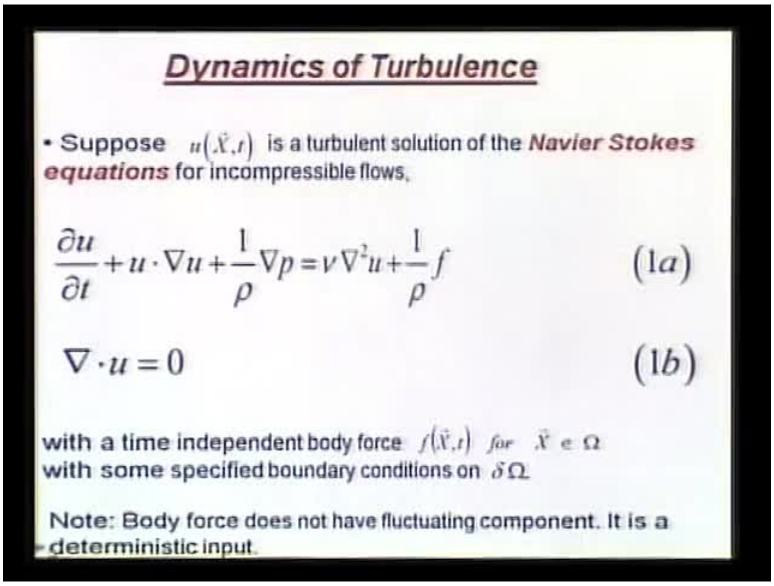
Instability and Transition of Fluid Flows
Prof. Tapan K. Sengupta
Department of Aerospace Engineering
Indian Institute of Technology Kanpur

Module No. # 01
Lecture No. # 36

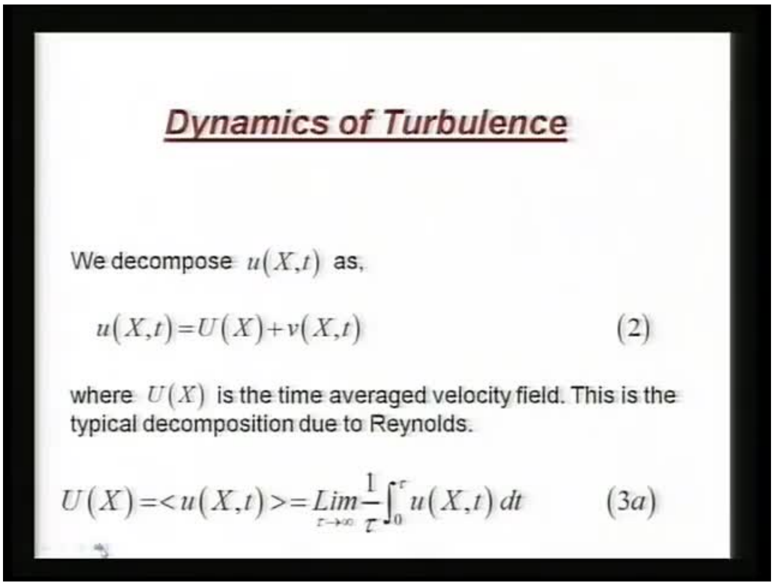
We are in our last lap on discussing about dynamics of turbulence.

(Refer Slide Time: 00:22)



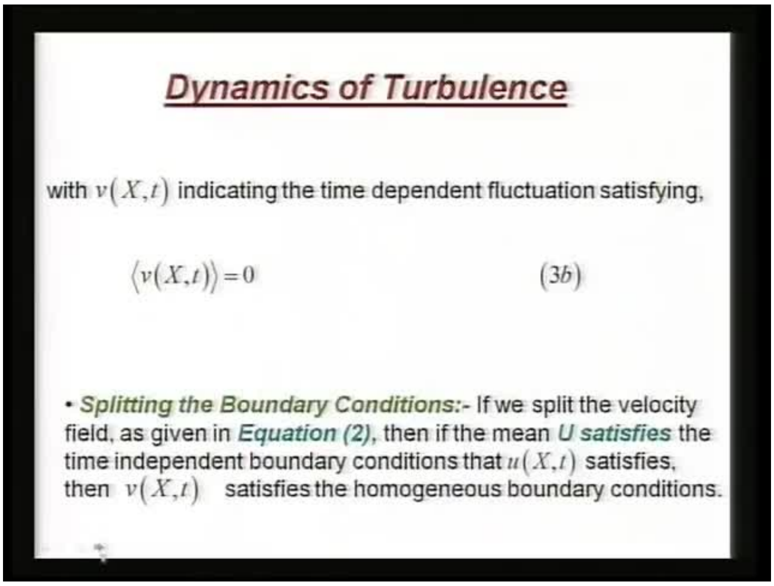
We say the turbulence will continue in phenomena governed by Navier-Stokes equation. This is the most generic form for incompressible flow. We could have a time dependent body force or it could be time independent. Even when it is time dependent, we noted yesterday, very specifically, that body force does not have a fluctuating component. It is essentially an input; deterministic input; so, we will give it in that respect.

(Refer Slide Time: 01:03)



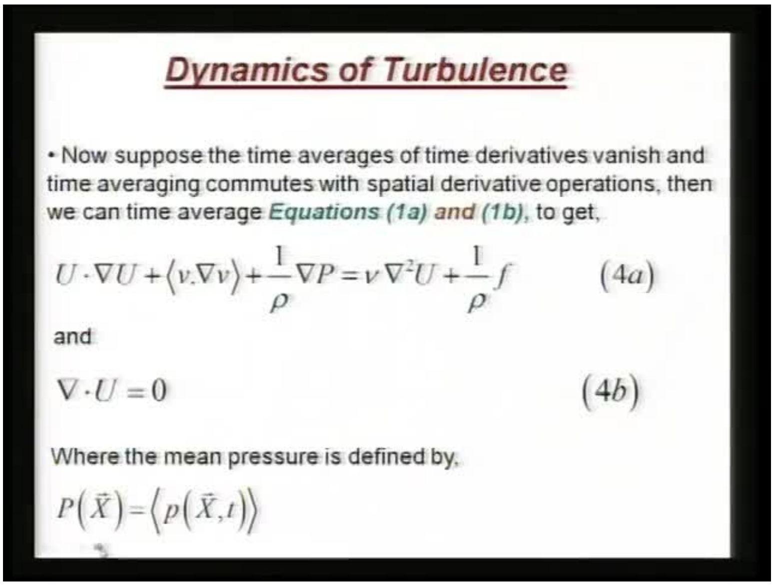
Then, of course, we performed this double decomposition of variable, as was originally suggested by Reynolds, which involves splitting the variable into a time averaged path on a fluctuation; time averaging is,, as defined here done over a span of time from 0 to tau and the tau going to infinity would perfectly define your time average..

(Refer Slide Time: 01:34)



Now, the other fluctuating component - we very specifically say that it is truly random; this was the point of view adopted by Reynolds as the answer to all those studies way back in 1880s. Today, we have a slightly different point of view. We know that all these flows although have fluctuating component underlying it, we have deterministic structure and that is what we discussed about in POD; from a stochastic dynamical system, we tried to come out with some kind of background deterministic structure.

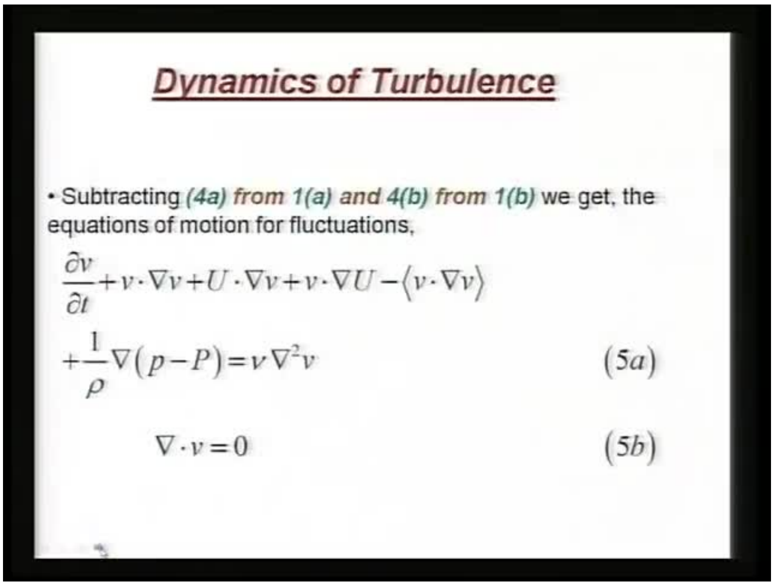
(Refer Slide Time: 03:11)



So, this is debatable then but, then we will proceed because most of our computing tool available today for turbulent flow is based on this assumption that the fluctuation is truly random. And we saw that along with this equation and with this kind of approach of defining the fluctuation as truly random, we can split the boundary conditions, put all the time independent boundary condition and the mean part that makes v or its any of its derivative to be satisfied by homogeneous boundary condition. And once we do that, we did derive in detail, the governing equation for the mean quantity.

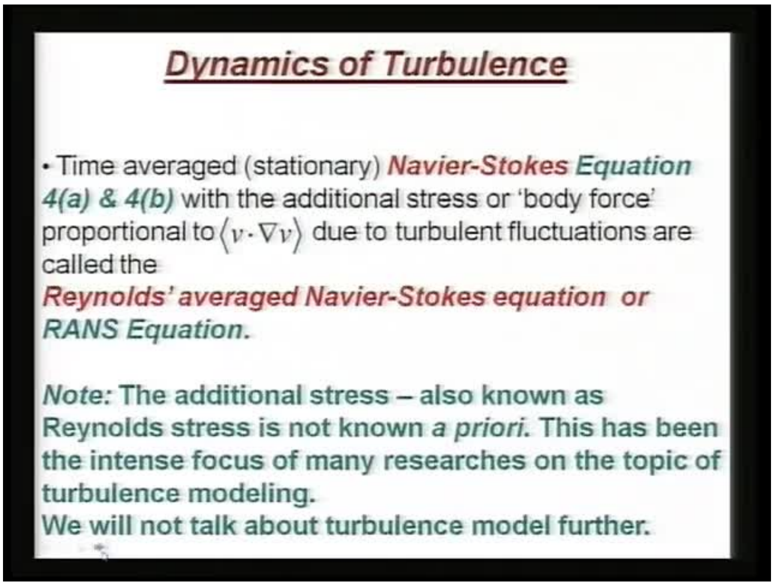
So, what we did? We took the Navier-Stokes equation; split this each of the variables into a mean part and a fluctuation, and then performed the time averaging over the whole equation. That led to this convective term for the mean and additional stress term came about which we called as the Reynolds stress. And this is the one area of research which supports that we need some additional information; per say, this equation is for the mean quantity; we do not have any sort of information apriori about the status of this. And what we figured then? That then to be able to solve this equation, we should have some kind of a model; that is what is called as a turbulence model.

(Refer Slide Time: 04:14)



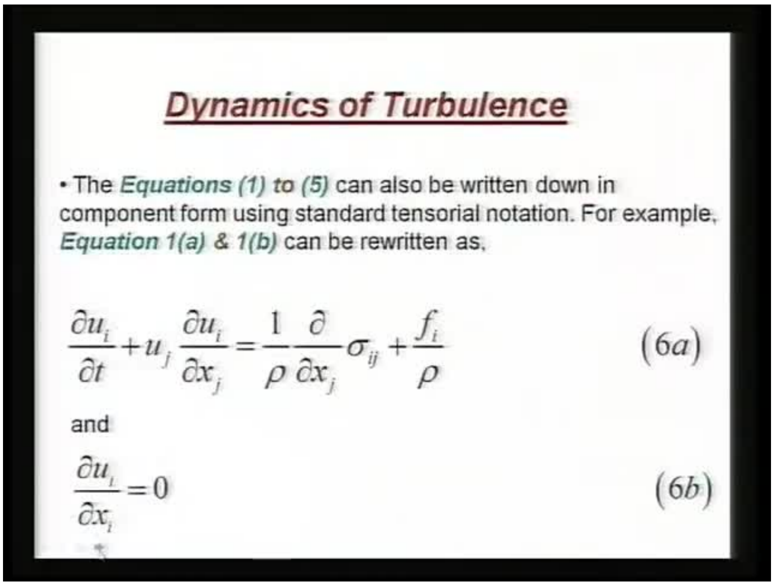
If we write down the equation for a mean quantity, similarly we can obtain the governing equation for the fluctuation and the governing equation for the fluctuation would be given like this. So, what you do is you take the full Navier-Stokes equation; from there you subtract the mean equation; that will give you this and what you see here that this fluctuation is then governed by the status of the flow as given by a mean flow and also by the pressure gradient. This is one interesting aspect of incompressible flow that you do not have any equation of state as such coming into play. What determines the dynamics is basically pressure gradient, not the pressure. So, this is one of the issue that we also have to keep in mind.

