	$M_0 = 0.8$	$\dot{m}_0 = 50$ kg/s
$C_{pa} = 1.005$ kJ/kg K	$C_{pg} = 1.148$ kJ/kg K	$\Delta H_C = 43,000$ kJ/kg
$\eta_{pc} = 0.85$	$\eta_{pt} = 0.88$	$\eta_b = 0.98$

So, we will now take an example of a turbo jet engine, and a which is flying at a flight mach number of 0.8 at an altitude of 15 kilometer. And the pressure ratio across the compressor with the polytropic efficiency of 0.85 or 85 percent, and the pressure ratio is 12. And turbine has a polytropic efficiency of eighty eight keep in mind always turbine is you know having higher polytrophic efficiency than the combustion right and that is it can be clear in this example, and at the combustion chamber exceeds the temperature is restricted to 1600 kelvin and the burner has a efficiency of 98 percent. And a total pressure ratio of 0.95 in ideal case it is one, but in this case it is 0.95 the convergent nozzle with an isentropic efficiency of 95 percent is used to produce thrust and the mechanical efficiency of this engine is assumed to be 98 percent.

And if the total pressure recovery factor in air intake is 0.91 we need to estimate the thrust produced by the engine if it consumes 50 k g's per second of air and besides this we can determine the specific thrust or specific conduction propulsive efficiency and thermal efficiency of course, the heat of combustion is given and. So, all. So, usually will be using C P of air as 1.005 kilo joule per k g kelvin and C P of the gas will be using one 0.148 kilo joule per k g Kelvin, and keep in mind that this C P will be using steel the end of compressor, and C P C will be using from the inlet of combustor to the nozzle. And this is for our calculation, but when you use a computer programme, you can use a you know C P varying in each component right it can be incorporated very easily, but in man calculation it will be quiet comparison.

So, we will be restricting to this two values of C P and gamma, gamma if you look at 1.4 and gamma g that is you know hot portions harder components will be using the1.33. So, I would insist that before really you carry out any analysis or any calculations you should keep this you know t s or the P V diagram in mind or I would have give to draw that, but for your whole clarity because the station numbers and other things will be in you know properly used otherwise.

$\pi_b = 0.95;\ \eta_n = 0.88;\ \eta_m = 0.98$
$PRF = 0.91$

Solution:

We know that T_s is expressed as

$$T_s = \left[(1+f)V_9 - V_0\right] + \frac{A_9}{\dot{m}_0}\left(P_9 - \acute{P}_0\right)$$

We know that flight velocity is given as

$$V_0 = \sqrt{\gamma RT_0}.\dot{M}_0 = \sqrt{1.4 \times 287 \times 216.7} \times 0.8 = 236.06 \text{ m/s}$$

But, the nozzle exit velocity is to be determined by

$$V_9 = \sqrt{2C_p\left(T_{t5} - T_9\right)}$$

In order to evaluate T_{t5} and T_9, we need to estimate the temperature and pressure at each station number.

You will be getting confused. So, if you look at here the t s diagram has a zone and this 0 station 0 to two is nothing, but your air intake and two to 3 is your compressor and station 3 to 4 is your combustion chamber. And then 4 to 5 the expansion in the turbine it can be comprised of both low pressure turbine and high pressure turbine and 5 to 9 is your nozzle right. So, and we can get the specific thrust you know in the format like which is equal to one plus f V 9 minus V naught plus a 9 by M naught right and and multiplied by P 9 minus P naught in the bracket right. So, if you look at here of course, we need to find out this V 9 we need to we need to know actually V naught because M naught is given right altitude is given you can find out we will in a moment we will do that and we know the P naught this is known right it is known, but P 9 is not known and a 9 is not known cross sectional area of the nozzle at the you know nozzle exit is not given and M naught dot is not given mass flow rate how much is entering you really do not know right. So, how to go about it of course, the f you can find out you know already how to evaluate f right and because in ideal cycle it will be similar if not same and V 9 you know we can evaluate right.

So, let us look at how we go about it the V naught is very easy to determine, that is nothing but a naught which is root of gamma RT naught M naught M naught is given to you. And t naught is from you altitude; you will get gamma is 1.4287 is your specific gas constant you will get you know and mach number is 0.8 you will get 236.06 meter per second right. So, the nozzle exit velocity is to be determined and we will go as usual like

you know V 9 is root of two C P T t 5 minus t 9 right. If you look at this diagram T t 5 a you know is to be determined, but T t 4 is known right can you really get this from this data because T t 4 is given as 1600 kelvin I am I right, can you look at you at this thing it is given as 1600 can I get without really looking at each point what will be pressure, and temperature is it possible yes or no certainly no, because it is the trout is not given if it is the trout is given you can get directly. So, there we need to go through each you know station number or component inlet and exit and find out pressure and temperature right and then we can get because particularly if I want to know this T t 4 then I will have to balance this work done by the turbine right which will be consumed by the compression in case of turbo jet. So, that we need to look at it and here we will be doing all those things calculate you know start from the station 0 onwards. And see how we will go it is the usual procedure what we had followed for ideal cycle, but we will be incorporating various values of the efficiency special losses as we go along intake.

