Instability and Transition of Fluid Flows

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Module No.# 01

Lecture No.# 30

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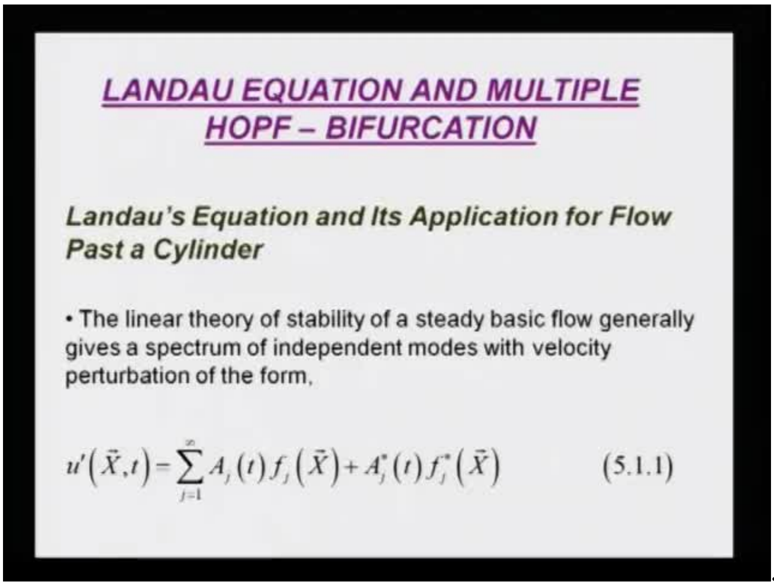

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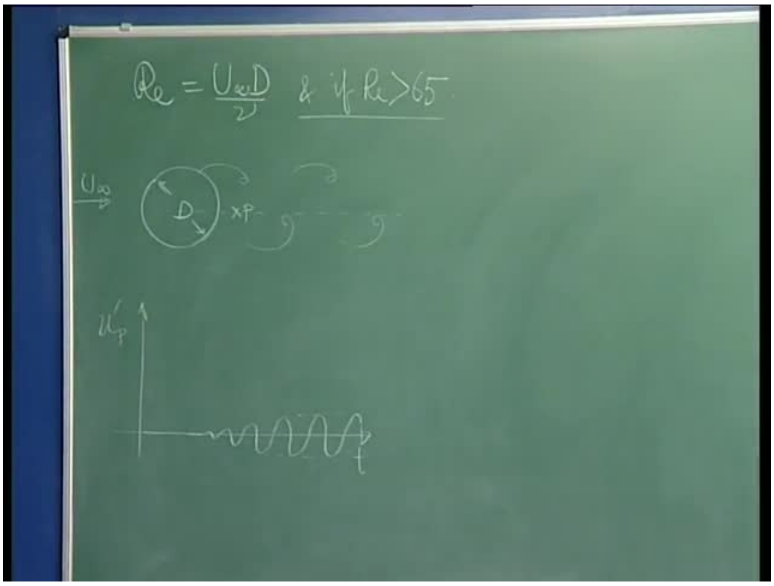

So, we started talking about fluid dynamical systems which primarily display temporal instabilities. As a very important example, we are looking at flow past a cylinder and what happens is, the flow would like this and if I talk about flows, for which if I define the Reynolds number in terms of U infinity D by nu, where D is, of course, the diameter of the cylinder, and if Re, to be on the safer side, I will just put it, let us say about 65 or so, then, what we see is, vortex shedding takes place. Vortex shedding takes place and if you take a point, in the near way, along the center line and find out its time trace, you are going to see a time trace of this kind.

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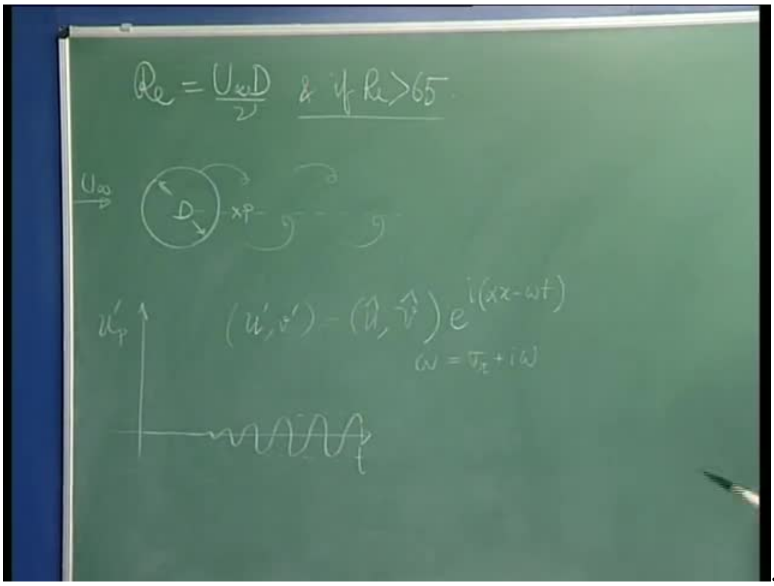

So, this is a, let us say, a stream-wise component of disturbance velocity that will… So, what happens is, we notice that, these disturbances picks up slowly with time, and if you really find out the envelope of this amplitude, you will find that, this is an exponential growth that you are noticing. Initially, you have an exponential growth and that actually gels with what we have been talking about in the past that, the disturbance quantities would be given in terms of, let us say, their amplitude, Fourier Laplace amplitude, kind of what we are showing it by carat and then, you would have the phase given by this; but this is not truly a real phase in the sense, alpha and omega can be complex.

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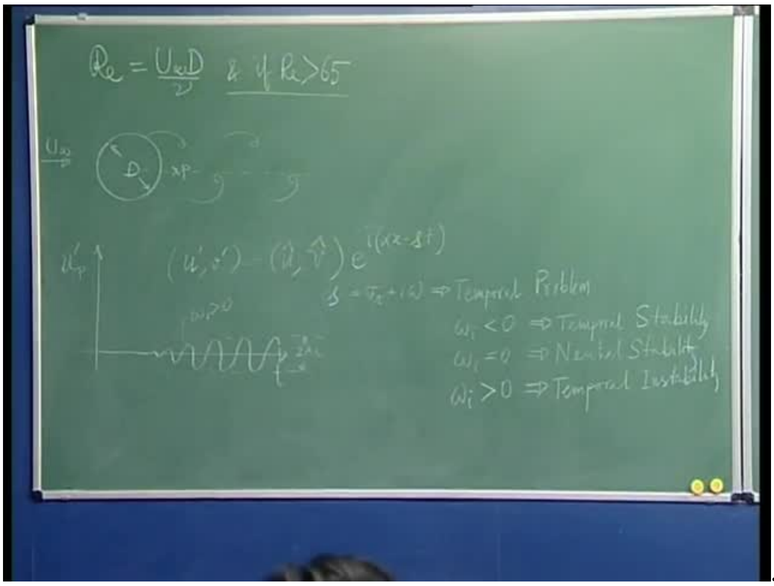

So far, we have focused upon situations, where alpha was considered complex, omega was real and now, we are switching our attention to cases, where alpha would be real, perhaps, and omega will be complex; that defines your temporal instability. Now, if that is so, if omega, as I write here, I will, let us say, write it in terms of a real and imaginary part, and then, let me call this a different quantity; let us call this as s, because that is what we will be using. So, let us put it like this, say alpha x minus s t and s has a real and imaginary part. So, this corresponds to your temporal problem. This is what we have done. Now, within this canopy, you can identify various cases. For example, if omega i is less than 0, then, what would you have? You would have temporal stability; that you can substitute it here, and because of this minus sign, and there is i and i, that makes it that. So, omega i negative means, we will have disturbances which decay with time.

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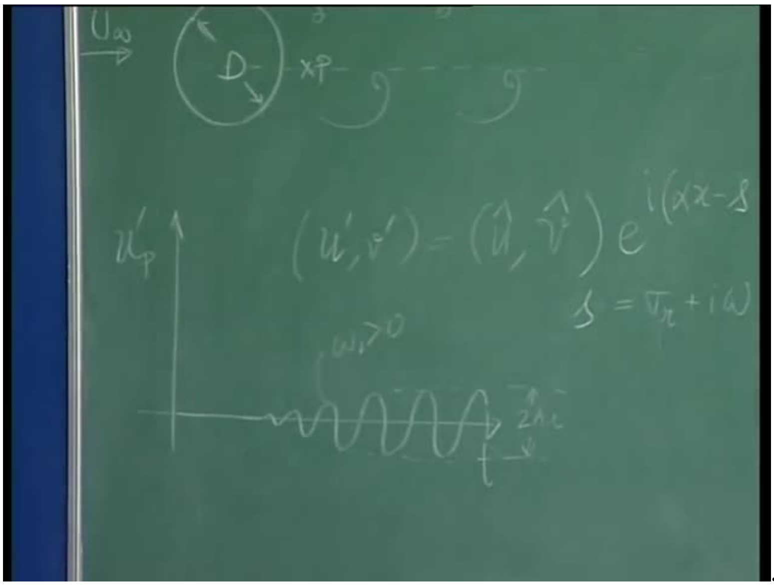

This is quite well known. This corresponds to neutral case. The case that we are talking about, would correspond to where the omega i is greater than 0. And then, we will have temporal instability and that is what you are seeing here. In this part of the flow evolution, in the early part, this is the case, where omega i is positive. And, what you notice about this growth of this disturbance is that, initially it takes off and because it is exponential, so, you need a very small trace amount of background disturbance to kick it up. And, once it kicks up, then, it grows exponentially. But what is noticed, in this flow is that, this growth is not unbounded, like what your linear theory would suggest; instead, what you see is a kind of non-linear saturation.