(Refer Slide Time: 05:20)



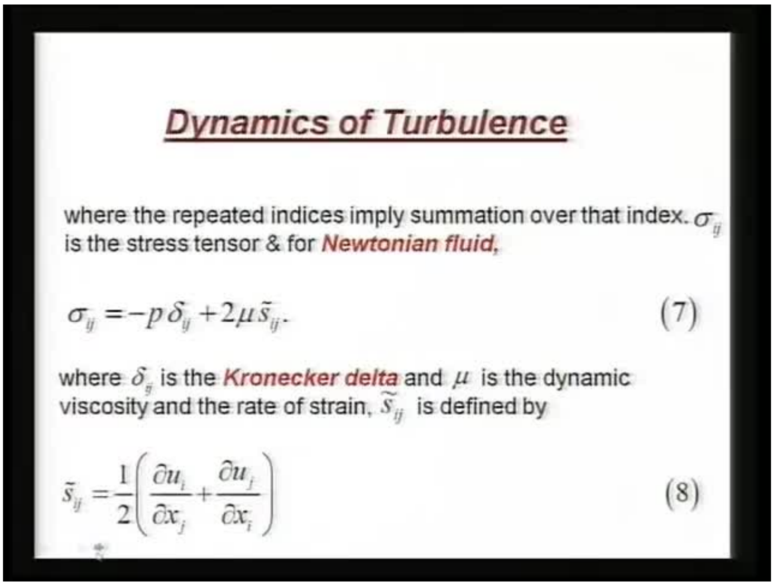
And once we decide what this is, then we can sum it up that the time average which is the stationary Navier-Stokes equation is what is called as the RANS or the Reynolds averaged Navier-Stokes equation or which requires modeling of this additional stress term for the fluctuation.

(Refer Slide Time: 05:50)



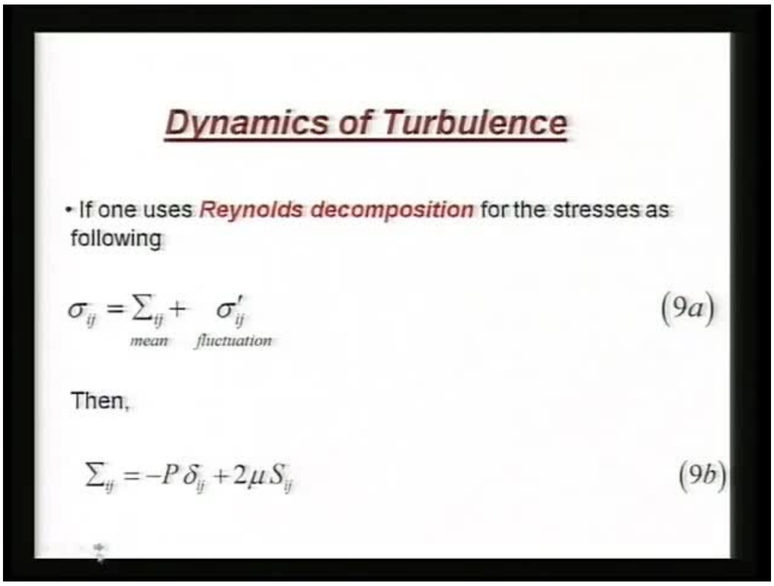
So, we basically would like to comment theoretically what is the status of this additional stress term without specifically going into any particular model. Now, we could write down those governing equations also in the tensorial notation in this particular fashion with the Cauchy equation written like this in terms as a stress.

(Refer Slide Time: 06:15)



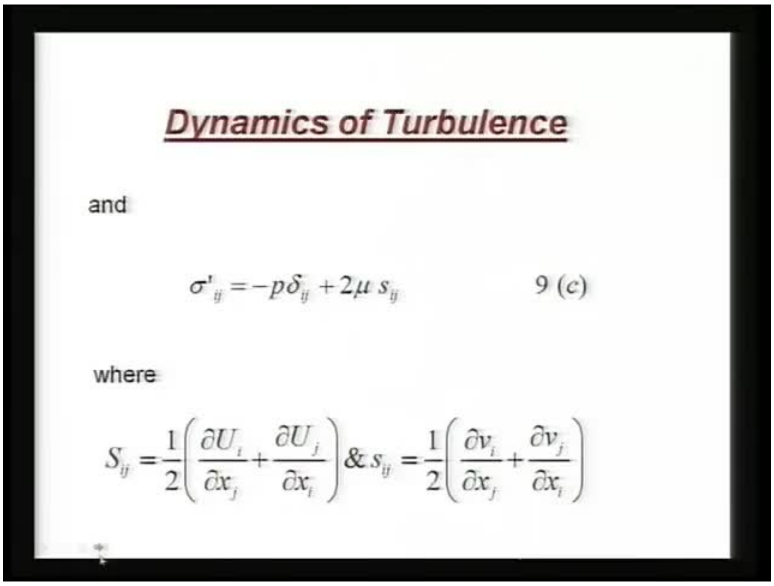
The stress itself is split into a mean and a fluctuating component and that is what we write here; the stress itself has two components; the hydrostatic part given by the pressure and the total strain, total strain - rate of strain, which is defined here in terms of the velocity field.

(Refer Slide Time: 06:39)



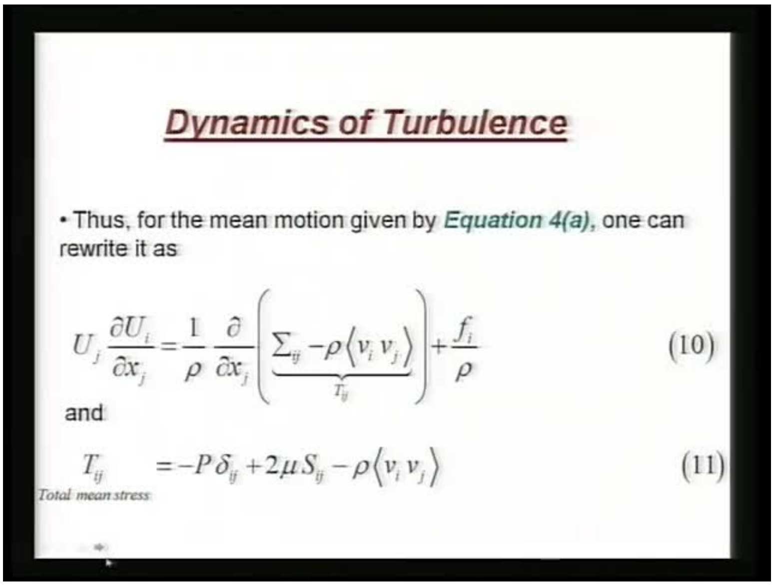
Once we have this, what we could do is, as I said that, you could split the total stress into a mean part on the fluctuation part; the mean part would come from the mean pressure hydrostatic pressure and the mean strain rate given by equation 9B.

(Refer Slide Time: 06:56)



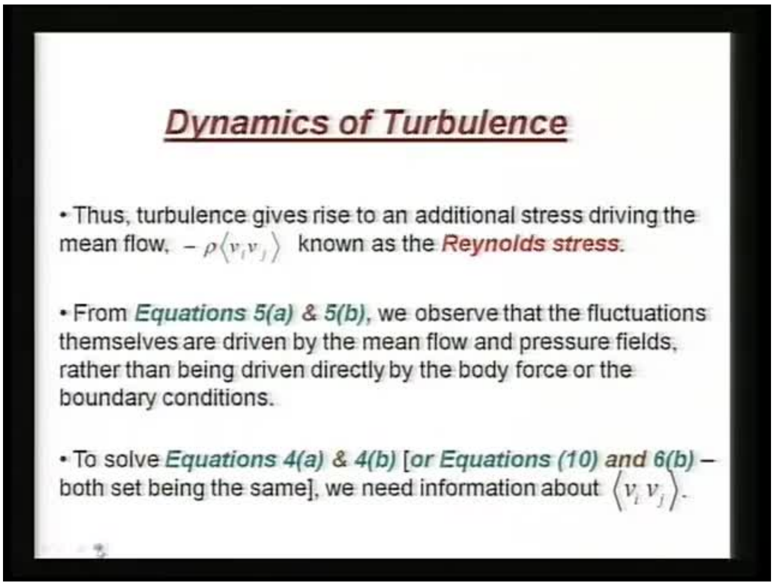
And the fluctuating stress could written in terms of fluctuating pressure and the fluctuating rate of strain. So, they are those given below. The strain rates are defined there.

(Refer Slide Time: 07:08)



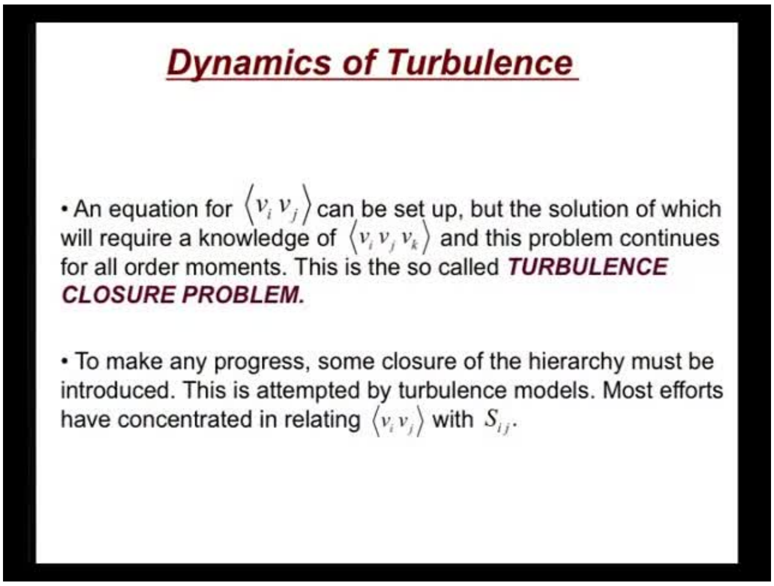
So, there is this alternative form of time averaged equation which you could write it in this form; the convective acceleration is balanced by the stress gradient on the body force and the stress itself is written in terms of the mean hydrostatic pressure and a path proportional to the mean strain rate and Reynolds stress term.

(Refer Slide Time: 07:36)



Now, we did talk about the fluctuating quantity that it is driven by the mean flow and the pressure field. Body force does not come into play; neither would the boundary condition have any say on determining the turbulent fluctuation. And we need the information about the Reynold stress.

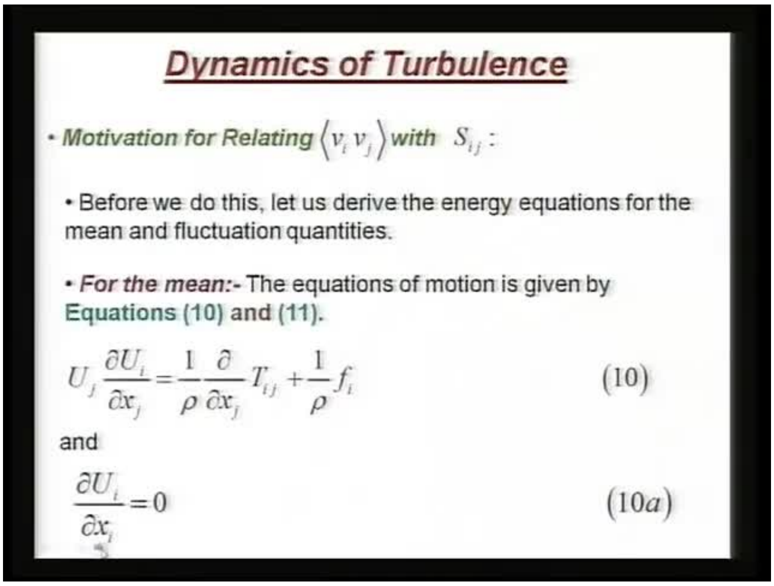
(Refer Slide Time: 08:09)



And this would be interesting to note that if we want to write down a governing equation, dynamical equation for this stress, we see that that equation will involve a triple co-relation term and this is what we called as a turbulence closure problem; a turbulence closure problem - meaning thereby that at no level we should be able to close the system; every system will have a high reductive relation term. And I also explained to you that people had been looking at it from a different perspective, with the hope that at some higher level, this stress time higher correlation may drop off. That is what people have done experimentally, but not to much great success.