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So, let us look at intake a intake total pressure and temperature at the station two can be evaluated as T t 2 is equal to T t naught is equal to T naught 1 plus gamma minus 1 divided by 2 M naught square. We you put this values of gamma 1.4 M naught is 0.8 t naught is given you will get 244.43 kelvin right which is same as that, now an ideal cycle let us evaluate the pressure ratio which is even for the isentropic one can think of right. And like if you look at this is P t naught by P naught T t naught by t naught gamma a gamma minus one. So, you can get 1.52 correct and getting this thing P t 2 is equal to P t

2 divided by P naught into P naught you will get these values, right. But keep in mind this will be basically P t naught kind of thing, this is basically P t naught right this would not be right. So, P t P r f will be 0.9 right, P r f is nothing but your P t two by P t naught right. So, this P t two is nothing but P f r into P t naught right that will give me 15.53 is that clear.

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Compressor:

The total pressure across the compressor is given as

$$\frac{P_{t3}}{P_{t2}} = 12 \Rightarrow P_{t3} = \frac{P_{t3}}{P_{t2}} \times P_{t2} = 186.43 \text{ kPa}$$

Using the polytropic efficiency of the compressor, let us now evaluate the total temperature ratio T_{t3}/T_{t2} as

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{\frac{\gamma-1}{\eta_{pc}\gamma}} = 12^{0.336} = 2.305$$

$$\Rightarrow T_{t3} = \frac{T_{t3}}{T_{t2}} \times T_{t2} = 563.53 \text{ K}$$

By carrying out energy balance across the combustor, we can find out the expression for fuel air ratio as

$$f = \frac{C_{pg}T_{t4} - C_{pa}T_{t3}}{\eta_b \Delta H_C - C_{pg}T_{t4}} = \frac{1148 \times 1600 - 1005 \times 563.63}{0.98 \times 43000 \times 10^3 - 1148 \times 1600} = 0.0315$$

So, the compressor the total pressure across the compressor is given by pt 3 P t two is twelve. So, P t 3 is equal to P t 3 by P t two into P t two. So, I told you know this thing we know this ratio we can directly get that values of P t 3. So, using polytrophic efficiency in the compressor we can evaluate the temperature ratio right the temperature ration we know T t 3 by T t 2. So, we know this polytrophic efficiency gamma is 1.4. So, you will get this ratio 2.305 right, and when you multiply it by this T t two with the this temperature ratio across the compressor, you will get as a temperature at the exit of compressor that is T t 3 is equal to 563.53 kelvin what will have to do. Now, basically will have to carry out an energy balance across the combustor and because we need to find out the fuel ratio; that means, air fuel or fuel air ratio right. So, this is basically fuel air ratio right f is equal to C P g T t 4 minus C P a T t 3 and eta b delta h C minus C P g T t 4. So, if you look at this is basically, if I look at that you know.

It is only eta b is coming into picture otherwise it is similar to that often ideal cycle right all those values we know, and then we will find out this is a quiet a very low value that is fuel air ratio, right.

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Then, we can determine $\dot{m}_f$ as

$$\dot{m}_f = 0.0315 \times 50 = 1.575 \text{ kg/s}$$

Turbine:

As there will be pressure loss in the combustor, the pressure at the turbine inlet P_{t4} becomes,

$$P_{t4} = \pi_b \times P_{t3} = 0.95 \times 186.43 = 177.11 \text{ kPa}$$

The total temperture at the exit of the turbine is to be evaluated. We know that the function of a turbine is to supply the requisite power to the compressor. Hence, we have

$$\dot{W}_c = \eta_m \dot{W}_t; \quad \Rightarrow \dot{m}_a C_{pa}(T_{t3} - T_{t2}) = \eta_m (\dot{m}_a + \dot{m}_f) C_{pg}(T_{t4} - T_{t5})$$

From the above equation, we can estimate the turbine exit temperature as

$$T_{t5} = T_{t4} - \frac{C_{pa}}{C_{pg}} \frac{1}{\eta_m (1+f)} (T_{t3} - T_{t2})$$

$$= 1600 - \left(\frac{1005}{1148}\right) \frac{1}{0.98(1+0.0315)} (563.53 - 244.43) = 1323.7 \text{ K}$$