So, this saturation amplitude is what we called, the ((study)) as the 2 A e. So, basically, that is what we are talking about; the disturbance that we are seeing, will have a temporal variation, whose amplitude is given by A subscript e. e indicates another equilibrium state. So, we start off with a case, where we do not have any disturbance. Then, we reach another equilibrium state, where we have a saturated amplitude. And, this is what you see in vortex shedding. So, what you are seeing as a vortex shedding, as I told you yesterday, it is like your fluid dynamical pendulum. What you see is that alternate shedding of vortices; that means what; you have some separation bubble forming here, re-circulating region, which grows differentially on either side; and once it reaches a certain amplitude, that is detached; while the other one, which was small so far, takes up the (( )). It is almost like your pendulum. Do you know, interplay between potential and kinetic energy, gives you an equilibrium state. Here also, these two vortices does the same thing.

So, one grows, the other one remains stagnant; and when the one that is growing, I mean, achieves a certain threshold amplitude, then, that is shed, then, the other side it starts growing. So, this is thus a case, where we have pointed out that, here, that non-linearity is playing that role in moderating the linear growth. And, the linear theory of stability of the steady basic flow will give us a spectrum of modes, with velocity perturbation of this form.

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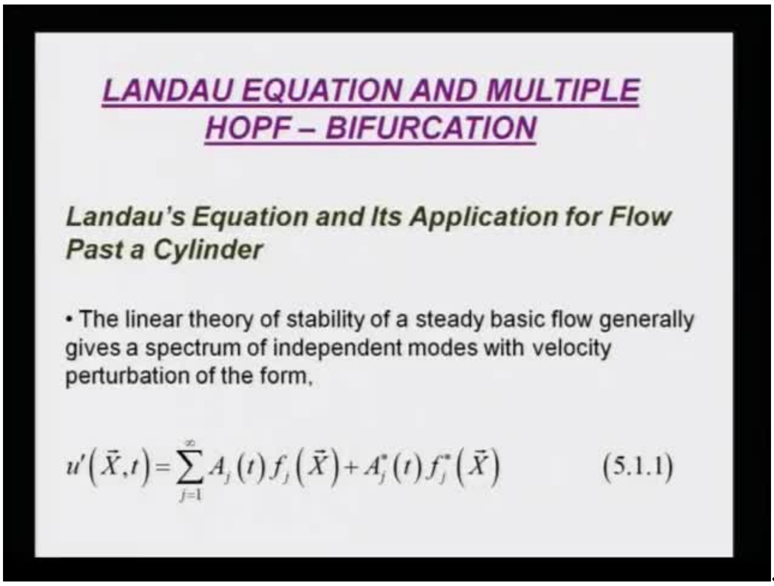

So, what we have done is, basically, a Galerkin projection and we have split out the space dependence and the time dependence. This A of t is like, what we are talking about here. So, A of t tells you, a temporal variation and the f corresponds to those modes, that we are talking about, the Eigen functions; model representations of the Eigen functions. Now, Landau have pointed out, he wanted to propose this as a model for turbulence, because by that time we are talking about 30s; in 30s people understood, at least people of the caliber of Landau, they understood that, this whole process of transition from laminar to turbulent flow begins with an instability. And, here is an example, this instability does not take it to unbounded growth by the primary instability alone; that is moderated by non-linearity and then, you get to an equilibrium state. And now, what is this state, this equilibrium state. This equilibrium state is no more steady; it is periodic.

And, as I told you yesterday that, there is a developed theory called Floquet analysis, which essentially studies instability of systems, whose basic equilibrium state is periodic.

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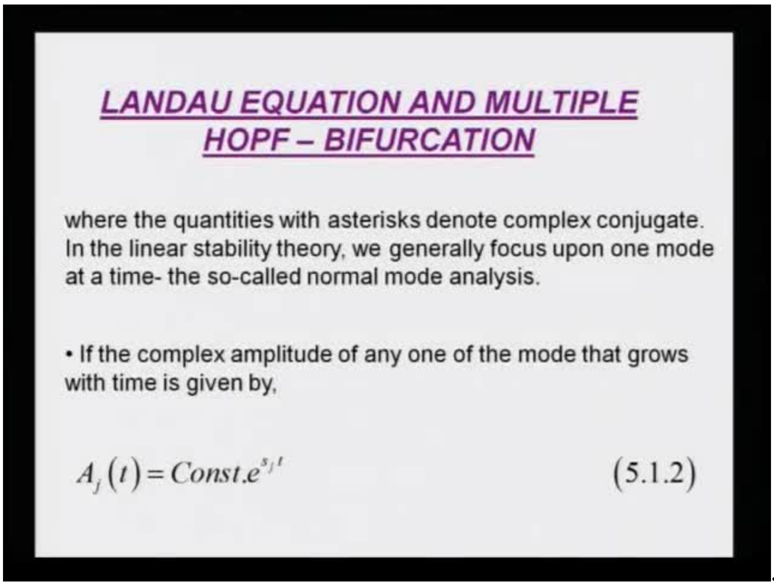

So, now, if I do that, I can now study the secondary instability of this periodic state. And, Landau's model was that, you would have the primary instability follows by secondary instability; then, you will have tertiary and so and so forth. This, all of this cascades into, eventually, your turbulent flow. Now, I also mentioned yesterday that, this view has been repudiated later on. People talking about class dynamics, that talk about systems, where you probably do not need to go all the way upto infinite sequence of such a instabilities; a 3 or 4 stage is good enough to fill up this spectrum completely. But let us begin from, how it had appeared. So, Landau proposed an equation, but let us, first of all, look at the initial stage, where we have exponential growth of disturbance.

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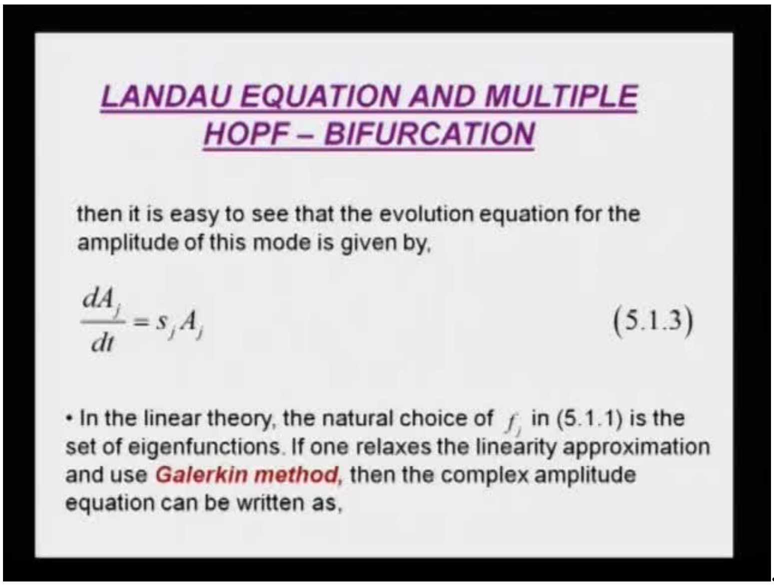

So, if I call that amplitude the jth mode, in terms of A j of t, that is some e to the power s j of t. Now, if that is the description of the jth mode, I can take a time derivative of it and divide the derivative by A j; I get this equation. That is what your equation 3 looks like. This is pretty much a consequence of linear instability. So, what is read actually, if I put this A j on the left hand side, I am going to get some d dt of l n a is equal to s j. So, s j is some kind of an exponent of the growth. Now, when we look at linear theory, the corresponding space dependent function are those sets of Eigen functions; and, if now, we relax the linearity approximation and use Galerkin method, then, the evolution equation for the complex amplitude, that is at A j, is written in the following form; is given like this.

So, what we are talking about, we are in search of a description, which defines this vortex shedding. So, linear path is not good enough. We must supplement it by non-linear action; that causes this saturation. So, do understand that, here, the non-linearity is playing the role of stabilizing an unstable system, which is seen to be unstable in the linear mode. Now, this is basically, catch-all term, which shows all kinds of non-linear action. And, the non-linear action means what, we are starting here, the evolution of the jth mode. So, the non-linear action can come above because, there are multiple modes. See, one of the aspect that, we studied beforehand, was for the linear theory. We know linearity assumption; I have seen in a super-position. That was one of the reasons that, we talked about normal mode analysis, where we studied individual modes; however, when you are talking about a non-linear dynamical system, superposition does not hold.