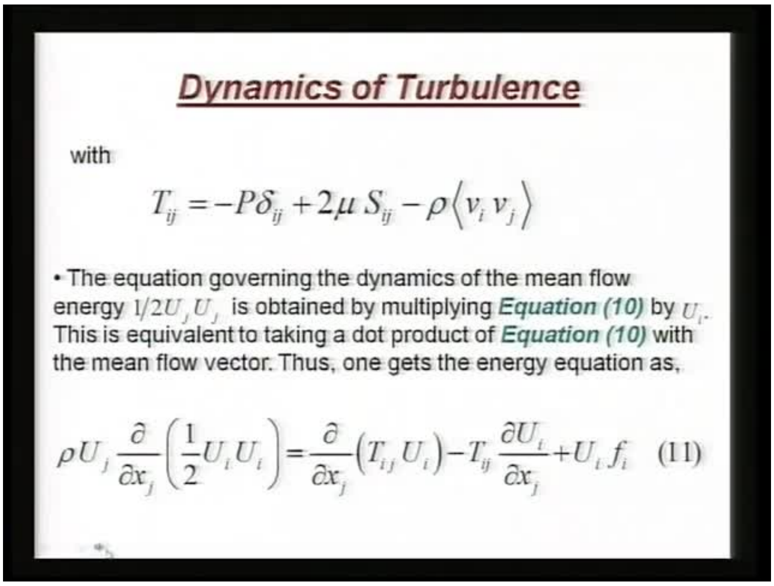
The complementary aspect would be that you do not try to write down this equation and try to get the triple correlation etcetera, but go down once; you look at this and try to relate it with the mean strain rate. This is, essentially, what is done in most of in the computations.

(Refer Slide Time: 09:23)



After we influence that we try to relate the mean stress mean strain rate with this Reynolds stress. To basically understand how we can do that, we need to understand the energy equation that defines the mean flow and the fluctuation quantities. When you look at the mean flow, these are the governing equations that we have seen before.

(Refer Slide Time: 09:55)



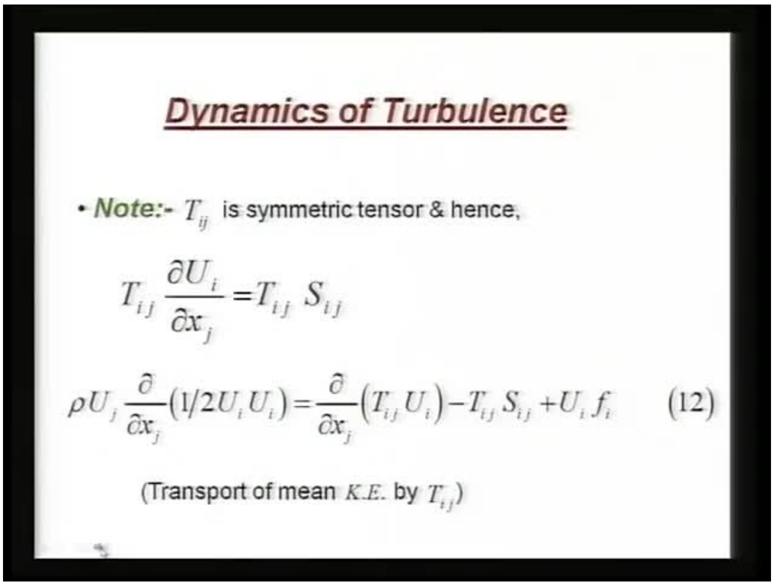
To derive the energy equation for the mean flow, what we do is take a dot product of this equation with the velocity vector and then we will get an equation for a quantity like this

- half of U j times U j. So, that is your kind of kinetic energy, specific kinetic energy and once you write that take the dot product and write it down, you get this.

What does it tell you? See, of course, we did that this does not depend on time, but just for the sake of completeness, I could add a del del T of half U i times U i.

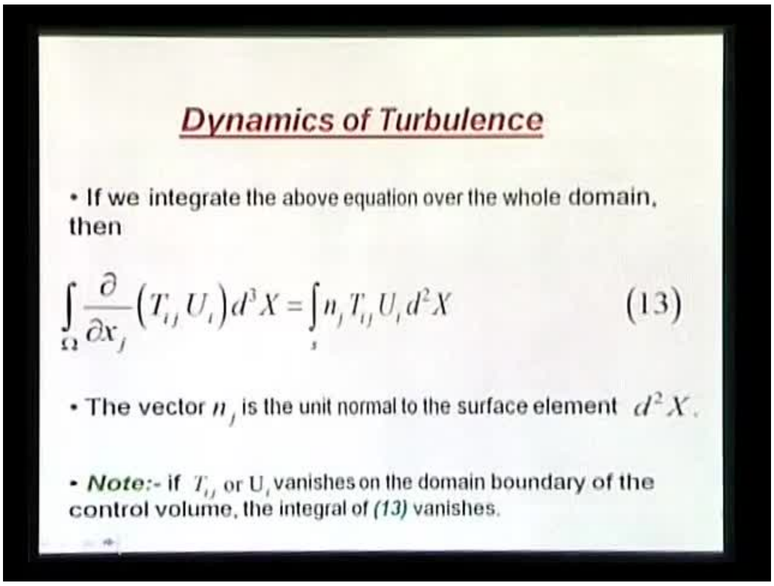
So, then, the left hand side will be what? The substantive derivative of the kinetic energy, specific kinetic energy; that is driven by this (Refer Slide time: 10:46). What is this term? This is like your gradient transport because here j is repeated; j is repeated. So, it is like del dot - the divergence term. There will be this additional term that will come about; we will see what it is. This is basically stress; this we can see insulated to the strain. So, this is stress times strain; that takes away the energy from the mean motion and this is the work done due to the body force (Refer Slide Time: 11:27).

(Refer Slide Time: 11:32)



So, look at look at that term, I told you that which involves the product of the stress times this velocity gradient. Since this stress tensor is symmetric, so this term we could just simply write it like this because what is S ij? S ij is half of this plus del U j del x i. So, when I take the product, they will be synonymous. So, this is what we are getting and this is what we are noticing, but this is basically your transport of mean kinetic energy by T ij. That is what we are looking at in this equation.

(Refer Slide Time: 12:35)

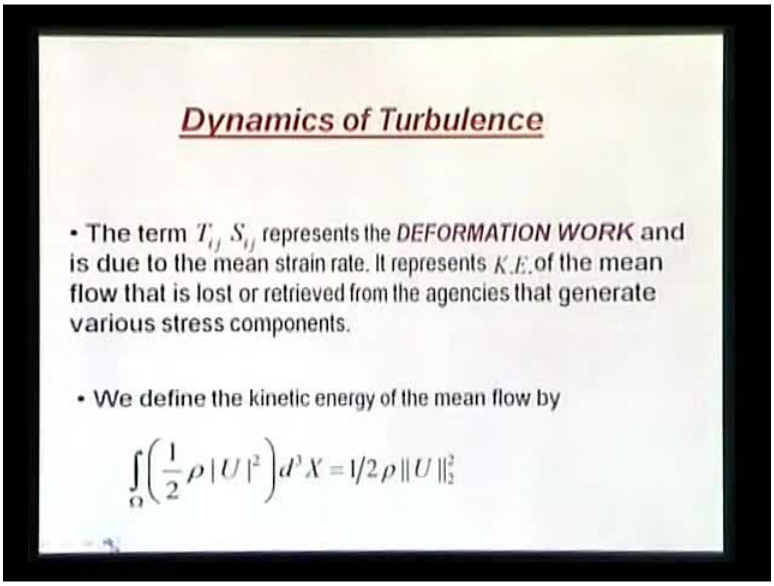


Now, what we need to do is we could look at the energy over the whole domain or we could look at it at a point wise. So, let us take a point of view of viewing the energy over the whole domain. Basically, it is all the two dimensional space that we integrate. What does this term give you? This is from the Gauss divergence theorem.

So, this divergence term is nothing but the fluxes; those are coming in through the control surface. So, if this is done over the whole thing, this is your control surface integral n j is nothing but the unit normal of this surface element, area element, which your term got as d 2 X. Now, you can observe that if you have taking a domain which is very large enough, you go very far away then what happens is that this tau i j is the stress; stress may vanish.

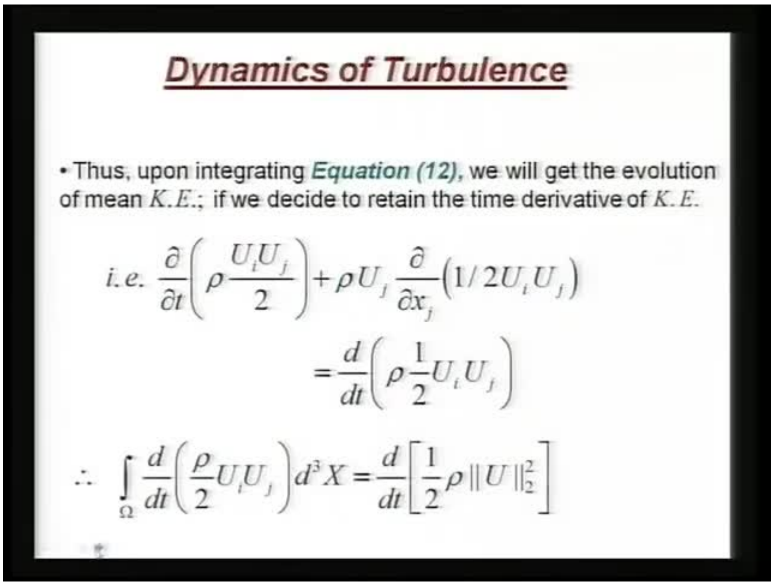
If I go to the uniform flow part, the stress may vanish there. Or we could look at the solid body where this stress is non-zero. But because of no slip condition the velocity itself will vanish. So, in either of the case, whether you are looking at the solid boundary or the far field boundary, the left hand side will contribute to 0. So, when you are looking at this, then what do you notice? That kinetic energy that is transported by T ij, if you sum it over the whole volume, it does not really (()) with the flow to other but, overall consider the whole a volume; this does not do anything.

(Refer Slide Time: 14:26)



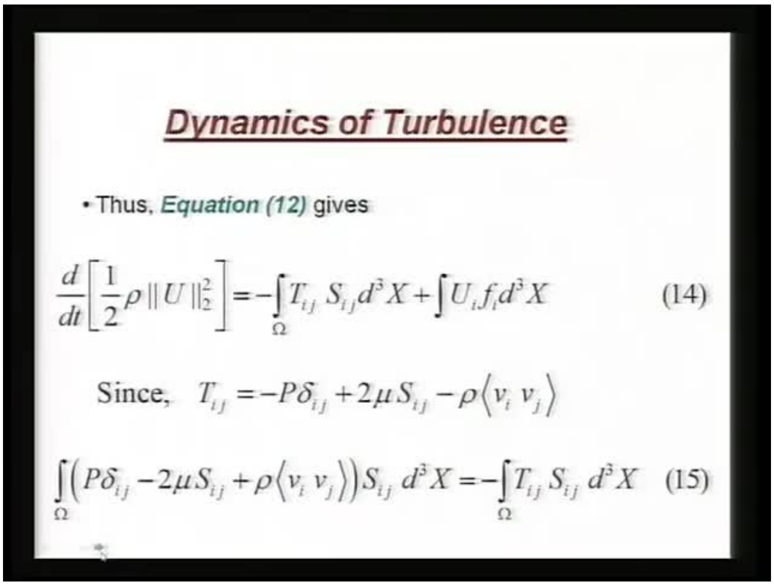
This term, the other term that the second term on the right hand side in it is to a product of stress time strain rate. This is what we are very familiar with; this is your deformation work - an elasticity we have studied. That is what we get stress time strain is your deformation work. If there is no deformation, if there is no strain rate, then this term will be 0. So, it represents basically kinetic energy of the mean flow that is either lost or retrieved from the agencies that generate various stresses. So, what we could do is we could define the kinetic energy also in terms of a known… These are definitions; so, do not have to worry too much about it. It is half rho U square integrated over the whole volume, we write. It itself has a half times rho and this; so, that defines your norm for the mean velocity field.