So, then we can determine what will be the f because we know air fluid air fluid entering into the engine is fifty k g per second. So, the mass flow rate of fuel will be 1.575 k g per second. So, turbine you know we need to look at it because we need to find out T t 4 and for that we will have to find out P t four. So, and P t 4 is not same as the P t 3 because there will be losses in your combustors right. So, then we can find out the P t 4 is equal to pi b into P t three. So, that is basically 177.11 kilo pascal in ideal cycle it could have been same as that P f P t 3 that is 186.43 kilo pascal right there is a difference now because of losses incurred in the combustion chamber. So, the total temperature at the exit of turbine is to be evaluated right for that we must you know what you will call. Now how much power is given to ran the combustion for that we will learn that w dot t is equal to eta M into w dot C right, in ideal cycle eta M is equal to one, but in this case it is having some values of I think 0.98 right. So, if you look at I, if I write down this turbine this will be this is a you know eta M over here into turbine right because not all the work from the turbine will go to an 0.98. Of course, if it is eta M is 0.98 this will go right M naught a C P a T t 3 minus T t 2 is equal to eta M m naught a plus M naught f C P g T t 4 minus T t five. So, from this expression we can get a relationship between T t 5 which is equal to T t 4 minus C P a divided by C P g one of eta M one plus f t 3 minus t 2. So, we

know all this values right and we can determine that happens to be you know thirteen one 323.7 right, If you at the difference is you know around something two hundred seventeen 377 kelvin difference is there, right.

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Then, the total pressure, P_{t5}, can be determined as

$$\frac{P_{t5}}{P_{t4}} = \left(\frac{T_{t5}}{T_{t4}}\right)^{\frac{\gamma_g}{(\gamma_g-1)\eta_{pt}}} = \left(\frac{1196.1}{1600}\right)^{\frac{1.333}{(1.333-1)0.85}} = 0.422$$

$$\Rightarrow \quad P_{t5} = \left(\frac{P_{t5}}{P_{t4}}\right) \times P_{t4} = 0.422 \times 177.11 = 74.76 \text{ kPa}$$

Nozzle:

The critical pressure ratio across the nozzle is determine as

$$\frac{P_{t5}}{P_c} = \frac{1}{\left[1-\frac{1}{\eta_n}\left(\frac{\gamma_g-1}{\gamma_g+1}\right)\right]^{\frac{\gamma_g}{\gamma_g-1}}} = \frac{1}{\left[1-\frac{1}{0.95}\left(\frac{1.333-1}{1.333+1}\right)\right]^{\frac{1.333}{1.333-1}}} = 1.914$$

But, the actual pressure ratio across the nozzle with respect to the ambient pressure is

$$\frac{P_{t5}}{P_0} = \frac{74.76}{11.2} = 6.67$$

So, the total pressure we need to determine and we can find out because we know this T t 5 now and T t 4 is known. So, by using this relationship of course, for the non isentropic case event like because here using the eta P d it is polytrophic efficiency for turbine P t 5 by P t 4 is the you know can be related to temperature ratio across the turbine power to the gamma g divided by gamma g minus 1. So, you can multiply this values keep in mind we are using 1.33 instead of 1.4 per air and you know what is it.

So, that happens to 0.422. So, P t 5 we can determine very easily because P t 4 is known and this is already known. So, get that and for the nozzle you know we need to determine whether the nozzle is chocked or not right and if you look at we are basically considering as a as to be a conversion nozzle right and for that we need to determine the critical pressure ratio across the nozzle right. So, what we will do we will have to you know look at this expression, which we had derived in the last lecture P t 5 divided by P C equal to one over in the bracket 1 minus 1 over eta n in the bracket gamma g minus 1 divided by gamma g minus 1, then at the whole bracket power to the gamma g divided by gamma g minus 1.

So, if you look at if I put this think this is happens to be 1.914 right, of course it may vary little bit depending on the efficiency of the nozzle right generally efficiency of the nozzle is quite high right because the accelerating flow. So, the losses and other things will be not much, but; however, if there is a thrust formation over expansion under expansion and like you know kind of things or there is a separation right due to you know other attachments then or it is a kind of a thrust vectoring kind of thing. So, when you are doing. So, this kind of happens then it n will be different.

So, if you look at otherwise mostly it will be around 1.914 right these values provided eta n value is 0 0.95 right, so but actual pressure ratio across the nozzle with respect ambient pressure. What it would be that we need to determine that is P t 5 by P naught is P t 5 value, you know P naught is given will find this 6.67. That means, is it nozzle is choked or it is not choked it is much higher P t 5 P naught is higher than the P t 5 by P C therefore, nozzle is choked, right.

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Since $\dfrac{P_{t5}}{P_0} > \dfrac{P_{t5}}{P_c}$, the nozzle is considered to be choked. So, the static temperature at the nozzle exit T_9 becomes

$$T_9 = T_c = \left(\frac{2}{\gamma_g + 1}\right)T_{t5} = \left(\frac{2}{1.333 + 1}\right) \times 1323.7 = 1134.7 \text{ K}$$

Now, the static pressure at the nozzle exit P_9 becomes

$$P_9 = P_c = \left(\frac{P_c}{P_{t5}}\right)P_{t5} = 38.9 \text{ kPa}$$

By using equation of state, the density of gas at the nozzle exit is estimated as

$$\rho_9 = \frac{P_9}{RT_9} = 0.119 \text{ kg/m}^3$$

Hence, we can evaluate V_9 as

$$V_9 = C_9 = \sqrt{\gamma_g RT_c} = \sqrt{1.333 \times 287 \times 1134.7} = 658.87 \text{ m/s}$$