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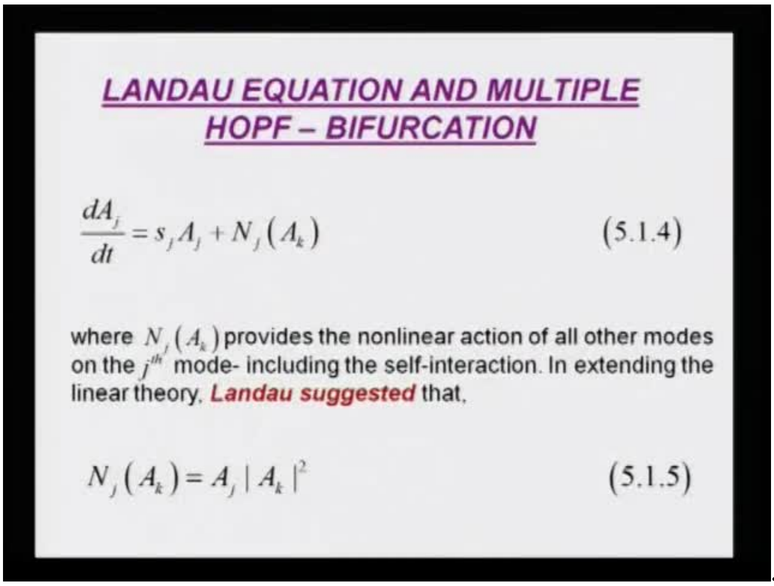

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So, each mode is affected by the other mode, and that is in a formal way, has been written down in a operator form. So, N j is a non-linear operator, working on the jth mode and that is created by all possible modes present. So, A k is written. So, it should be something like, some overall possible case. Now, Landau himself suggested that, this is what it should be. In fact, let me confess, Landau did not really suggest this. This came about with farther refinement of Landau's model by Stewart Watson and that, you see also in the monograph of Drazin and Reids and also the book that, I have just written it (( )). And, this is a kind of afterthought, because this tells you, how the jth mode interacts with the kth mode. In fact, what Landau himself did was somewhat different. Landau said, look, if I look at the time trace, then, what I am looking at is a single peak. You know, it is a periodicity; so, what does it mean? One of the mode is the dominant mode.

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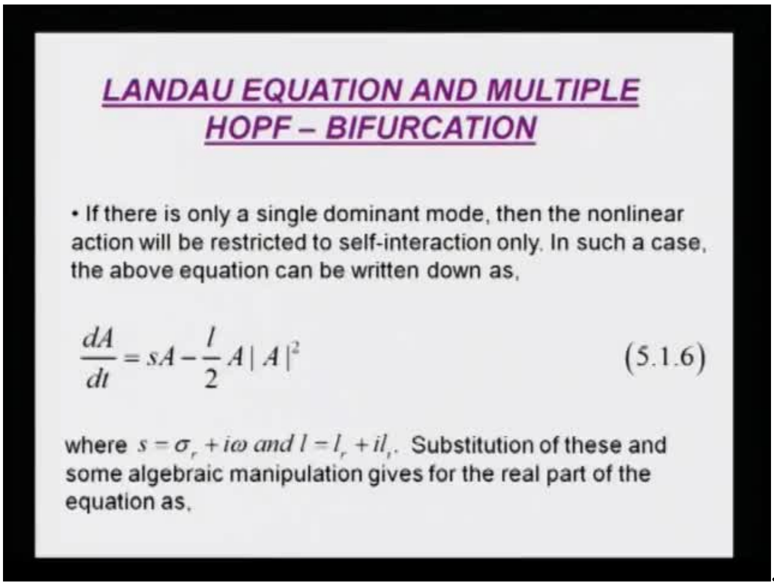

So, basically, then, I really do not have to rummage through all this js. I should be able to look at only one and let us call that amplitude as A. And, what Landau suggested was this; that A, which is the most dominant mode is the governed by this. So, this part, we understand, comes directly from linear theory; this is the Landau's projection of the non-linear action. And, what is this? A is acting upon itself. So, this is essentially, what I will call it as self-interaction term. Now, we have said that, this s is then linear exponent, growth exponent. So, it has a real part and an imaginary part. And, what landau said that, non-linear term is multiplied with respect to a constant. This is a property of the flow that we are considering; so, in this case, the flow past a cylinder. So, this constant itself is complex. So, it has a real part and then, imaginary part; put together is what is called as a Landau coefficient.

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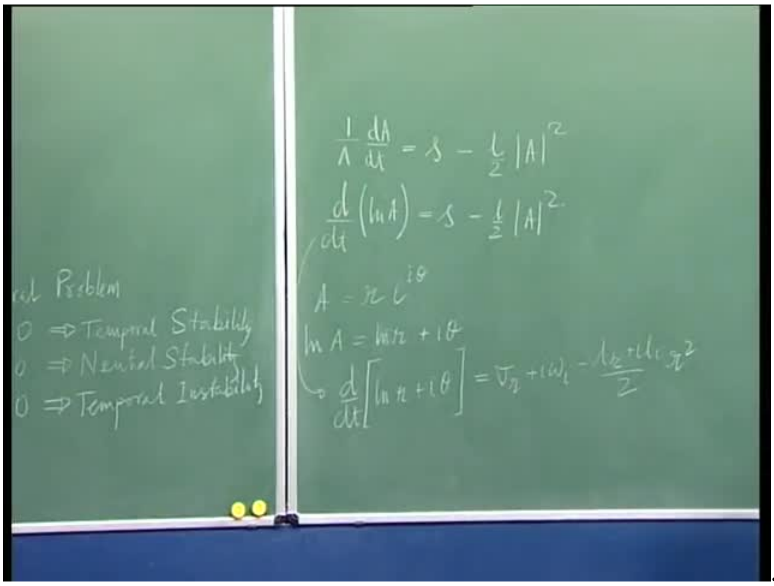

Now, what I could do, I could take a look at that equation. If I divide both sides by A, then I get, 1 upon A dA dt, is equal to s minus l by 2, and this is this. So, this also could be written down in this form, you will agree with me. So, if I do like this, now, A itself is complex. So, I could have a polar representation. So, if I write A as, from radiant vector times i theta, r times i theta; then, what happens? Of course, you can see ln A is ln r plus i theta. So, I could put it in there. So, what I get from here, then, I will get d dt of l n A; I will write it as l n r plus i theta and what about s? s we have written already there, as a real part and imaginary part. And, this I will write it as l r plus i l i by 2. And, what about mod A? mod A is r. So, I will have r squared.

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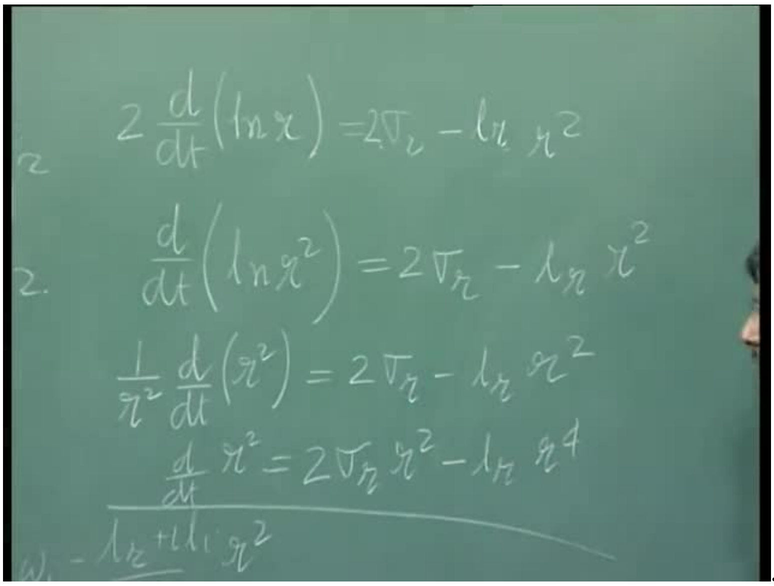

So, now, you are actually in a position to split it into real and imaginary part; that was the whole idea of writing it in this fashion. So, if I do that, what I get? I get an equation like this. The real part we will give me, what, d dt of l n r, then, I have sigma r and what about here, I will get minus l r by 2 r square. So, I could take away this 2 here, from everywhere; I could write it here as this. So, I could write d dt of 2 l n r; I can put this 2 inside; 2 l n r is… So, what I could do, I could write this as 1 over r square d dt of r square, is it not? Now, you can see that equation. Here, if I multiply by r square all through, I get, d dt of r square is 2 r r square minus l r r to the power 4; and, that is your equation 7.

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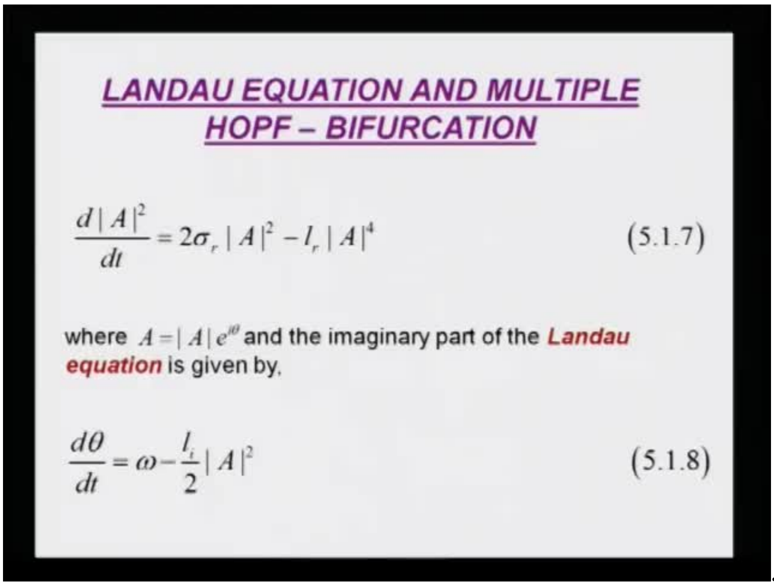

So, that is your derivation of this. It is interesting though, that Landau never put his, this three steps, algebraic steps. He just simply wrote that, the amplitude should be given like this and of course, in his original prescription, l was just l r. And, the imaginary part also, now, we can write it down, is it not?