(Refer Slide Time: 15:29)



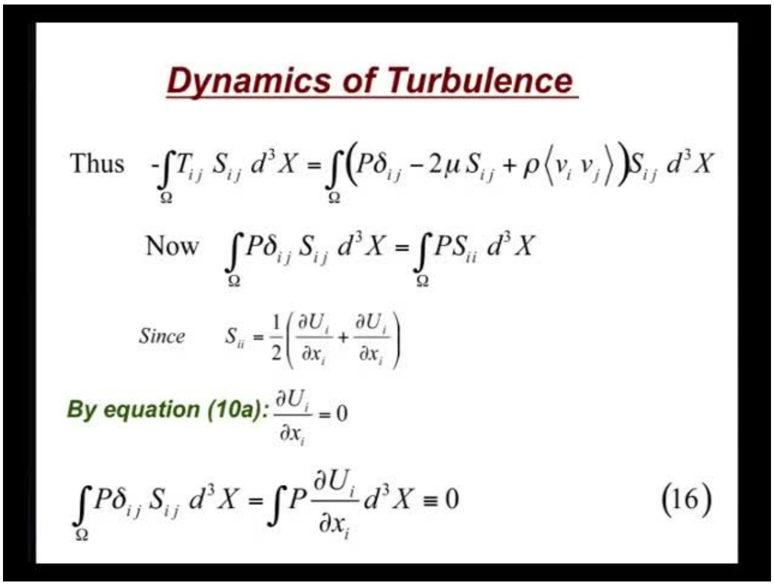
And then we could write down the equation in this particular form. As I told you we could add on this term without loss of any generality. Then what happens is this plus this will give you this total derivative (Refer Slide Time: 15:47). So, that is what we have written. So, that again, we can perform the integral over the whole volume and define it in terms of the norm.

(Refer Slide Time: 16:03)



So, this is basically the time rate of change of mean energy; that is what we are talking about. So, that would basically then come from this two components - deformation work and the body force. And we can take a closure look of what constitute this deformation work. I could write down the stress in terms of the hydrodynamic pressure, hydrostatic pressure, the mean strain rate and the Reynold stress, and this is what is done.

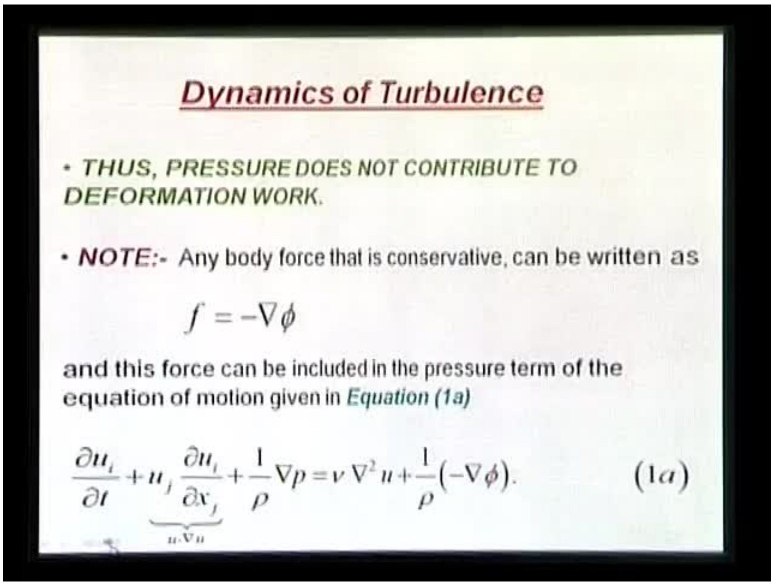
(Refer Slide Time: 16:46)



So, basically, then T ij S ij is integrated over the volume. We will have these three components and let us look at one of the components namely, the pressure term. How does that sum hydro static mean pressure relate to this S ij? Note that this has a delta ij. So, if I am doing this, this will only contribute from the trace of the rate of strain. So, (()) the diagonal element of the S ij matrix; so, that will be S ii. So, S ii is by definition is this. And what is this? This is this (Refer Slide Time: 17:17), and what is this? This is mass conservation equation for incompressible flow; this is 0.

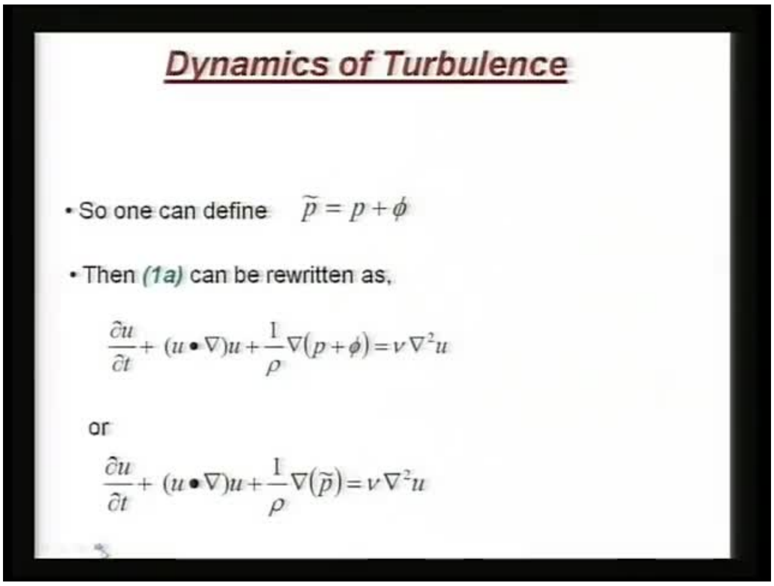
So, what happens then? Then this quantity does not contribute; so, pressure does not contribute anything to deformation and you can guess it also, because it is a hydrostatic pressure, it is isotropic, you cannot create a strain rate. Why? You can only create a strain rate in what way? By changing the column. But we are talking about incompressible flow. That is why this does not contribute. You saw that because it comes out from S i i. If we are looking at compressible flow, S ii is non-zero. Then, pressure will contribute to deformation work. But the moment you talk about the incompressible flow, you exclude the possibility of any compression or dilation dilatation.

(Refer Slide Time: 18:18)



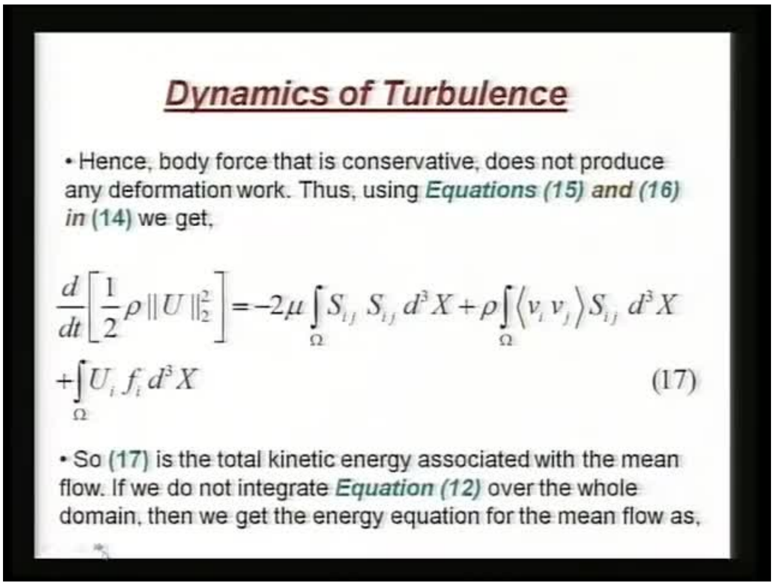
Then, of course, you would not get that. Note also that when we are looking at body force and consider that as conservative. If it is so, then you can write it as a gradient of a scalar, with a minus sign indicating it is conservative. So, if I do that, I could rewrite the dynamics of the overall quantity the Navier-Stokes equation; so, instead of f, we have written minus this. So, what you could do is we could take this and this because we have two gradient terms: one is due to the pressure; other is due to the body force.

(Refer Slide Time: 19:06)



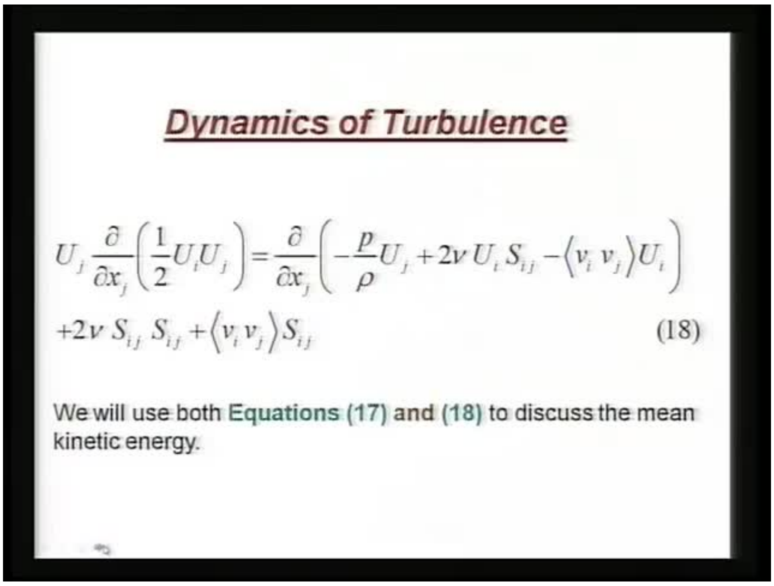
And we can actually redefine an augmented pressure field which has the hydrodynamic part plus a contribution coming from the body force and this is what we are going to get. We have already shown that for incompressible flow, the pressure does not contribute to deformation work. So, having a conservative body force, that also would not contribute anything to deformation work. So, this is something we must understand.

(Refer Slide Time: 19:43)



Then, what we are left with is that rate of change of mean motion; kinetic energy would be given in terms of two terms. What is this? This is a viscous deformation work; that is what where mu is, and S ij times S ij is a quadratic in the mean strain rate and this is the work done due to the Reynold stress. And this, of course, if we keep talking about non-conservative body force, it will (( )) also be there. So, this is one of the way of writing out the total kinetic energy associated with the mean flow.

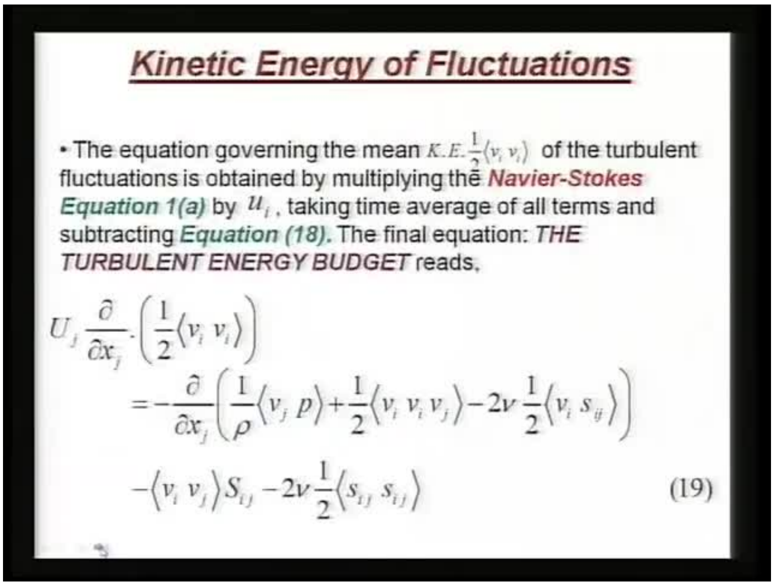
(Refer Slide Time: 20:42)



We could also write it as a point property instead of integrating over the whole domain and then we would be writing it like this. This is also something we understand. Once again, you notice what? This is that your gradient transport. So, this is the divergence form; there is j; there is j here (Refer Slide Time: 20:57).

So, if we integrate over the whole volume and if we can somehow show that this vanishes either in the solid boundary or in the far field boundary, this will not contribute. What term would contribute to this from the mean strain rate and the eddy viscosity? So, either of this form would be amenable in our discussion for mean motion; mean kinetic energy.