So, the static temperature of the nozzle you know exit t 9 becomes basically t C that is the critical temperature right that is two by gamma g power to T t 5. So, you will get that 11134.7 kelvin, how can you change this when the nozzle is choked right if it is choked you cannot really change your mass flow rate mass flow rate remains constant, but how can we change that we want to have suppose the pilot wants to change, that how will he do that there is no thrust this is a conversion nozzle. So, you will have to basically

change the exit area or you you will increase the upstream pressure right I can change the pressure. And then do that then only I can allow the more mass to flow, otherwise I cannot right of course, there is the several implication suppose it is choked and you are trying to push mass flow rate. So, what will happen and there is several implication of it. So, let me not get in to that. So, now the static pressure of the nozzle exit pressure P 9 becomes P 9 is equal to P C and P C P t 5 into P t 5 will get the 38.9 kilo Pascal, whereas your P naught was much lower than this right.

So, the using this equation of state you can find that density row 9 is equal to P 9 RT 9 keep in mind P 9 is same as P C is not P 9 is not same as the P naught as we are done in the ideal cycle. So, we can evaluate this V 9 which is nothing, but root over gamma g RT C which is independent of the upstream right, because the mach number is equal to one right. So, you can you know that what is the speed of sound. So, you can write down very easily root of all gamma g RT C, you need not to that expression part I wrote in the beginning that you should keep in mind which I happens to be 658.87 meter per second.

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Let us evaluate $\frac{A_9}{\dot{m}}$ using the continuity equation as

$$\frac{A_9}{\dot{m}} = \frac{1}{\rho_9 V_9} = 0.0127 \text{ m}^2\text{s/kg}$$

$$T_s = \left[(1+f)V_9 - V_0\right] + \left(\frac{A_9}{\dot{m}_0}\right)(P_9 - P_0)$$

$$= \left[1.0315 \times 658.87 - 236.06\right] + 0.0127(38.9 - 11.2) \times 10^3$$

$$= 795.77 \text{ Ns/kg}$$

In order to estimate TSFC, we will have to evaluate the fuel-air ratio as

$$TSFC = \frac{f}{T_s} = \frac{0.0315}{795.77} = 39.61 \text{ mg/N.s}$$

Propulsion efficiency, $\eta_p = \dfrac{2V_0 T_s}{a_0^2 \left[(1+f)(V_9/a_0)^2 - M_0^2\right]}$

$$= \frac{2 \times 236.06 \times 795.77}{236.06^2 \left[(1.0315)(658.87/295.07)^2 - 0.8^2\right]} = 95.82\%$$

So, now we need to evaluate a 9 by M naught a 9 by M naught, if you look at nothing, but your row 9 V 9 V 9, you know rho 9 you know, because you know pressure you know temperature ideal gas law you can use right. You can get that values which is happens to be 0.0127 meter square second per k g right. So, even though you do not know the a 9 and M naught you can get, because you can evaluate the V 9 and row 9

right, that we did not discuss, when we are discussing the real cycle analysis particularly carrying out the parametric studies right. And here, but while solving problem we need to understand this is some simply you know mass conservation equation. So, then we can get t s equal to one plus f, f; already we have evaluated V 9 minus V naught plus a 9 by M naught this portion. We have already evaluated P 9 by P naught substitute those values, you will get seven 59.777 Newton second per k g right, you just substitute this values you will get right keep in mind, that when you are doing this you need to be you know careful about the pressure, because it will be in s i unit, this is in kilo Pascal, right. So, therefore, you will have to otherwise, you will incorporate you know blender or error in that. So, in order to estimate t s f C, we can estimate the fuel air ratio, you know already we have done that t s f C is equal to f divided by specific thrust it happens to be 39.61 milligram Newton per second if you look at this is in s I unit. So, for milligram we will have to convert it will very small to that multiplied by thousand. So, the propulsive efficiency you will get which is you know in terms of specific thrust into the slide velocity divided by this portion in the denominator and when you substitute this values you will get 95.82 percentage, right.
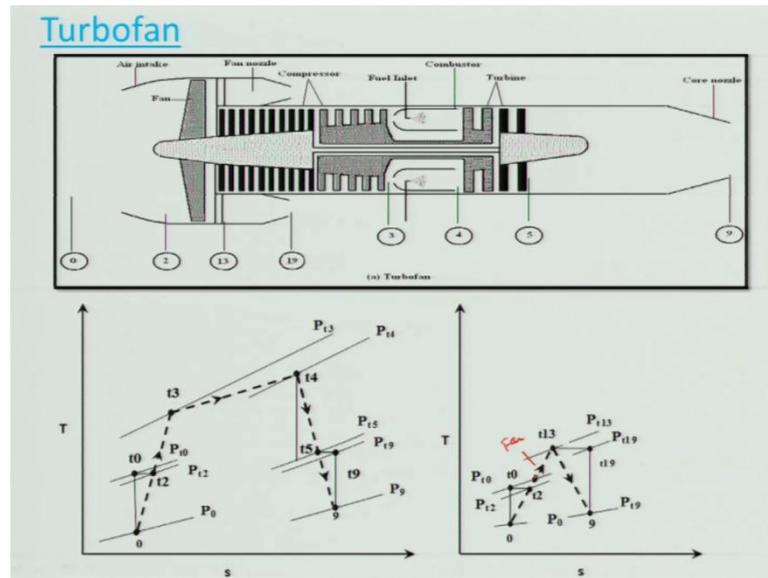
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The thermal efficiency can be evaluated as