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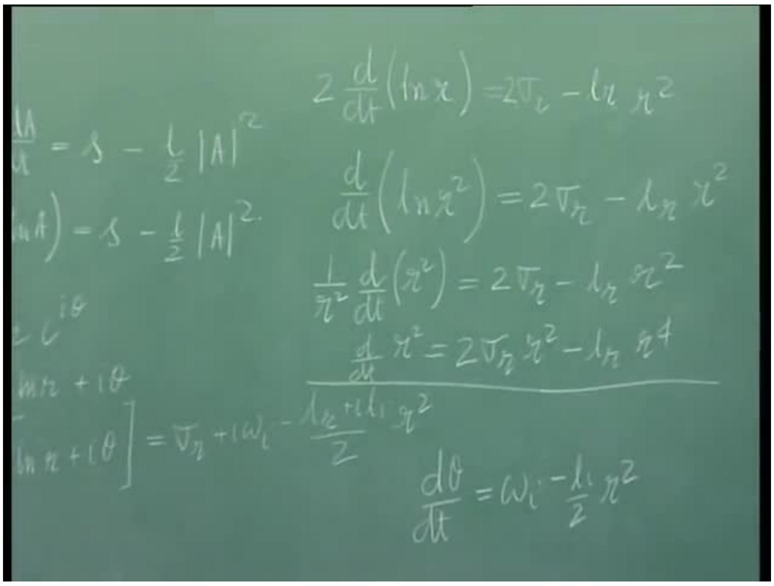

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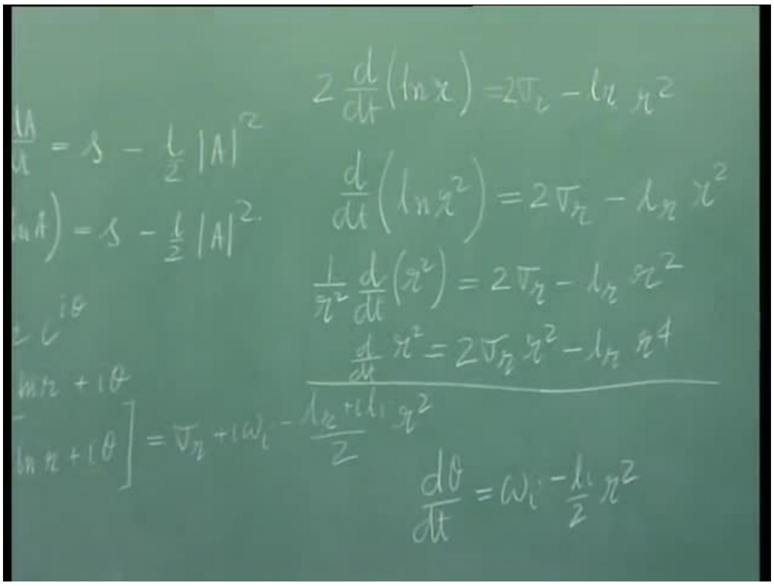

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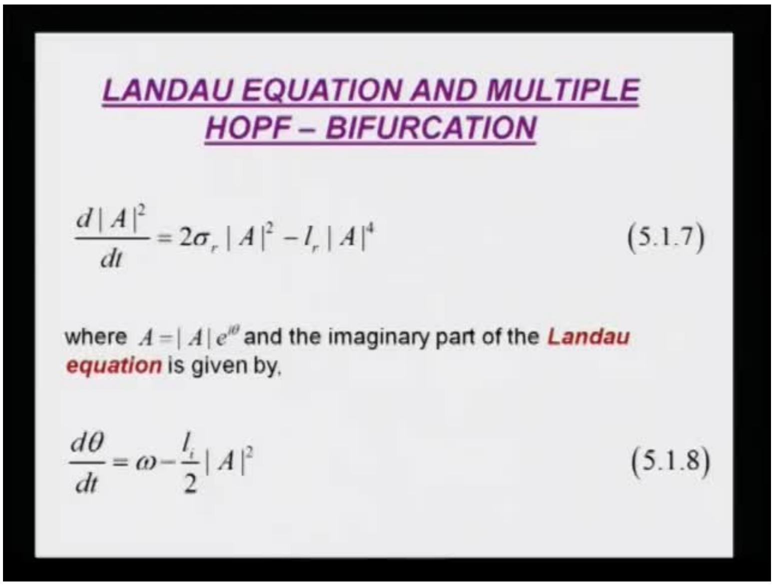

Now, what does the imaginary part give us? Imaginary part would give us here… So, I can knock off the i. I will get d theta dt on the left hand side and on the right hand side is, omega i minus l i by 2; that is what we have written there also. Now, we do not know how Landau obtained this equation and not this, but this is how it is. There are two things that comes out. That, this equation is almost like what I say in science, how subjects have developed, that first you have a key; then you go around looking for the lock; that is how you see, if how Navier-Stokes equation was written; people kept on finding out simplified cases for which solution exists. So, solutions are like key and then you go around and see, in which physical phenomena, that model works out.

So, Landau also perhaps, took that kind of an approach, where he looked at this equation. This equation is what, it is a non-lineaR equation; it is a non-linear ODE; but the good news is, this is exactly solvable. So, because it has a exact solution, and that solution has a very interesting feature, which can explain this saturation, is what is important about this equation. But needless to say, if you look at the phase variation, phase variation is interesting. This is like your Strouhal number; this omega, that you are writing here, is like your Strouhal number, that you would be actually measuring. If you put a probe here, and get the signal dual 50, you will see a dominant frequency. That frequency is, of course, this. That is this, but what about this part? This part is a very interesting part; because what does it say, that your actual frequency, not only depends on what you get from your linear theory, but it also gets modified due to non-linearity; and you can very clearly see, this frequency is amplitude dependent; it depends on A square. So, this has captured the imagination of many researchers and lots of people, actually, in studying this particular flow, reports results on variation of this Strouhal frequency and the background disturbance amplitude, etcetera and that may have, it is a clue in this equation.

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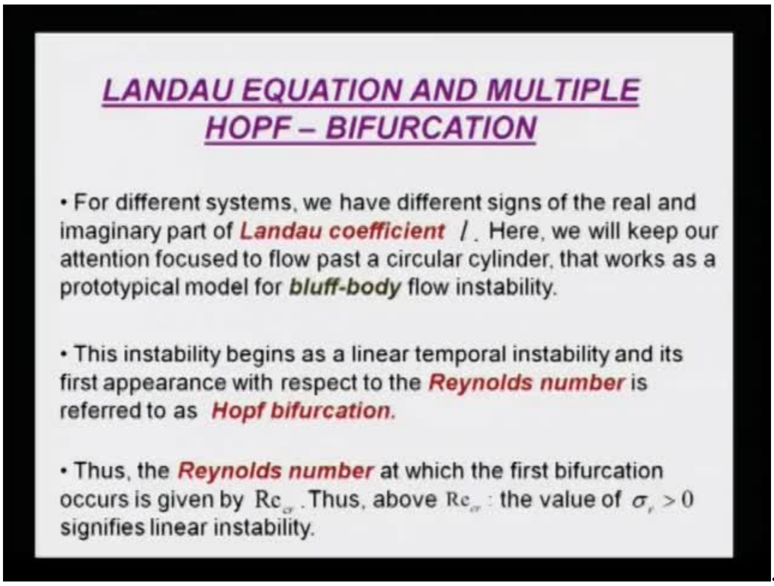

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So, I suppose, having gone up to here, we can talk about the following; that is, Landau coefficient, it is still a multiplicative constant. And, we do not know what kind of real and imaginary part it will have. It also depends on the sign of the real quantity, because we have written down that equation; the sign itself will determine what is it actually; what does it say? The time rate of change of the amplitude is determined by the linear instability, but if the sign is different, then, what does it do? If l r is positive, it reduces the d dt of r square. So, it actually decreases the linear instability. So, l r positive is the case, where the non-linearity stabilizes; but if l r is negative, you can very clearly see that, it could give rise to added growth due to non-linearity. So, Landau was very much aware of this.

So, what he actually proposed, it, he was also interested on many things, through this single equation. He was hoping that, you will be able to explain that non-linear saturation in case of bluff body flow like flow past a cylinder. He was also hoping to solve this problems, which had shown stability through linear analysis. You can very clearly see, the case that, this part could be negative, the linear part. This sigma times r square, this could be negative. So, what does it mean, it is linearly stable; however, if l r is negative and this quantity overrides this, then, it will be non-linearly unstable. So, flows like pi flow, weight flow, Poiseuille flow, that is where, maybe, one would be interested in a negative l r. I will not talk about that, do not have time and there are not much of advancement of that have gone in.

It is time, somebody starts looking at, with the help of DNS, they can start looking out for the Landau coefficient for some of these flows. This is also another open challenging in research problem, I can suggest to any of you willing to continue in this field. It is very interesting because, this is a need of the hour; there are lots and lots of fluid dynamical systems, which still await a proper answer. So, this could be the case that, one should be solving for those and then, get the answer. So, what we will be talking about, in this case, is flow past a circular cylinder, because, I suppose, this is a sort of a canonical problem, for which many engineering discipline looks out for an answer.