(Refer Slide Time: 21:31)



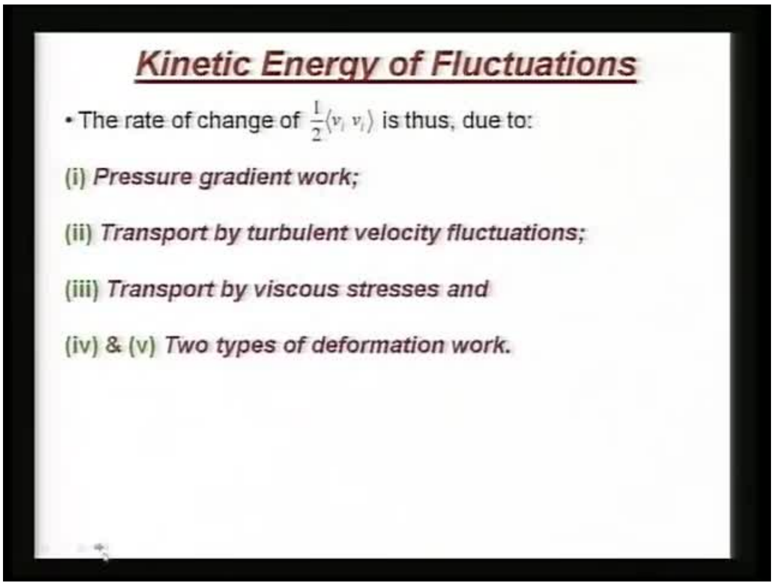
Now, we could turn our attention to kinetic energy of the fluctuation. We talked about the mean version. Let us now talk about the kinetic energy of the fluctuation. Here, we would be talking about the mean kinetic energy of the fluctuation given by half of v i dot v i and this is the time averaged quantity. So, we indicate it by the enveloped bracket.

Now, if I look at the Navier-Stokes equation and multiply by u i, that is basically taking a dot product to the velocity vector. Then we take the time average of the resultant equation. From that, we subtract the kinetic energy equation that we have written for the mean motion in the previous transparency, which you wrote it for a single point (Refer Slide Time: 22:31).

So, this equation is also written for a particular point. So, from this equation is obtained by subtracting this mean part from the instantaneous; that gives you this. So, this is what it is. So, you find out that the turbulent energy budget so it is calling. We are calling it turbulent energy because it is associated the fluctuation and there we have this convective term. We could also add a time dependent part of this stress. But do understand that Reynold stress term could be time dependent; it is not that it is a sort of an ornamental term which is identically equal to 0.

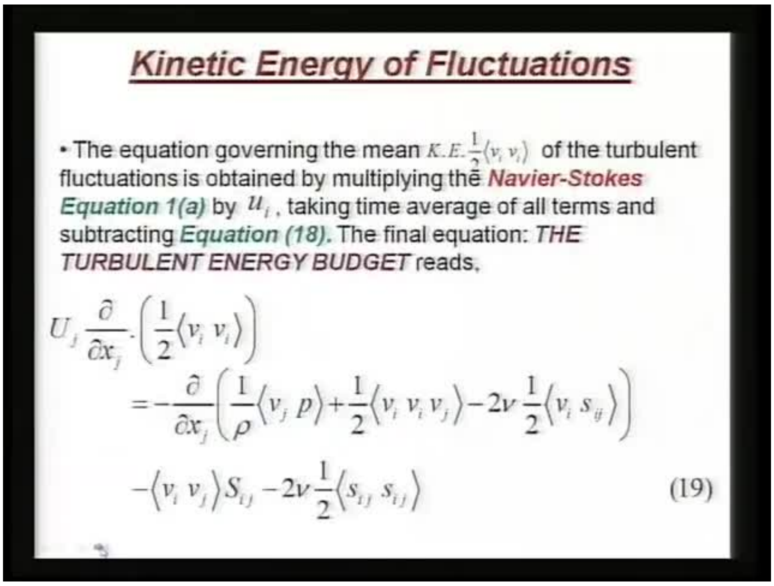
Here, it could be necessarily be there and once again we have these three sets of terms: one is the gradient transport term; we can see it. The second one is what the Reynold stress interacts with the mean strain rate and what is this? This is d 2 - the viscous the spatial; that is what it is 2 mu nu times S ij times S ij; S ij lower cases ij that stands for fluctuating strain rate. So, that is what we are going to get.

(Refer Slide Time: 24:00)



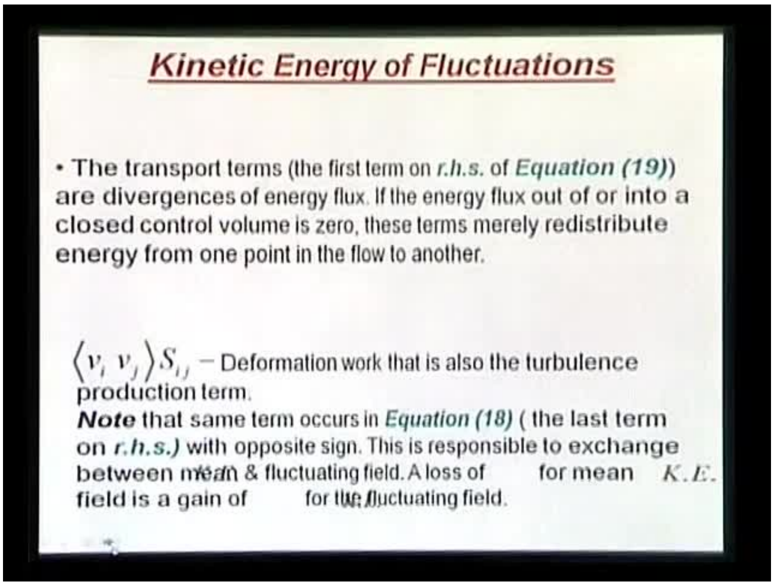
So, if then, look at it. Then, the kinetic energy of fluctuation can be dropped about by pressure term, pressure gradient term. Then, we could talk about transport by turbulent velocity fluctuation from your transport by viscous stresses and two types of deformation work. So, let me just move back one step and then this is what we are talking about.

(Refer Slide Time: 21:31)

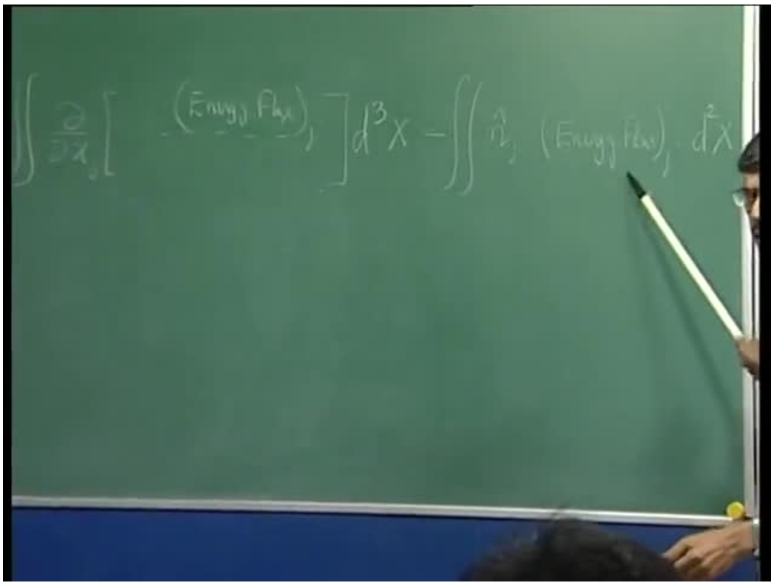


So, we talking about the pressure gradient term that is coming above and this is the transport by turbulent fluctuation term and this is your viscous losses and these are two sets of deformation - one is due to the mean; other due to fluctuation; that is what. We just cataloged it in as a summation of three 5 terms that come about.

(Refer Slide Time: 25:00)



(Refer Slide Time: 25:25)

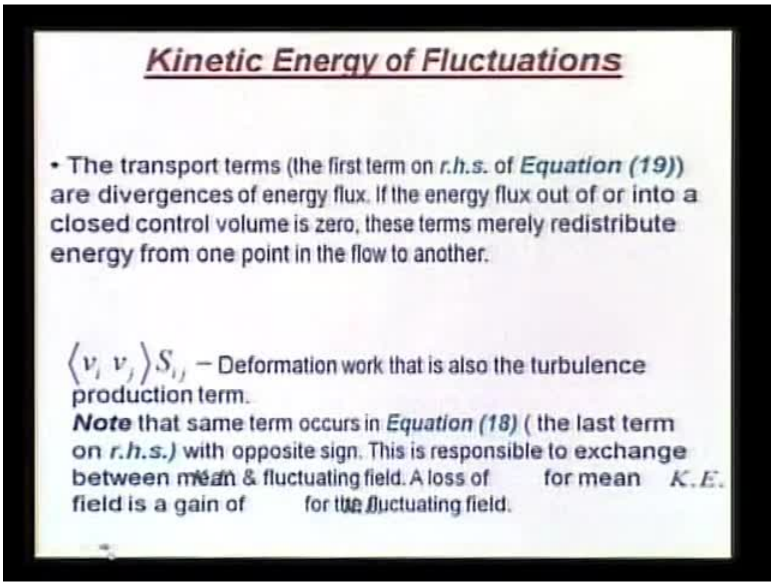


Now, if I look at the transport term that is the del x ji of that whole quantity a real (( )) divergence of a energy flux; that is what we have. So, basically we are writing here is nothing but, the del del x j and some quantities which are this. So, this is what we talked about as the energy flux; well, energy flux means, we are talking about the fluctuating energy. So, this is the divergence of this is your divergence operator (Refer Slide Time: 25:54). In this we are talking about the jth component; so, that is what we are getting.

So, if I talk about this energy flux out of a into a closed control volume, this is going to be 0 because if I take a very large volume outside, again the fluctuation would go to 0 there. If I go far away from the physical body, on the body the fluctuation themselves have 0.

So, what happens is this term will not contribute if I integrate it over whole volume. So, that is that is because you write it. So, this is your volume integral that you will write it as your area integral and then you will be writing say n j unit vector into vector flux, that energy flux that your integral and this will be this. And this quantity is going to be 0 and all the boundaries…

(Refer Slide Time: 25:00)



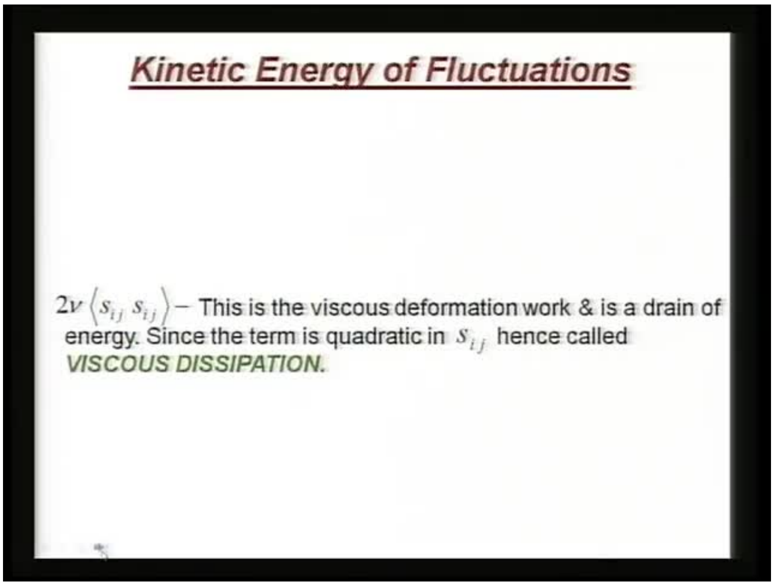
So, then you are looking at a solid boundary or the farther boundary. Also, that we are saying that contribution will be 0, this term only redistributed energy from one point in the flow to another. The two sets of deformation work that we had, one corresponded to the Reynolds stress times the times the mean strain energy. Well, this basically tells you all this is getting created and where is it getting created from? The mean field itself.

So, this is some kind of a production term. This is how turbulence is created. So, energy is transported from the mean to the fluctuation field. And if you actually look at this term and the similar term in the mean motion, you will see they are of opposite sign. So, whatever is lost to the mean motion is there here in the fluctuation energy.

So, that is why we are writing this we had seen that what is happening I mean it is not like what we did in the study of instabilities in the instability studies we just presume that the disturbance growth occurs because there is always there unlimited source of energy from the mean, but here the disturbance quantity the fluctuation quantities are of the same order of magnitude as the mean.

So, we will have to look at it together side by side and that is what I am suggesting. I am showing it to you that this kind of production term that we see in the kinetic energy of fluctuation, we get a same term, but with opposite sign in the mean energy equation. This is the same thing. So, basically, what we are getting is exchange of kinetic energy between mean and fluctuating quantities, and gain for one is a loss for the other.