$$\eta_{th} = \frac{\left[(1+f)V_9^2 - V_0^2\right]}{2f\Delta H_C} = 14.46\%$$

And similarly thermal efficiency will find for this example that is which happens to be one plus you know 14.46, because all those quantities we can know. So, we can substitute those values and we can get.

Now, what we will do we will basically look at the turbo fan engine right and we will not really carry out the same we did, but the salient features we will get and most of them will be similar to the turbo jet or the turbo fan which we had carried out for ideal and even the real cycle in case of turbo jet. So, this is a schematic which we have already seen let us look at it again it is having an air intake that is from station 0 to 2 and 2 to 13 is your fan and 13 to 19 is your fan nozzle, right. And what do you call basically if you look at two to 3 itself will be the compressor for the core engine the pressure ratio of the across the compressor in 2 multiplied by fan will give pressure ratio due to compression right and 3 to 4 is your compression chamber. And 4 to 5 is your turbine.

And of course, 5 to 9 is your nozzle right basically it is 7 to 9, but it is tele 5 which we need not to worry about nothing is happening here. So, the process is as you know in the t s diagram it is you know this is for the core engine the left hand side of the figure that is 0 to two is your what do you call compression in the air in the 2 to 3 is the compression in compressor. And 3 to 4 is heat addition and 4 to 5 is the expansion in the turbine and 5 to 9 is your expansion in the nozzle keep in mind, that for the fan it is similar one. That is basically compression in the air intake, and the fan this is basically fan portion station 2 to 13 and expansion you know in the nozzle that is 13 to the 9.

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So, the total thrust will be consist of two thrust one is primary stream thrust and the other is secondary stream thrust, right. So, one can think about you know that thrust will be basically total thrust will be M dot C V 9 by minus a naught keep in mind that this your primary you know thrust. And contributed due to momentum, and is having a pressure term that is a 9 P 9 minus P naught, similarly this is this portion is for the fan right that is M dot f V nineteen minus V naught is for contributed due to momentum change in the nozzle, expansion due to nozzle. And the a nineteen P 19 minus P naught is basically the pressure thrust in the fan due to expansion in the nozzle fan nozzle basically.

So, the total specific thrust you can you know express in terms of alpha, alpha is nothing, but your by pass ratio which we have already defined right and one plus f V 9 divided by a naught minus M naught M naught is basically mach number flight mach number and similarly in the case of this your what do you call pressure thrust right in core engine right and similarly for the fan. So, this is for the fan engine and this portion is your momentum thrust and this portion is your pressure thrust in the case of fan pressure thrust right and of course, even alpha is 0. Then it is nothing but your turbo jet engine right when alpha is equal to 0, because when alpha is 0; these terms the these terms will just go away right.

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By carrying out similar analysis, we can obtain an expression for both V_9/a_0 and V_{19}/a_0 as

$$\frac{V_9}{a_0} = \sqrt{\frac{2}{\gamma_g - 1}\left[\left(\pi_n \cdot \pi_t \cdot \pi_b \cdot \pi_c \cdot \pi_d \cdot \pi_r \cdot \frac{P_0}{P_9}\right)^{\frac{\gamma_g - 1}{\gamma_g}} - 1\right]}$$

$$\times \sqrt{\left(\frac{\gamma_g R_g C_{pa}}{\gamma_a R_a C_{pg}}\right) \frac{\tau_\lambda \tau_t}{\left(\pi_n \cdot \pi_t \cdot \pi_b \cdot \pi_c \cdot \pi_d \cdot \pi_r \cdot \frac{P_0}{P_9}\right)^{\frac{\gamma_g - 1}{\gamma_g}}}}$$

$$\frac{V_{19}}{a_0} = \sqrt{\frac{2}{\gamma_a - 1}\left[\left(\pi_r \cdot \pi_d \cdot \dot{\pi}_F \cdot \pi_{Fn} \cdot \frac{P_0}{P_9}\right)^{\frac{\gamma_a - 1}{\gamma_a}} - 1\right]}$$

$$\times \sqrt{\frac{\tau_r \tau_F}{\left(\pi_r \cdot \pi_d \cdot \pi_F \cdot \pi_{Fn} \cdot \frac{P_0}{P_{19}}\right)^{\frac{\gamma_a - 1}{\gamma_a}}}}$$

So, we will just look at by carrying out similar analysis we can get an expression for V nineteen right which looks to be quiet, but I have just put it. So, that you can see it, but you can we have already learned how get this kind of thing it is having various terms like pi n pi t pi b pi C pi d pi r, of course P naught by P 9 right and this you know you can if it is choked. Then it will be different value it will be P C by P 9 right and if it is not choked then it will be P naught is equal to P 9; that means, P naught by P 9 will be one, right.