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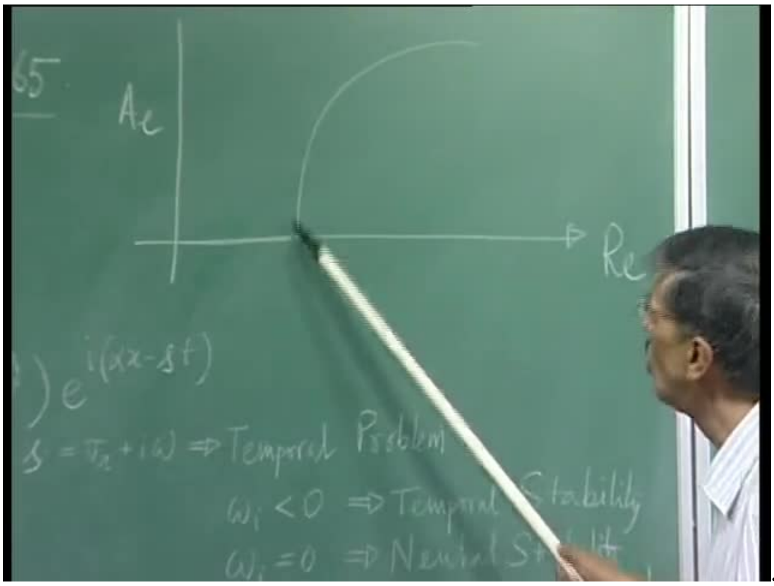

So, we will only be talking about l r as positive. We said that, the instability begins as linear instability, and the first appearance of this is created by Hopf-bifurcation. If you recall, yesterday, I did talk about the Hopf-bifurcation. So, basically, what we talked about that, if I plot in the parameter space, in this case, it is a simple problem; the only parameter is Reynolds number; and on this side, I will plot, let us say A e. And then, what I said yesterday that, upto some value of i, we will get this kind of a thing. So, where the linear instability begins, the amplitude goes almost like, vertically up. So, this is like 90 degree, and that is what is called as the Hopf-bifurcation. The Reynolds number at which the first bifurcation occurs is indicated by the symbol Re critical Re cr. So, what does it mean, that above Re cr, this real part is positive, because, it is unstable, linearly unstable; so, that is the harbinger of linear instability.

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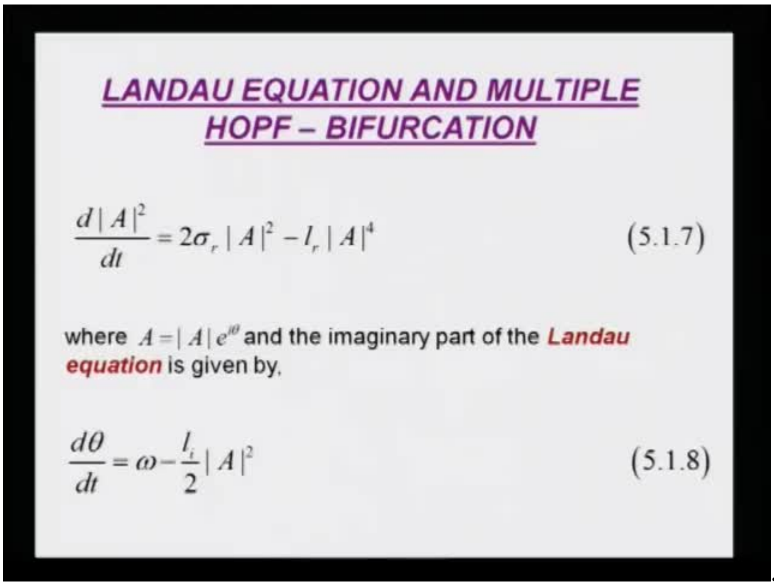

Now, once that happens, we do get some interesting solution. What we would be talking about, let us say, if I have this, I did not write this solution. We have this equation 7. Now, what happens is, we have seen how the linear growth rate is moderated by non-linearity. Now, when I reach this plateau, means what; what happens to this d dt of A square? It becomes 0; if that is 0, then I have reached an equilibrium state. So, I can put A equal to A e here. So, if I put this equal to 0, that gives me a value of equilibrium amplitude. And, that directly comes from here. You can see, from here, if that is equal to 0, then you can see, there are two solutions; one in of course, r square; another is 2 sigma r minus l r into r square. Now, this equal to 0, and that you can talk about. And, you very clearly see, one of the solutions is this and that is this.

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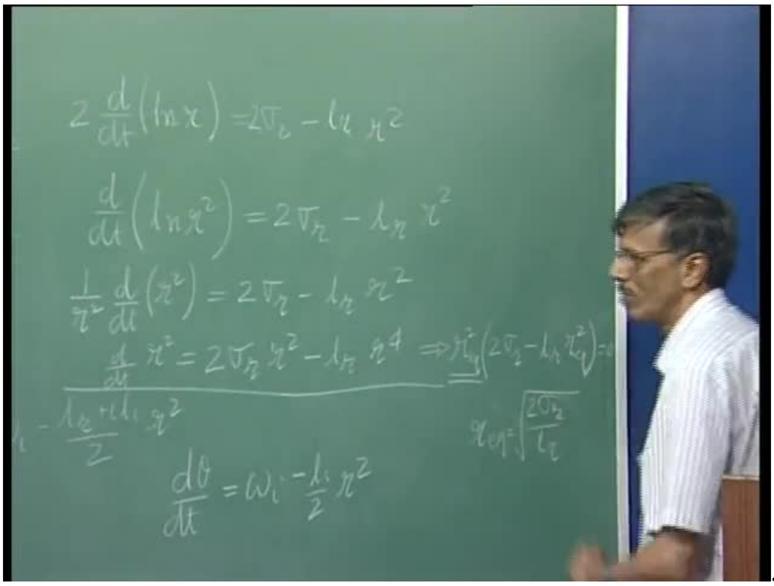

So, at any Re, I have one solution that corresponds to Re equal to 0, and the other solution comes from here and that is your this. So, I will write it like this. So, Landau's equation, without even solving, helps us identify the equilibrium state. But you can actually solve it. I will leave it as a sort of a short homework. It will take you a couple of minutes to solve it as a function of A. What you do, as I told you, divide by A square and then, it becomes very easy. You will be able to solve it. And, you will see that, solution A as a function, mod A square, as a function of t, would be a decaying function of time. So, for, after a long time. So, initially, you will get this part, but for a long time, it will saturate. The time dependent part slowly tapers off; that is why Landau took this equation. So, I want you to write down this equation. If you cannot, if you can get the solution, you can come back to me, I will tell you, how to do it.

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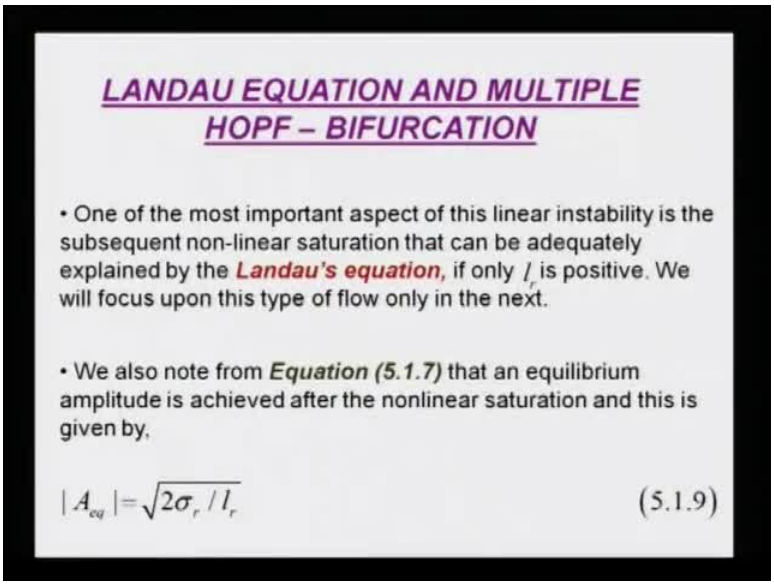

So, we talked about this and this is what I said that, equilibrium amplitude corresponds to this case. Very interesting. So, what you get is, the equilibrium amplitude is given by sigma r divided by l r. How is sigma r as a function of Re? I have crossed Re critical, that is why sigma r is positive. So, sigma r, that we are writing here, should be proportional to Re minus Re critical, is it not.

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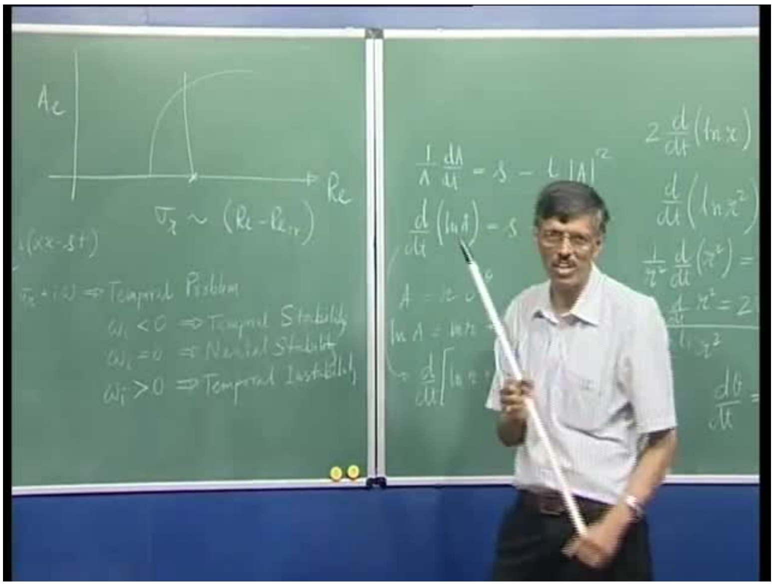

Because, flow will become more and more unstable; Reynolds number keeps increasing. And, if sigma keeps growing like this, what happens to the equilibrium amplitude? It goes like this, and what is the nature of this curve, parabolic; because, you can see here; A e goes as square root of Re. So, if you now, go to the lab and do the experiment, all you do is, you have a hot wire probe, you keep measuring, keep increasing the Reynolds number, and you get this; draw this curve. And, you have found out, where that Re critical is and if you have this curve, you can take any two points, and you can obtain this. In fact, some experimental is did by Strykowski; working with Professor Sreenivasan, at here; they actually solved this problem experimentally, and tried to comment about, what happens to omega r, l r, etcetera. We will see, what those results are, as we go along. Now, I would show you a comment, I will show you a comment attributed to Landau.