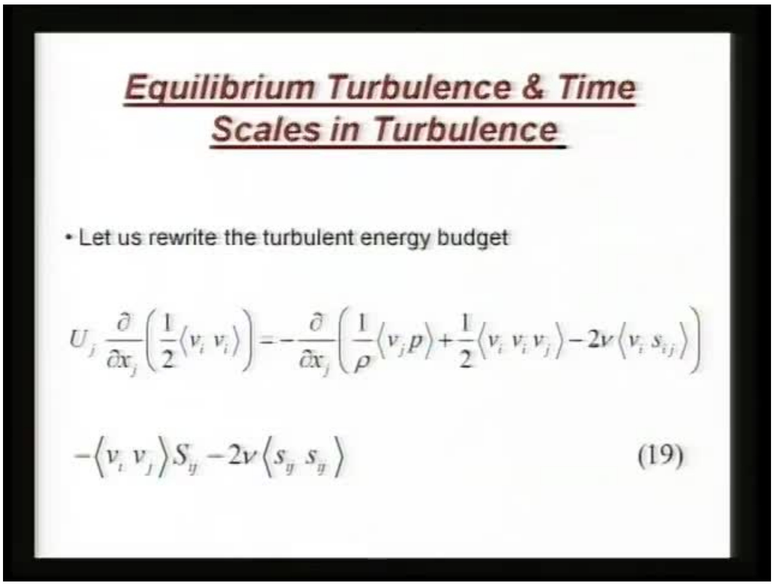
(Refer Slide Time: 29:29)



So, we could then finally, focus upon the last term. The last term is this. This is nothing but, 2 nu kinematic viscosity times s ij times si. This is truly the viscous deformation work and is a drain of energy. And what is it this? This is a quadratic term. So, it is always going to show you as a loss. So, that is why we will be calling it as a viscous dissipation.

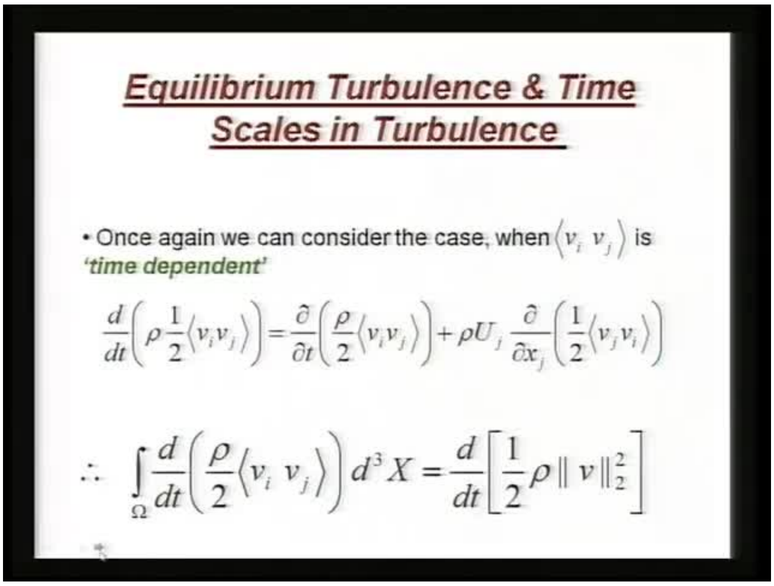
I do not need to emphasize it again to you that this term contribution coming from the fluctuating strain rate will be much more than a mean strain rate. Why? Because the fluctuating quantities are occurring over the smaller scale. So, the rate of strain tensors are going to predominantly stronger compared to the mean eddies; mean eddies will be larger; fluctuating eddies will be smaller. So, that is why, do not think that this a lower order term. In fact, it is a higher order term. This is one of the reasons the turbulent flows are considered to be more dissipative than corresponding laminar flow because the presence of this term. So, this is something you must keep in mind.

(Refer Slide Time: 31:01)



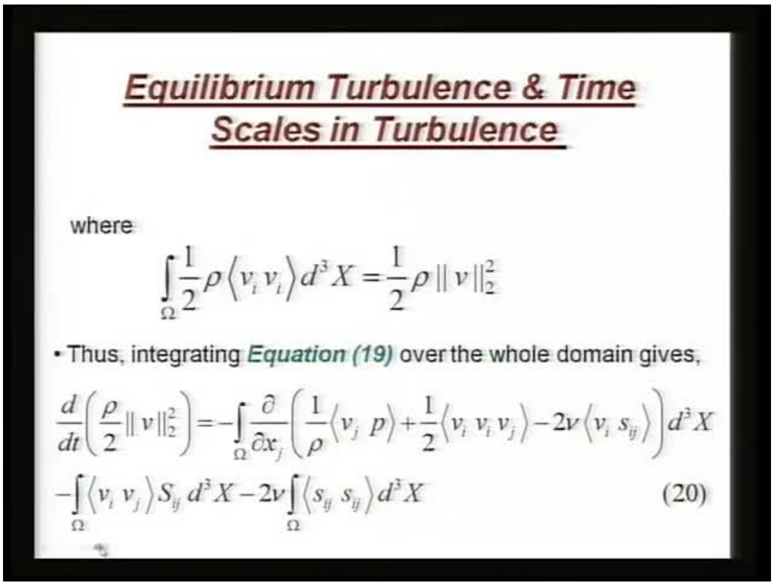
Now, what we are going to do is, now we will go over to the next part for discussion. We are now being able to write down in all the things that we wanted to do. The turbulent energy budget is given by this equation. We just now see in this term; so this term is what we have written here and this was a turbulence production term and this is a viscous dissipation term. That is what we see when we look at a turbulent kinetic energy.

(Refer Slide Time: 31:44)



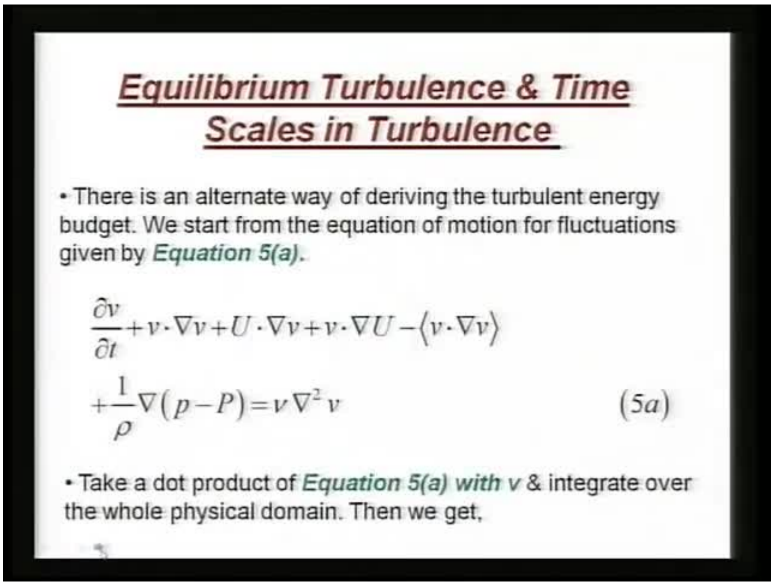
So, consider the case when this Reynold stress is time dependent. Then, we can talk about this rate of change of that turbulent stress would have this local convective term and that if we integrate over the whole volume once again, we will have (( )) in defining a norm for the fluctuating quantity.

(Refer Slide Time: 32:20)



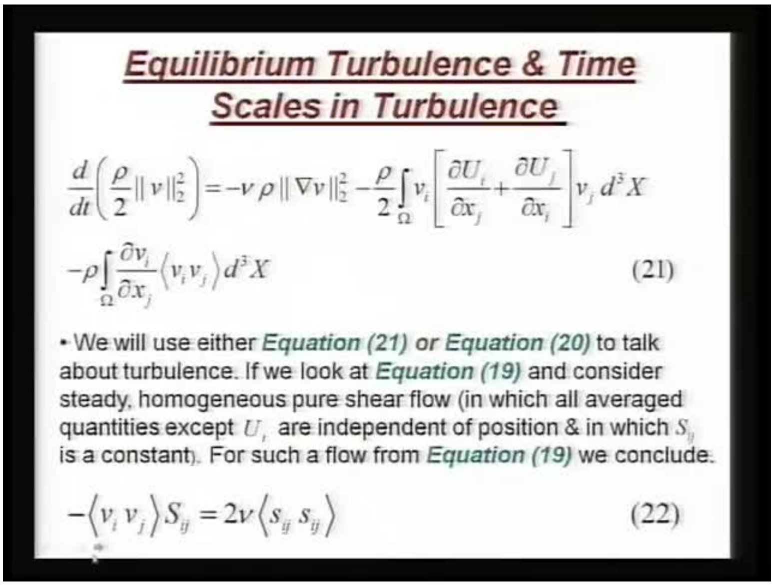
So, this is the approach that would be taking and then what we are going to get is the corresponding turbulent energy budget written over the whole control volume; just now what we talked about was a point properties, but now we are integrating over the whole domain and this is what we are going to get.

(Refer Slide Time: 32:55)



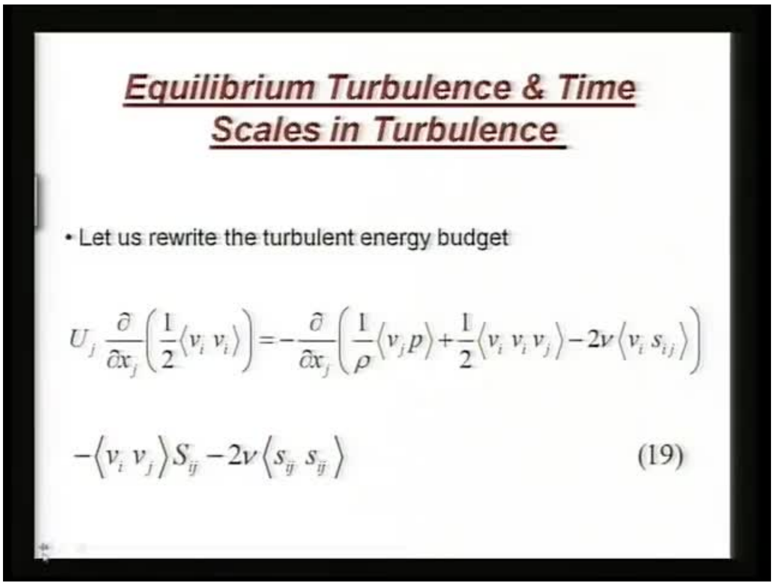
So this is essentially the same equation where integrated over the whole volume. So,, once we have that then what you could do is we could talk about deriving the turbulent kinetic energy budget by looking at the corresponding equation written by the vectorial notation like this; this is what we derived yesterday.

(Refer Slide Time: 33:29)



So, now, if I take a dot product of this with respect to v, then we get the following equation. The following equation would have contributions coming like this. So, there are two ways: we define our turbulent kinetic energy fluctuation in terms of this integrated equation; this is equation 21 or the previous one and we can talk about. Now, if we go back and take a look at equation 19, equation 19 was the following equation. Well, let us say over and see what that equation is.

(Refer Slide Time: 31:01)



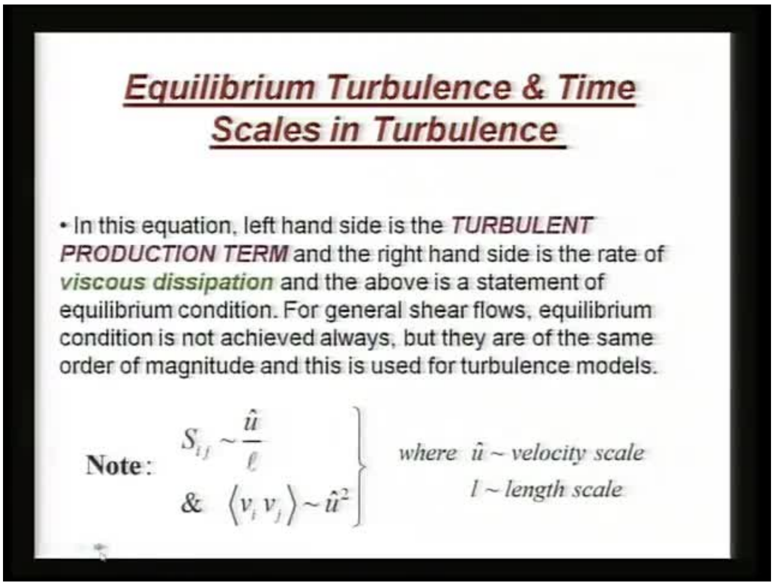
Yes. Now, what we are talking about is a time rate of change of the turbulent energy fluctuation. It is given by this gradient transport given by this production term and the dissipation term.