So, that is he conditions and of course, if it is a generally conversion diversion nozzle is not being used in case of turbo jet engine, but turbo jet sorry turbo fan engines it is not being used generally conversion is used. So, either it will be chocked or it will be un choked kind of thing, but in case of a turbo jet engine this P naught by P 9. You know will be given provided, there is a convergent nozzle or it has to be estimated from someone right, whether it is a over expanded or under expanded it will be different value like. So, similarly we can get a similar expression for V 19 by a naught.

By carrying out an energy balance between the turbine, compressor and fan, we have

$\alpha = 0$, Turbojet Engine

$$\tau_t = 1 - \frac{1}{\eta_m (1+f)} \frac{\tau_r}{\tau_\lambda} \left[(\tau_c - 1) + \alpha (\tau_F - 1) \right]$$

where η_m is the mechanical efficiency. The expression for fuel-air ratio f for the turbofan engine as

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\frac{\eta_b \Delta H_C}{C_{pa} T_0} - \tau_\lambda}$$

Hence, the relation for TSFC can be estimated by

$$TSFC = \frac{f}{(1+\alpha) T_s}$$

The expressions for propulsive and thermal efficiencies can be derived in similar ways as

$$\eta_p = \frac{2M_0 \left[(1+f) V_9/a_0 + \alpha (V_{19}/a_0) + (1+\alpha) M_0 \right]}{(1+f)(V_9/a_0)^2 + \alpha (V_{19}/a_0)^2 + (1+\alpha) M_0^2}$$

And which if you look at it is having another parameter that is pi f and similarly tau f apart from because the fan pressure ratio and then fan temperature ratio has to be given. So, by carrying out energy balance between the turbine compressor and the fan you can get this expression tau P which is you know equal to one minus 1 by eta M 1 plus f tau r divided by tau lambda into tau C minus 1 plus alpha tau f minus 1. If you look at again similar thing when alpha is 0 right, when alpha is equal to 0 then you know turbo jet engine, right. So, this term will go away. So, it is same as that of turbo jet what we had derived earlier. So, the eta M is the mechanical efficiency the expression for fuel air ratio will be same as that of the turbo jet engine there is nothing much change, because you are adding fuel.

In the core engine alone, but there might be turbo jet engine where you can you know add some fuel it can be possible, but generally it is not being used I can have a combustor in the fan stream in to that you can imagine, it can be done, but generally people are not tried out that till. Now to my knowledge the best domain knowledge right, but the relation for the t s f C can be estimated in the similar way f by 1 plus alpha t s there is not much difference between there is no difference at all between this expression and the ideal cycle analysis there is no difference.

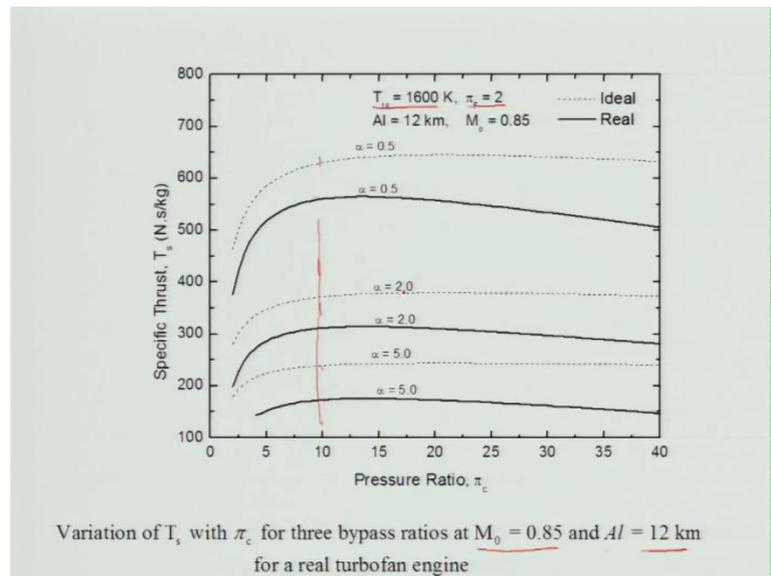$$\eta_{th} = \frac{a_0^2 \left[(1+f)(V_9/a_0)^2 + \alpha(V_{19}/a_0)^2 + (1+\alpha)M_0^2 \right]}{2f\Delta H_C}$$

Overall Efficiency,

$$\eta_0 = \eta_p \cdot \eta_{th}$$

Similarly if there is difference it is only eta b right expression propulsive in thermal efficiency will be similar in nature for the ideal cycle right because we are using the same expression. So, so as to thermal efficiency. So, overall efficiency is the multiplication of the propulsive efficiency and the thermal efficiency.