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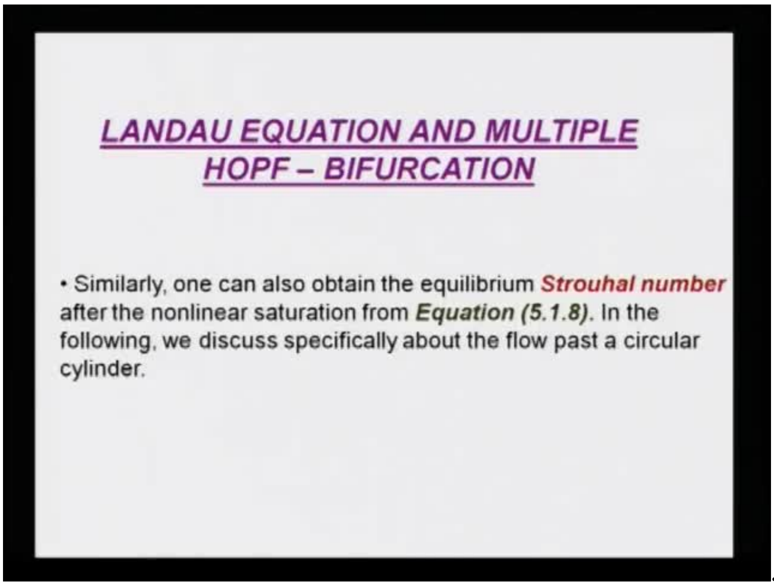

He did not think very much about the ability to find out the imaginary part; that is this Strouhal number. He says, that remains indeterminant; but we have written down the equations, so, we know better, now. But this Strouhal number also, depends on the non-linear saturation and the amplitude, it depends on also l i. So, if I do study flow past a circular cylinder, experimentally, or by many accurate numerical method. And, you know that, we do that. We do solve equations with great deal of care and accuracy. So, we do get results and we actually produce what these parameters could be; we should be able to show that.

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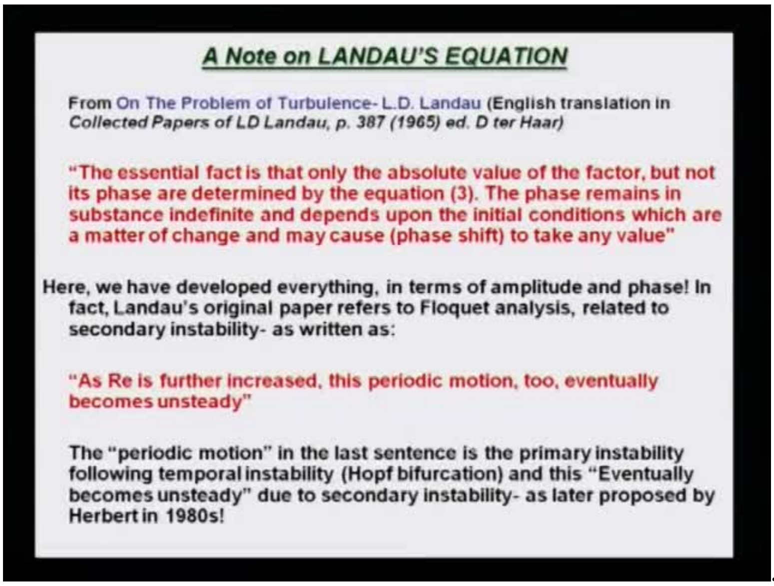

See, as I told you, there are some excerpts from what Landau wrote originally, from his collected papers, translated. He wrote that, the essential fact is that, only the absolute value of the factor, but not its phase, are determined by that Landau equation. That complex equation, that was written. He did not even write down the complex equation; he wrote only this part, this equation, this part. And, he was not aware of the existence of this; that is what he saying that, phase is not determined. The phase remains, in substance, indefinite, and depends upon the initial conditions which are a matter of change and may cause phase- shift to take any value.

Just in hindsight, he is not correct. Here, instead, we have developed everything in terms of amplitude and phase. In fact, Landau's original paper refers to Flouquet analysis, that is what we talked about, which relates to the secondary instability. And, he wrote it that, as Re is further increased, this periodic motion, that we get the vortex shedding, if we keep on increasing the Reynolds number, then, that also again become unstable; that is what he means by unsteady, means, but periodic or the period changes to something else.

So, that is what he means. The periodic motion in the last sentence, is the primary instability following the linear temporal instability by a Hopf bifurcation. And, this eventually becomes unsteady is basically, the secondary instability. Interestingly enough, in 80s, Professor Herbert, when he was in Germany, he used this, to study the secondary instability of flow past a flat plate. And, there were, quite a bit of interesting work done, and I will not go through that aspect of secondary instability of external fluid mechanics, but that is totally based on what we are talking about, is a Floquet analysis.

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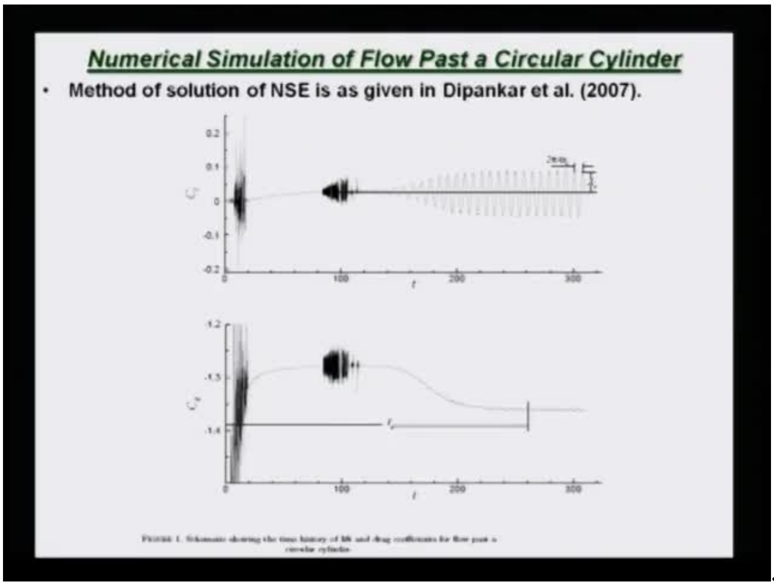

Suppose, so, the scenario is something like this, that, I have a flow over flat plate let us say. We created Tollmien-Schlichting wave and we have a parameter combination, such that, we have a neutral stability. So, if I have a neutral stability, what I will get; I will get a periodic solution. That periodic problem is susceptible to background disturbances; and, that is what studied by Herbert in 80s. We do not have time, we will not talk about it, but that is what Herbert studied, following this original idea of …Now, show you some bit of numerical results or maybe I should go, get you a better quality picture, but still, let us try to understand, what we are seeing here.

If it is basically, numerical solution of two dimensional Navier-Stokes equation, what we have plotted here is, cl versus time; cd versus time, and what you notice that, the cl versus time, it shows some kind of very high frequency oscillations. But then, again, that quenches, and then again, it suffers that, and afterwards, slowly this instabilities that we are talking about, they pick up, and you get to those kind of equilibrium amplitude and the frequency, etcetera. The drag also shows similar kind of corresponding high frequency oscillations, but once, this non-linearity sets in, and you go to a next equilibrium state, you notice an interesting thing. The drag reduces. This almost go with the aphorism people say that, nature always finds out the least energy solution.

So, it is like this; although in a transient phase, the drag grows high, but when you reach equilibrium state, you actually again come down to a low drag configuration. So, this kind of non-linear stabilization is a kind of nature's way of optimizing and bringing it to a low drag configuration, and this kind of periodicity is also seen here.