Now, let us talk about equilibrium turbulence. What do we mean by equilibrium turbulence? Let us understand it now. If we are looking at the general flow, it looks like this, but let us look at a very special case; a special case where we are talking about a steady, homogeneous pure shear flow; steady in the sense, we are talking about in the beam. Now, if we are talking about homogeneous pure shear flow, that by definition will imply that all the average quantities except U i are going to be independent of position and in which S ij is constant; that is what we talked about.

So, S ij is not 0, but it is a constant. Like a quite flow, we add the shear same everywhere. So, this is what we are talking about. We are talking about homogeneous pure shear flow; say, S ij is a constant. Now, then what will happen? If you look at that equation that we wrote, there were this gradient term that will not be there. We are talking about the steady state. So, the left hand side is 0; then what is left, is only the production term and the dissipation.

So, for such a flow that we have, kinetic energy of fluctuation equation looks like this. So, this is what we mean by equilibrium turbulent flow, where there is no net change; whatever is produced is perfectly dissipated; equilibrium achieved; so, in most of the turbulence models that we see in existence, they originate from this concept that we have some kind of a production, being kind of balance by dissipation. And once you have that, you can talk about certain properties.

(Refer Slide Time: 37:18)

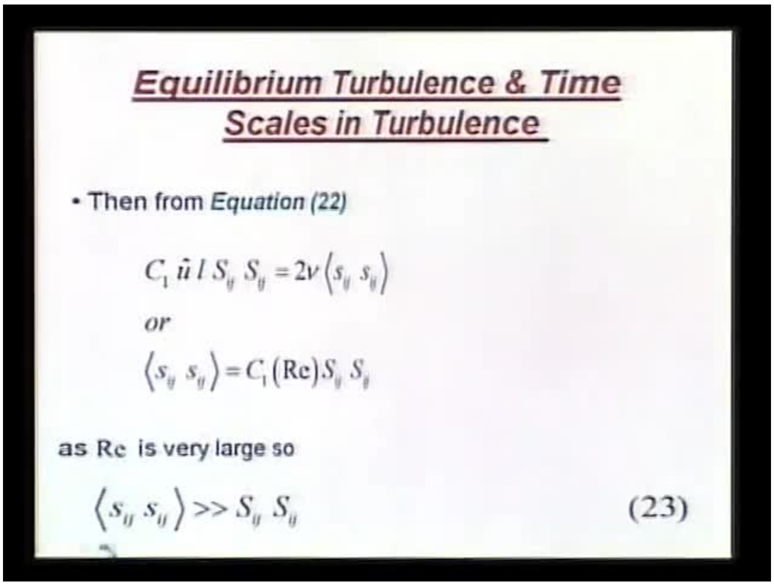


So, what we have written there? On the left hand side, we have written the turbulent production term and the right hand side, we have written the rate of viscous dissipation and that itself is a statement of equilibrium condition; whatever is produced is lost. So, thus, when we go over from this very specific case of homogeneous pure shear flow to the general shear flow, well, we acknowledge that this equilibrium condition is not satisfied all the time. Nonetheless, nonetheless, when you look at the order of magnitude of production and dissipation, they are of the same order; they are of the same order of magnitude and this observation is used in all turbulence models; this is the main thing.

So, now, the production term involves S ij. So, S ij I could write it like this; it is like del capital U by del x. So, we are talking about the mean motion. So, if we define a velocity scale which I called U cap and l is the larger length scale, the mean motion scale, then S ij is going to be like this (Refer Slide Time: 38:64). And, additionally, you also said that when it comes to turbulent flow, the fluctuation quantities are of the same order of magnitude of the mean quantities.

So, in that respect, then what is the order of magnitude of the Reynold stress? That is also of the other of u square. So, we make use of this order of magnitude analysis and then we see what we get? Once we get that statement of equilibrium production is equal to dissipation will give us this.

(Refer Slide Time: 39:21)

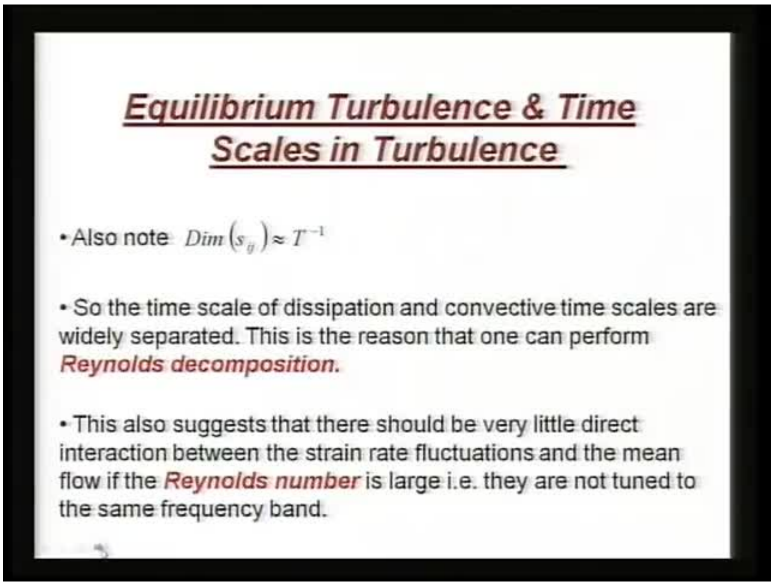


So, this is the dissipation term. We have kept it is, but the production term we have purposely altered here because what we had here? We had v i v j times S ij. So, v i v j is u square. So, that we could write it in terms of this - u hat l into s i. You just sort it; a simple rearrangement and manipulation. Once you do that, then what? We see that s ij times S ij, its time average is given like this; now, this itself I can see that nu can go downstairs u l by nu will be a Reynolds number; that factor 2 has been absorbed in c 1.

So, what we are seeing is a very interesting observation that I already been emphasizing time and again, that if you are looking at turbulent flow which is necessarily a very high

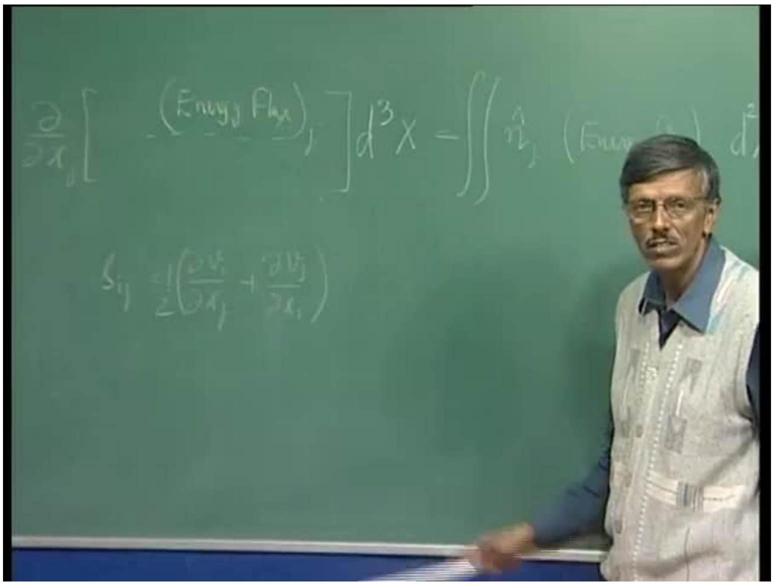
Reynolds number, flow then what happens? The fluctuating strain rate, that time average is much larger compared to a mean strain rate. This is the origin or the way we can estimate this.

(Refer Slide Time: 41:02)



Now, additionally also you note the dimension. What is the dimension of strain rate?

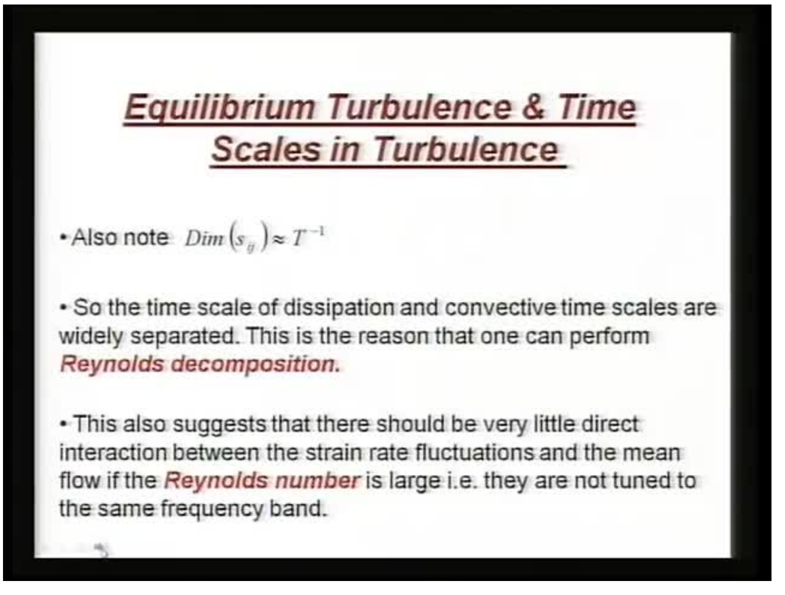
(Refer Slide Time: 41:16)



So, the strain rate that we have written here, this is this. So, what is the dimension? This is the velocity; this is the length; so it is going to be dimension. So, rate of strain actually is that they might inverse of a time scale. That is why vorticity also has a same feature; vorticity also as a same kind of dimension. Is not it?

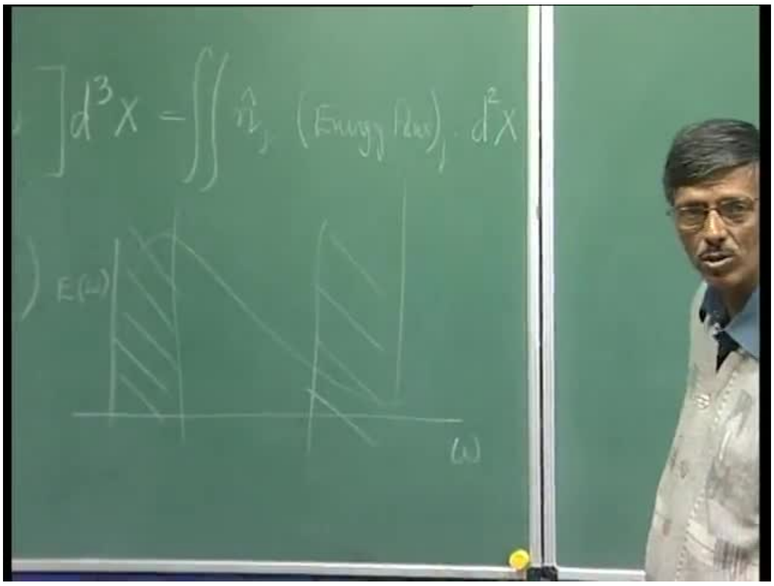
So, what you see, that in a sense turbulent flow, then defines a whole host of time scales. Depending on whether you are looking at the mean strain rate or the fluctuating strain rate, you get different times of types of time scale, and you find that the time scales of the fluctuating quantities and the time scale of the mean quantities, they are significantly different.

(Refer Slide Time: 41:02)



Because that is what we saw in the previous slide that S i j a lower case fluctuating quantity is Re times capital S ij. That means what? That means that this times scales of the mean motion and the time scales of the fluctuations, they are widely separated; they are widely separated and once again this should tell you why people do unsteady RANS equation. It eventually boils down to this fact that this time scales are so far apart, that I could do two types of time averaging operation: one will be at a very small time scale, those relates to turbulent fluctuations, or I could do a time average over the this thing. So, when I write u RANS equation, I am actually performing the time averaging over the small scales and then I am following the larger time variation. That is what we do.