Variation of T_s with π_c for three bypass ratios at $M_0 = 0.85$ and $Al = 12$ km for a real turbofan engine
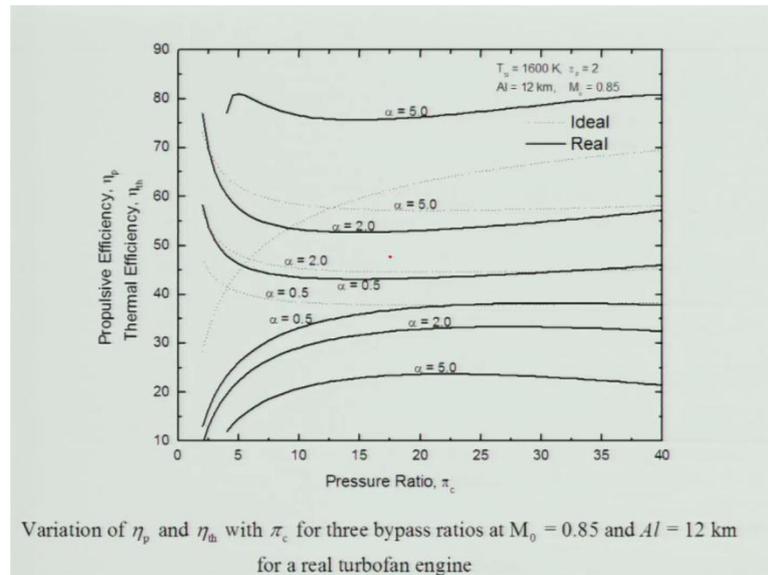
So, I am doing this you know relationship one carry out parametric analysis keep in mind that there are several parameters one can think of, because we know all those things we have carried out a parametric analysis for a turbo fan engine by considering alpha, that is

a bypass ratio changing from you know various values to the 5.2. Then one similarly fan pressure ratio can be changed similarly your flight mach number altitude and fan pressure sorry the pressure ratio across the compressor can be varied.

So, if you look at more number of parameters which can be varied then you play around look at what you can get out of it right. So but however, we will carry out certain you know certain parametric analysis by considering the flight mach number as 0.85 altitude as twelve you know twelve kilo meters for real turbo jet fan engine and T t 4. We are considering as 1600 kelvin, and fan pressure ratio two right and we are varying here the bypass ratio from 0.5 and then 2 and 5 3 bypass ratio. We have considered, but you can vary you could write a program just you know you give the values, it can calculate for you, you can see now here what I have plotted is a specific thrust versus pressure ratio across the combustion. If you look at that is dash line you know is the ideal one if I look at for the alpha is equal to 0.5 the specific thrust, you know increases with the pressure ratio and then reaches maximum value. And then decreases and you will get the similar one for the real cycle analysis only thing it you know it attains maximum value much before than the ideal.

One that is one difference, but most clearing difference is that it is having a lower value for the same pressure ratio, if I take this pressure ratio you know these thing it is having a lower values of the specific thrust right than the ideal one. And of course, it is the similar thing if I go for the alpha is equal to two then again it is having the similar trend only the real cycle analysis will give the lower specific thrust as compared to the ideal one. So, also for the alpha is equal to 5 we are seen similar trend for the turbo jet engine as well right and why it is. So, because you need to have the more mass flow rate for the same thrust right then only you can get because losses are more. So, your momentum and the velocity will be lower kind of thing.

Variation of η_p and η_{th} with π_c for three bypass ratios at $M_0 = 0.85$ and $Al = 12$ km for a real turbofan engine

So, you need o give or allow it to have more kind of mass flow rate to the engine, but whereas, if you look at the thrust specific fuel consumption, you will find that you know like in the all 3 bypass ratio 0.52 and 5, this thrust specific fuel consumption is higher than the ideal 1. Of course, the trend remain same right trend remain same in the sense like the thrust specific fuel consumption decreases with the pressure ratio across the compression, right. And it will be reaching some minimum values somewhere depending on the you have to find it out it is not very obvious from this figure, but however when you look at the data it will be clear right, and as I told earlier that, because the losses are higher in case of a real cycle. And which we have considered it is not that losses would not be there, but in real cycle we are considering it.

So, therefore, it is expected that thrust specific consumption for the same condition same condition I mean the pressure ratio across compressor turbine rate temperature fan pressure ratio everything remains same it is having highest value as compared to the the what do you call the ideal one right. So, let us look at this propulsive efficiency if you look at the propulsive efficiency I have plotted over here right for the alpha 0.5 if you look at the propulsive efficiency. Of course, increases little bit and then it decreases and again it reaches a minimum value in the end it, of course increases for alpha equal to five, but whereas for alpha is equal to two it also decreases. And also reaches minimum value there keep in mind that the propulsive efficiency in case of real cycle is much

higher as compared to the ideal one the ideal one is the dash line right and because as I told the velocity will be reduced.