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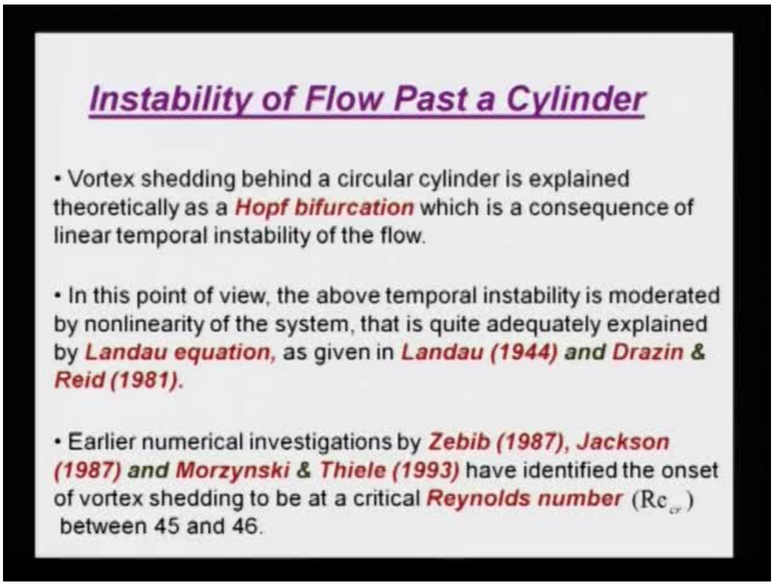

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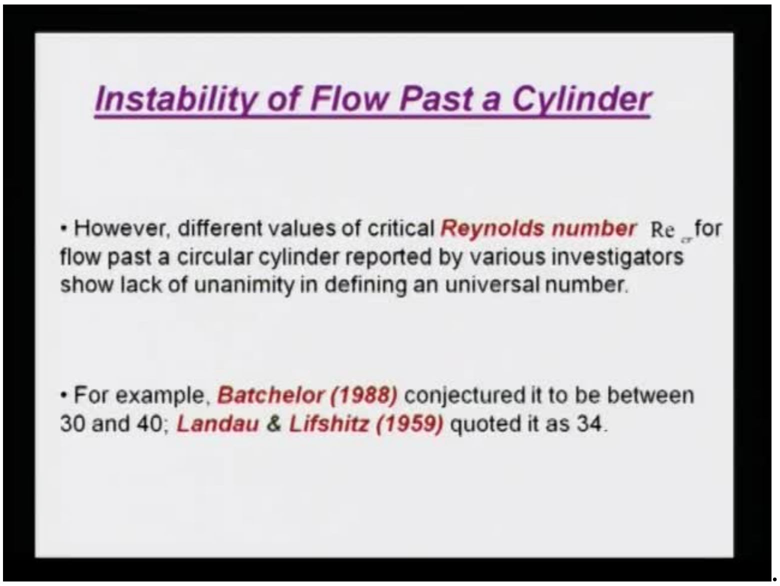

So, this is just to show you an example of a numerical solution, but let me just get back to what we were looking at before, and talk about what has gone on; how people have viewed vortex shedding behind a circular cylinder. People have called it Hopf bifurcation as a consequence of linear temporal instability. The above temporal instability is moderated by nonlinearity which can be explained by Landau equation. You can read it in Landau's original work or in Drazin and Reids monograph. Some numerical investigations have been done in mid 80s. Most of them were, either Galerkin finite difference calculations or finite element calculations, and all of them in synchronicity say that, this instability that you see from the solution of Navier-Stokes equation happens in the range of Re 45 to 46. This we can, like an urban myth, and all of us suffered, because of this insistence that, there is one such thing as Re critical. However, if we look at the experimental result, we find that, there is no such thing as a universal Re critical, like this numerical solution investigates, investigators showed. What really happens is that, different people have commented a different values; for example, if we look at Batchelors' book, he is conjectures that, it should be within 30 and 40; and Landau and Lifshitz actually quoted it as some 34, based on some unreported results; but it is there in Landau and Lifshitzs' book.

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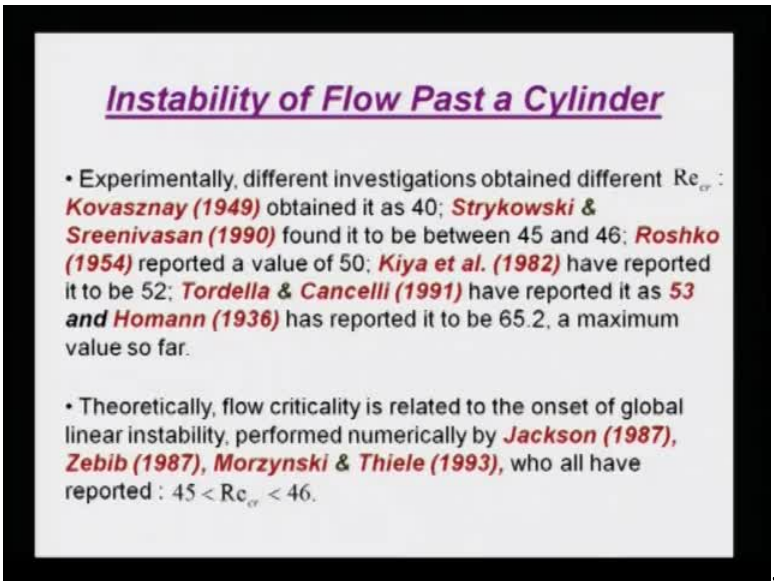

So, these are what you see in text books, what people have been talking about. If you look at the recorded experimental data, then, Kovasznay reported it to be as 40, but below that 45, magic number, and this numerical calculation. And, this was the work that I was talking about, Strykowski's work with Professor Sreenivasan, and they reported this results in late 80s; and it is something, they said that, it is between 45 and 46; is an interesting result. But if you look at Roshko's original work, in early 50s, in NASA, he reported a value of 50. This group in Japan, they reported it a value of 52, and Tordella and Cancelli has actually shown it to be 53. But this was what, yesterday we had talked about, Fritz Homann's work. Fritz Homann reported it to be 65.2, a very, a maximum value.

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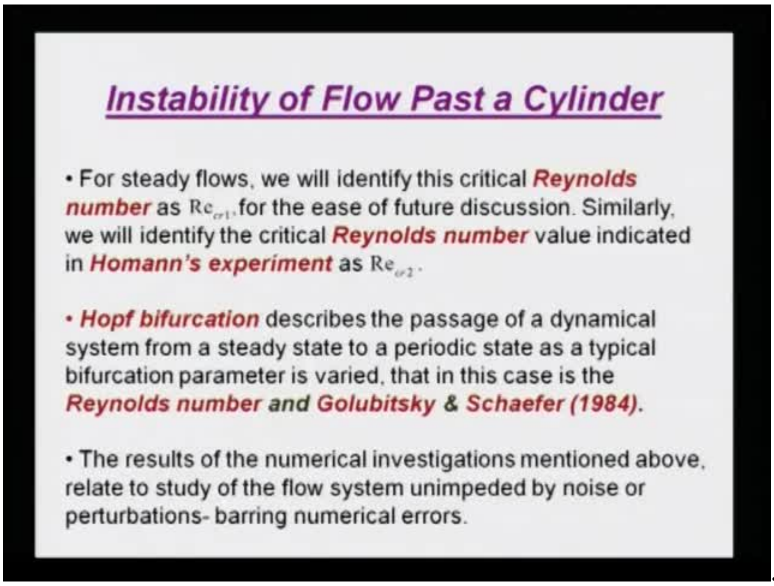

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So, if I want to talk about theoretically, flow criticality is related to the onset of global linear instability; that is what those three people claimed and they say that, this happens like this. And, we need to reconcile this two view point and we would do it shortly. So, what we see as the instability, reported by those numerical investigations, in the vicinity of 45, let us call that as Re critical 1. And, the value reported by Homann, let us call it as Re critical 2. Hopf-bifurcation actually describes the passage of a dynamical system from a steady state to a periodic state; as the bifurcation parameter, in this case, the Reynolds number, is varied.

So, in this is a nice reference; you can read about bifurcation; Golubitsky and Schaefer’s book. The results of the numerical investigation mentioned above, relate to study of the flow system, unimpeded by noise or perturbation, barring numerical errors. I think, the sentence looks very innocuous, but you have to understand it that, in experimental facilities, you always have background disturbances. Or for that matter, your cylinder is not a perfectly a circle; it can have surface irregularities.

That is not what you are doing numerically; but, when we do numerical calculations, we are subjected to numerical errors. And numerical errors, most of you know, come in various colors and shades. So, you have to think of it very carefully. It is not just simply saying it is truncation error, round off error; we know there are various sources of errors.

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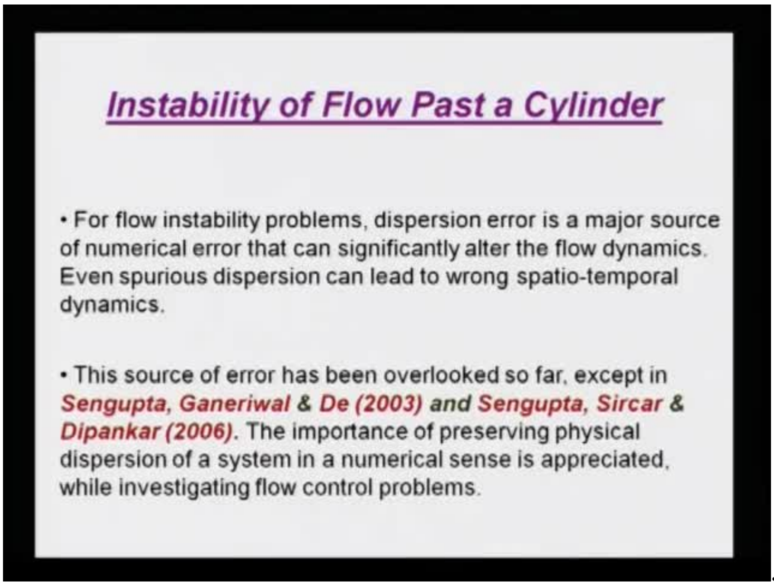

But we need to understand that, this is a major point of departure, when we try to reconcile experimental results with numerical results; because, we are not talking about same things; we are talking about apples and oranges. So, when we are talking of flow instability problem, dispersion error is a major problem. You see, what happens; we are talking about disturbances which are growing, evolving in space and time. How is it related? It is related through the dispersion relation, that is what we have been talking about. Numerically, most of the people were unaware of how this dispersion error affects calculation. That could be a major source of error. We can actually, create spurious dispersion and that, can lead to wrong spatio-temporal dynamics. And this, we have studied with great deal of care and we have identified that developing numerical method, we must preserve physical dispersion relation numerically.