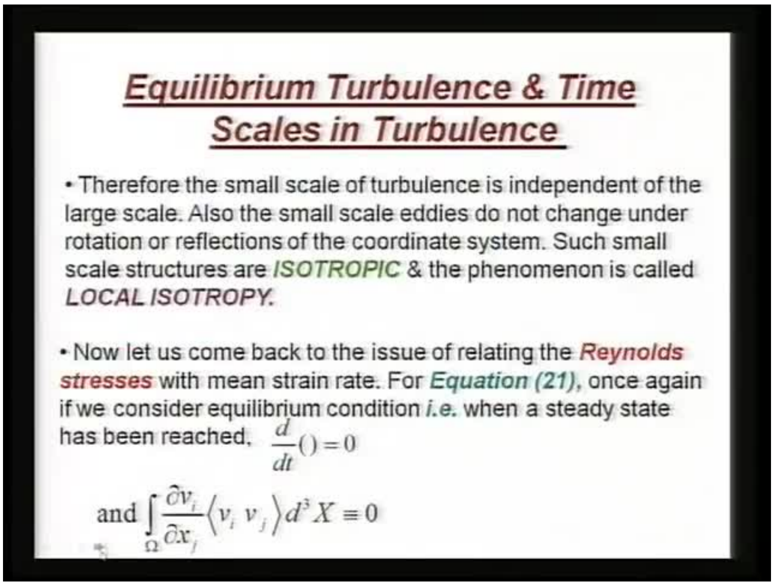
(Refer Slide Time: 44:01)



And if these two things are so much far apart from each other in terms of the scales, there is reason for one to believe that if I plot on this side omega and on this side energy of omega or I could write corresponding k, then what will happen is this sort of things will happen, that I have this part, the mean part and the fluctuating part - they are separated. Why the part and they do not interact with each other directly; it comes through all those indirect root of cascading and stretching we talked about.

So, that is precisely what we are talking about; that if we are looking at very large Reynolds number flow, these events are now tuned to the same band of frequency one works there and the other resides on the other side. And this is what makes Reynolds averaging, a worthwhile operation.

(Refer Slide Time: 44:59)

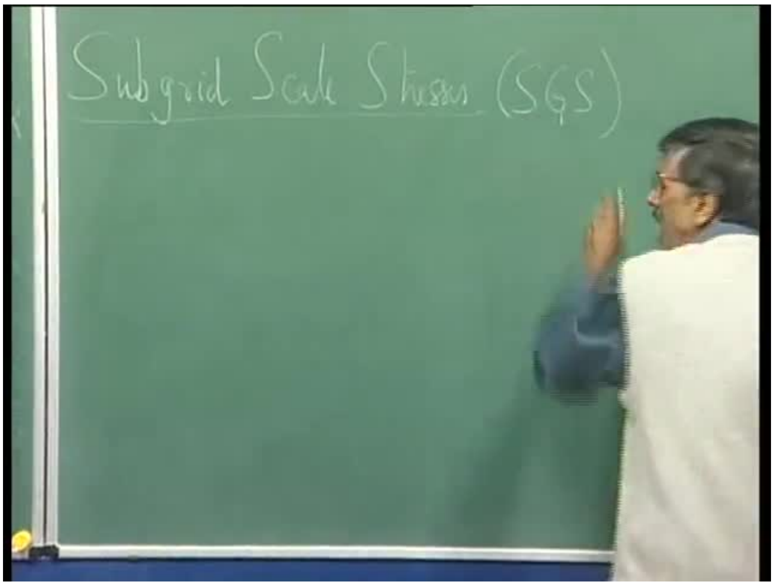


Now, this also would give you a justification, what happens to these types of motions? See, what happens is high frequencies associated turbulent fluctuations are related to very small like scales. So, that is what we are saying. We have very small scale of turbulence and that is independent of the large beam motion. These small scale eddies are getting created because of all those non-linear processes; they are smaller in size.

So, they could be assume to be also isotropic. They were all looking similar because you may have a fewer larger eddies, but by successive breaking down into smaller eddies, a smaller eddies at the lowest possible scale are going to represent a sort of a isotropic structure. This is what is called as a local isotropic assumption.

So, in all large eddies, stimulation early as calculations that we talk about. We make this assumption that the flows are different in the large scale, but when you are looking at it at the lower scale, there, the eddies have some kind of a universal feature. They are independent of rotation or reflection at the coordinate system, so that we can treat them as isotropic vertices. And then, once that is done, you can say, for all if different flows have same small scale quantities; I could have a single model for these small scale events and these are what are called as some weight scale stresses.

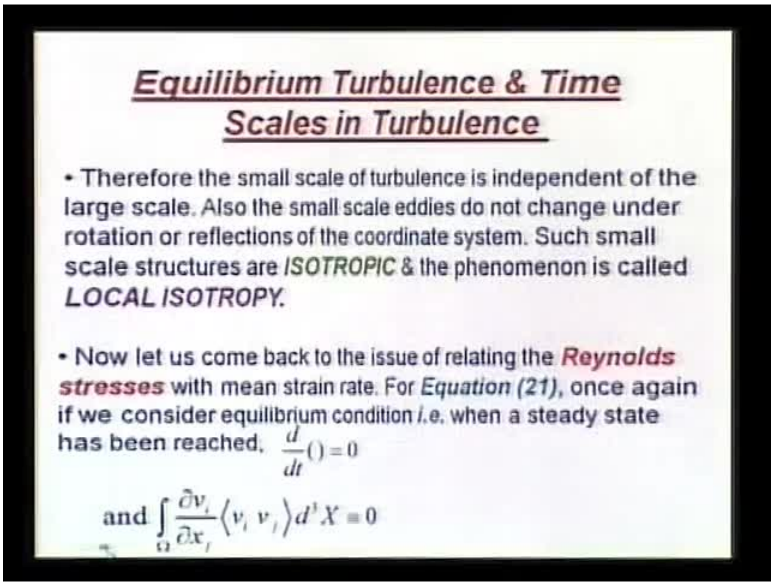
(Refer Slide Time: 47:15)



So, do understand that we are now looking at a different aspect, but essentially, the ideas are related; ideas are related because we have been, now, generally talking about the specific energy of the mean motions, specific energy of fluctuations, but what we see? That when we write down the Reynolds average equation, we talk in terms of time scales and when we are talking about larger dissimulation. We are talking about its behavior from space variation, but they are somehow related because we have seen the dimension of S ij is 1 over t.

So, you see, this is a sort of advantage, but if not done carefully, this can actually lead to confusion; so, let us not worry about this, but now, having done all of these that we can talk about Reynolds stress or the sub grid scale stress, then, life is little more comfortable. What we have talking about in largely dissimulation, we are talking about computing in with some precession that at the level of local isotropy, we do not need anything because we have gathered theoretical information that it is isotropic.

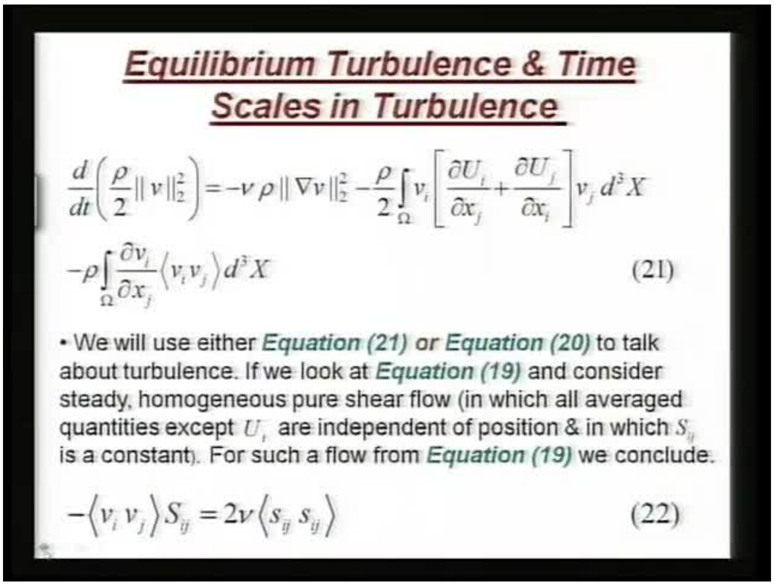
(Refer Slide Time: 49:25)



So, I do not need to needlessly resolve the flow, but finally I can use that information gathered already and put it in there and solve for the larger eddies which are not isotropic, which are not homogeneous; that is the whole concept. Now, we still have not followed where we started. We wanted to relate the Reynolds stress with the mean strain rate.

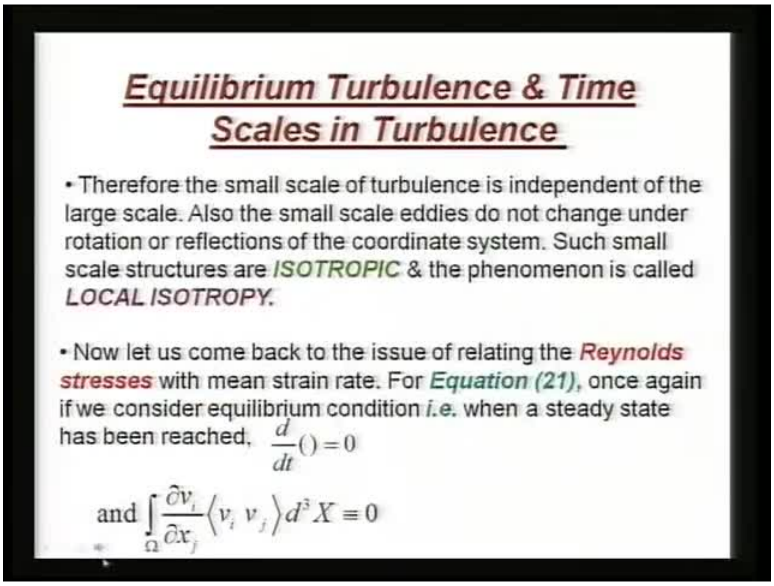
So, what happens? If we look at this equation 21, that was the turbulent fluctuation energy that we wrote in an integrated form of the whole volume. Now, for that flow, if I assume equilibrium condition, that means I say that d by d t of that quantity equal to 0, then the steady state would be reached. And what would we get? Well, let us go back and take a look at equation 21 for our convenience.

(Refer Slide Time: 50:10)



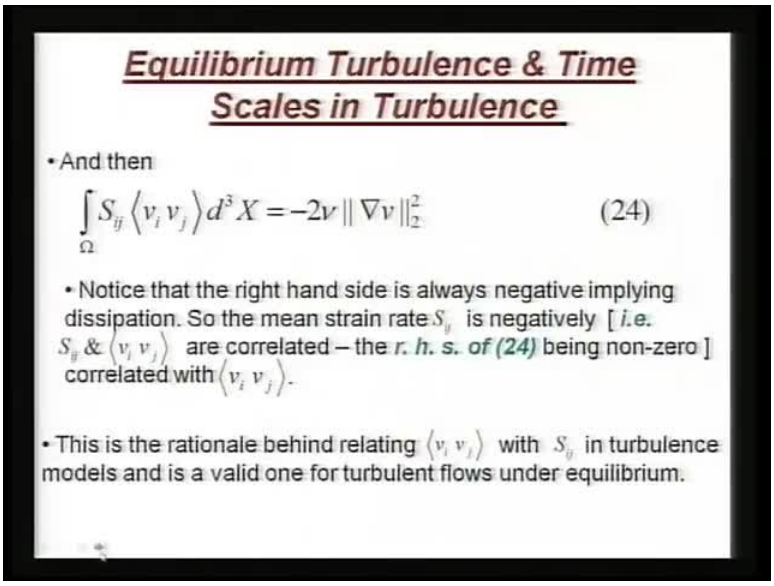
Here, it is. So, we are talking about a steady state has been reached. So, we are knocking of this quantity and this is your capital S ij and this is your v i v j. This is your production term and that is your sort of loss with respect to the turbulent fluctuation.

(Refer Slide Time: 49:25)



Now, if I go back and start talking about this, that this would… Then, if I time average that equation that I just now wrote, it will all come down to this term.

(Refer Slide Time: 51:17)



So, this is what, we are getting that del vi del xj times the Reynold stress integrated over the whole volume should be equal to 0. If I say that, then what will happen? This is what is going to be written. This is your production term and this is your dissipation term. Again, stating the same equilibrium condition, this is always negative implying dissipation and this is your mean strain rate.

So, what is it? This quantity is negative, but non-zero. That means what? The mean strain rate and the Reynold stress, when I integrate over the whole volume they do not become equal to 0. If they were 0, if the result is 0, then what would do? We conclude that these two quantities are not correlated over the whole domain. However, what we are noticing is, indeed, they are correlated and the correlation is a negative correlation. So, this is the reason that we always try to relate the turbulent stresses with the mean strain rate. This is the main stay of most of the turbulence models which have been proposed ever since the term of planting.

So, you understand all those algebraic models came about just from the simple observation of equilibrium turbulence that the mean strain rate is negatively correlated to the Reynold stress. And in this note, we will stop today.