So, therefore, you know propulsive efficiency will be higher in case of in a real cycle analysis, and also alpha goes on decreasing you know from 5 to this thing propulsive efficiency similar trend, you are getting whereas, for the thermal efficiency you know it is with the alpha it is will be remaining same kind of things. And it is what you call it is increasing and then decreasing and keep in mind that sorry I think I did a mistake like this thermal efficiency remains same and I think for the this thing and whereas, the overall efficiency, if you look at these are all the overall efficiency it is having similar values. Alpha is equal to 0.52 and point and 5 kind of things.

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Variation of T_s with M_0 for three bypass ratios at Al = 12 km for a real turbofan engine
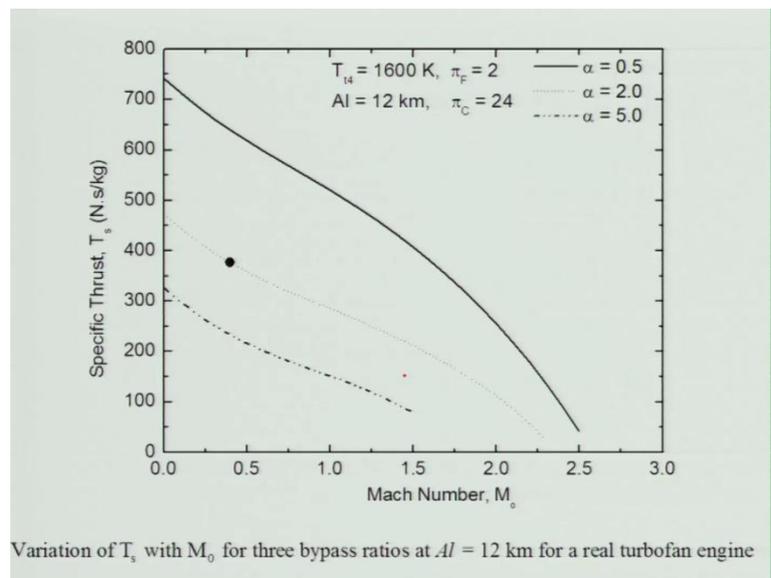
So, let us look at specific thrust for a with mach number what happens? Right. Specific thrust if you look at alpha is equal to 0.5, it is having a very higher values at the static thrust which is I mean mach number 0, and it goes on changing and you keep in mind the slope changes particularly in transonic region, you know the slope is like this. And then it is changing the slope here, it is different, and then here it is different just opposite. So, as the bypass ratio decreases the specific thrust gets decrease I mean I cannot compared with the ideal one, but you can see it is the very interesting thing which we did not do for the ideal cycle right this we have done for the real cycle. And for this we have considered the fan pressure ratio across combustion 24, sorry fan pressure ratio across

fan pressure ratio is two and the pressure across compressor is 24, we have considered that is for one case, but you can take for various cases.

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Variation of TSFC with M_0 for three bypass ratios at Al = 12 km for a real turbofan engine
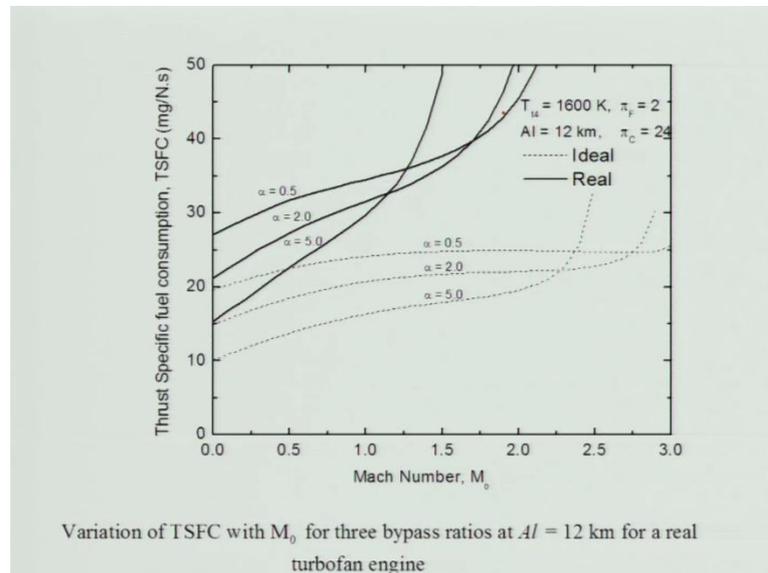
And look at it how it is happening, but if you look at thrust specific fuel consumption, you see that it is the real cycle is the solid lines alpha is 0.5, it is increases with the mach number and then you know again it changes the slope. And it increases as the mach number goes you know transition resumes higher mach number, it is the slope is very high. That means, at a higher rate the thrust specific fuel consumption increases and of course, as we you know increase this bypass ratio we see the different trends and get right and is it is similar to that of the ideal cycle. But however ideal cycle is having lower t s f C, which is obvious as I told that day we have not considered the losses, but the changes what is occurring here in this region. If you look at these are the transition region where the slope are changing as you are incurring the more amount of air, then naturally it is expected that the transition will be more towards the you know when the mach number beyond one right kind of things. So, with these we will stop over and I will take an example in the next in the turbo fan engine and look at it, how we can then we can move to the turbo prop engines.