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This was done because, we wanted to actually, study the work that was done by Strykowski and Sreenivasan. What they did was, if you have a cylinder, in its ((weak)) you put a smaller cylinder. So, basically, we are talking about, putting another cylinder here, somewhere, I do not know where it is; they did investigate. So, there is this. So, this is the main cylinder and we have this. So, this, we will call it as a control cylinder.

You know what, when you put such a control cylinder, many a times, if you position it correctly, and the Reynolds number is less than 100, you see vortex shedding disappears. This was, very interestingly found out by Kovasznay; you recall, I just now showed that, Kovasznay in 40s reported a critical Reynolds number of 40. He noted that, when you bring in a hot wire probe, to measure those fluctuations in the wake, that in some locations, the vortex shedding disappears. And, this observation of Kovasznay was picked up by Strykowski for his PhD thesis. They wanted to find out, what is this role of control cylinder; how you can control it. And, this was what, we actually studied numerically. And, they did those experimental studies. This is that famous JFM paper of 1990. Strykowski and Sreenivasan, they reported that, they showed not only that, you can change the shedding pattern by putting in a small control cylinder, you can do the same thing by putting in a heating element also.

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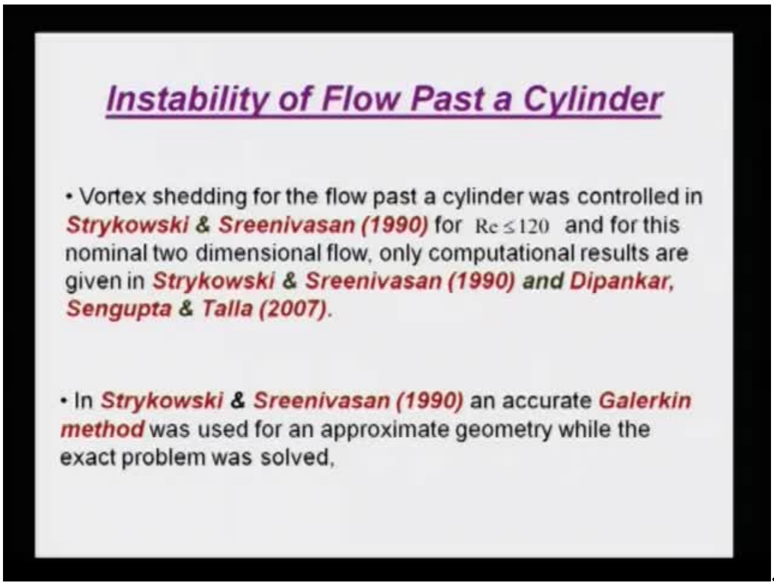

So, looks like, there are various ways of flow control. Unfortunately, though, such flow control works only for your Reynolds number. If you keep your Reynolds number restricted below 120, you can control it. So, this is the reason that, we looked at dispersion relation preservation scheme, because we wanted to control this. And, we actually reported results in this JFM paper, trying to explain, how flow vortex shedding can be controlled for Re less than 120, by just simply solving nominal two dimensional Navier- Stokes equation.

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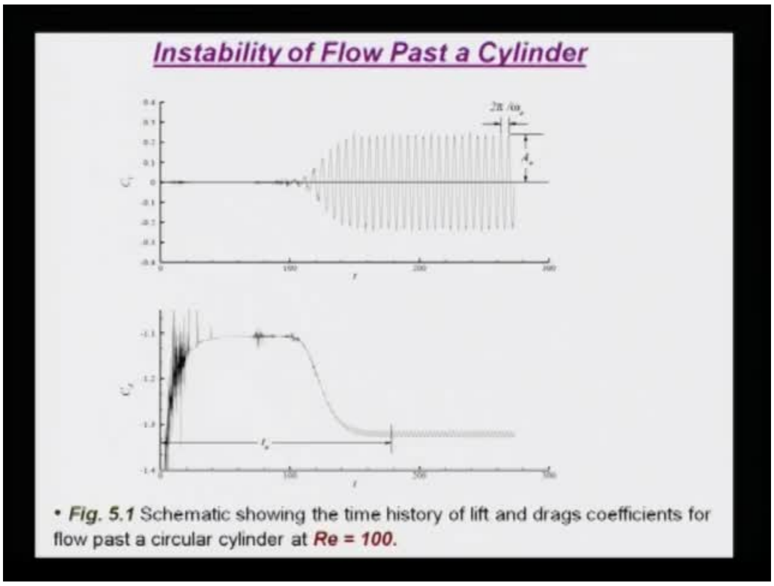

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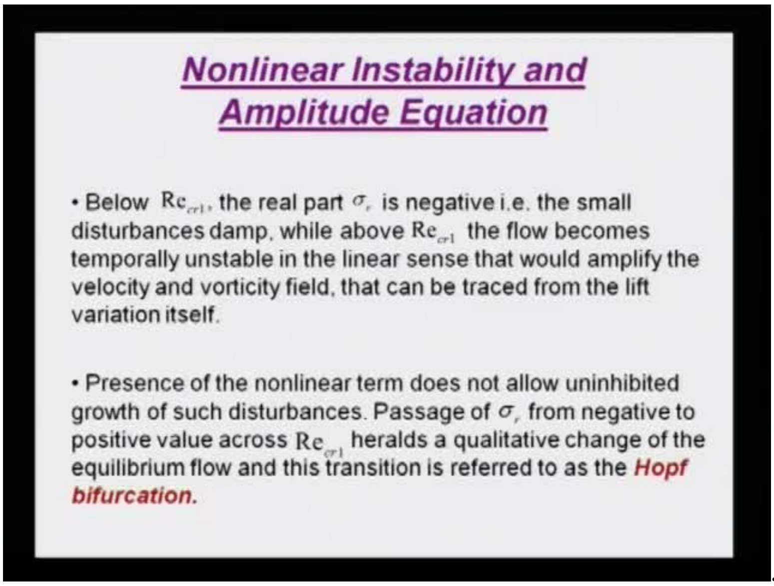

Well, with Strykowski and Sreenivasan, there were some computational results presented, but they did not really consider the actual geometry of the control cylinder. They had simply found out, a cluster of points where they did change the momentum transferred to mimic what is the effect of control cylinder. But the work that we had done in the 2007 paper, we did the actual, we solved the problem. And, this is what we see, very clearly. We explained that, if we are careful, we see that lift coefficient evolves like this; you have very high frequency oscillations; that is also seen in the drag coefficients; but then, once the linear instability of picks up and the nonlinearity saturates, then, you get your low drag values. I mean, these are not very low drag value, but they are quite significant. This is a function of Reynolds number. I think, we need to talk about numerical issues later. What I would like to do is, talk about what happened, is this issue of non-linear effects moderating instability and amplitude equation.

So, below Re critical 1, the real part of sigma r is negative. So, of course, any, you do not have linear instability; while above Re critical 1, the flow becomes temporally unstable, in the linear sense; that would amplify velocity vorticity field, and that, you can trace it from the left variation itself. Presence of the non-linear term does not allow uninhibited growth of such disturbances. Passage of sigma r from negative to positive value, across Re critical, heralds the qualitative change of the equilibrium flow and this is a formal definition of Hopf-bifurcation, which we would be following.

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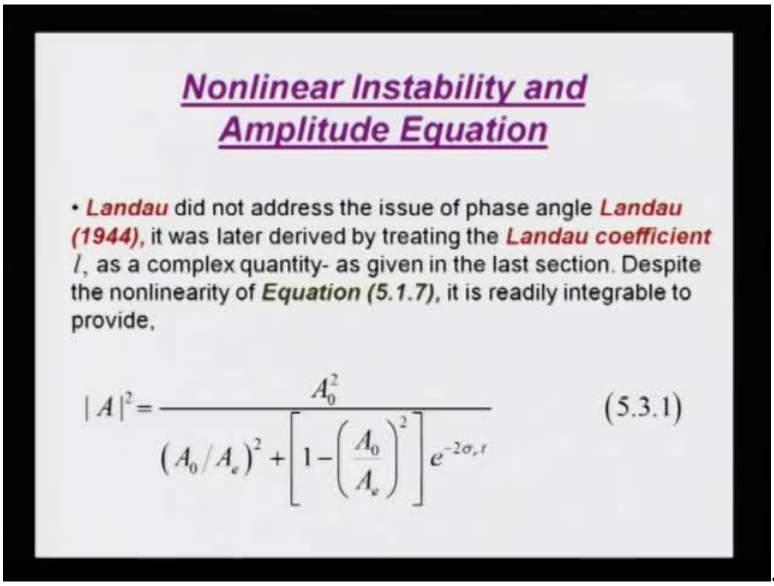

And, this is the Landau's equations' analytic solutions. As I told you, you can see the time dependence comes here from e to the power minus 2 sigma r t. Now, this is positive. So, the exponent is negative. So, as time increases, this quantity goes away. What is e naught? e naught is the initial solution. What is A e? A e is what we have written here, the equilibrium solution. So, what happens, for very large time, this goes off and this is A square A square goes off and you get A e. So, you get a solution, an equilibrium amplitude, which does not depend on initial condition. So, this was something, that is what everybody thought that, they should be able to do, that all experimental facility should show as a universal value; because it does not depend on A naught. So, we will stop here today; we will start from this point on and see, what can be done furthermore.