

**Lighter-Than-Air Systems**  
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**Lecture – 52**  
**Pressure Height**

Today I am going to cover a portion that we had not covered before the midsem because it was getting too theoretical and I thought let us give a break, but this is again part of the previous capsule. So, let us revisit pressure height.

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**PRESSURE HEIGHT**

- Definition of ( $H_{ph}$ )  
 Pressure altitude at which an airship's ballonets become empty
- Inflation fraction relationship
  - Ignoring superpressure,  $I_2 = \frac{P_{S1}}{P_{S2}} \frac{T_{A2} + \Delta T_{sh2}}{T_{A1} + \Delta T_{sh2}} I_1$
  - Under ISA conditions with zero superheat
    - $T_A = T_S$  and  $\Delta T_{sh} = 0$ ;  $I_2 = \frac{P_{S1}}{P_{S2}} \frac{T_{S2}}{T_{S1}} I_1$
    - $I_2 = \frac{\delta_1}{\delta_2} \frac{\theta_{S2}}{\theta_{S1}} I_1$       OR       $I_2 = \frac{\sigma_{S2}}{\sigma_{S1}} I_1$

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And first of all let us see if we can define pressure height and from now on we will call it as  $H_{ph}$ . So, this is the definition of pressure height about which you are all familiar. It is a height as the airship keeps climbing, we release the air from the ballonet and you reach a height at which the ballonet become flush and there is no air left in the balloon. So, that is the pressure height and the analogous definition will be the ceiling for an aircraft.

So, let us remember or recall the inflation fraction relationship applicable. So, if we ignore superpressure, we have seen earlier that the inflation fraction at any operating condition 2

$$I_2 = \frac{P_{S1}}{P_{S2}} * \frac{T_{A2} + \Delta T_{SH2}}{T_{A1} + \Delta T_{SH1}} * I_1$$

So, I have ignored superpressure, but I have retained the superheat terms. So, this is something that we had already derived earlier and just reproducing here for continuity. So, if you consider ISA conditions, so what will happen under ISA condition is that the ambient temperature  $T_A$

will become the standard temperature under ISA at that altitude called as  $T_s$  and the  $\Delta T_{SH}$  or the superheat contribution will be 0.

So, we get a very simple expression

$$I_2 = \frac{P_{S1}}{P_{S2}} * \frac{T_{S2}}{T_{S1}} * I_1$$

So, this simple relationship directly connects inflation fraction at any altitude with inflation fraction at some other altitude with the pressures and the temperatures. Once again, we are ignoring superheat, we are ignoring superpressure., we are considering ISA conditions and we are also ignoring in this case any lifting gas purity or the effect of humidity, etc.

This is a very simplistic explanation. Now under ISA conditions, the ratio of pressures  $\frac{P_{S1}}{P_{S2}}$  is called as  $\delta$  and the ratio of temperatures is called as theta,  $\theta$ . So, you can replace this by the standard relationships. And once again, we also know that the value of theta by delta is sigma or the density ratio. So, therefore inflation fraction at any operating condition under standard ISA will be equal to

$$I_2 = \frac{\sigma_{S1}}{\sigma_{S2}} I_1$$

So, this simple relationship connects any two operating altitudes with their densities.

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**PRESSURE HEIGHT AND GROUND-LEVEL INFLATION FRACTION**

$$I_G = \frac{P_{S_{ph}}}{P_{S_G}} \frac{T_{A_G} + \Delta T_{sh_G}}{T_{A_{ph}} + \Delta T_{sh_{ph}}} I_{PH}, \text{ Where}$$

- $I_G$  = inflation fraction at ground level
- $P_{S_{ph}}$  = atmospheric pressure at pressure height ( )
- $P_{S_G}$  = barometric pressure at ground-level
- $T_{A_G}$  &  $T_{A_{ph}}$  = ambient temperature at ground-level & PH
- $\Delta T_{sh_G}$  &  $\Delta T_{sh_{ph}}$  = superheat at ground level & PH

**What is the value of  $I_{PH}$  ??**

$$I_{PH} = 1$$

$$I_G = \frac{P_{S_{ph}}}{P_{S_G}} \frac{T_{A_G} + \Delta T_{sh_G}}{T_{A_{ph}} + \Delta T_{sh_{ph}}}$$

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Now, let us look at what happens when the airship for some reason has to fly above the pressure altitude? And also let us look at the relationships. So, I am replacing the same expression from the previous slide here. I am calling it as  $I_G$  it stands for the inflation fraction at ground level

equivalent to the subscript 1 in the previous slide. And I am putting PH as the subscript for 2 considering PH to be the pressure altitude or the pressure height.

So, once again we get  $I_G$  inflation fraction at ground level is equal to

$$I_G = \frac{P_{SPH}}{P_{SG}} * \frac{T_{AG} + \Delta T_{SHG}}{T_{APH} + \Delta T_{SHPH}}$$

is this clear? So, we have just taken the simple relationship between any two points and we have considered those two points to be the ground and the pressure altitude and we are assuming that one to one relationship exists. Please note here I have not assumed anything like ISA, although I have ignored superpressure and retaining superheat.

Now, what is the value of  $I_{PH}$ , inflation fraction at pressure altitude? 100% or 1, ratio is 1. So, therefore you can replace it by 1 and hence you can say that inflation at ground would be equal to just the ratio of pressures and the ratio of temperatures including the superheat is present because the inflation fraction at the pressure altitude is 1. There is no air left, the ballonnet is flush.

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**H<sub>PH</sub> & I<sub>G</sub> Contd.**

$$I_G = \frac{P_{Sph}}{P_{SG}} \frac{T_{AG} + \Delta T_{shG}}{T_{Aph} + \Delta T_{shph}}$$

□ Assume ISA conditions & Zero superheat at P

□  $T_{Aph} = T_{Sph}$  and  $\Delta T_{shph} = 0$

□  $I_G = \frac{P_{Sph}}{P_{SG}} \frac{T_{AG} + \Delta T_{shG}}{T_{Sph}}$  Hence  $\frac{P_{Sph}}{T_{Sph}} = \frac{P_{SG}}{T_{AG} + \Delta T_{shG}} I_G$

Substituting  $P_{Sph} = \delta_{ph} P_0$  and  $T_{Sph} = \theta_{SPH} T_0$  we get :

$$\frac{\delta_{PH} P_0}{\theta_{SPH} T_0} = \frac{P_{SG}}{T_{AG} + \Delta T_{shG}} I_G \text{ or } \sigma_{SPH} = \frac{T_0}{P_0} \frac{P_{SG}}{T_{AG} + \Delta T_{shG}} I_G$$

where  $\sigma_{SPH}$  = ISA relative density at pressure height.

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The same formula we copy ahead. Now, let us look at these expressions once again. So, if we assume ISA conditions and zero superheat, then the  $T_A$  will become  $T_S$  and the  $\Delta T_{SHPH}$  will vanish. So, therefore the equation will become

$$I_G = \frac{P_{SPH}}{P_{SG}} * \frac{T_{AG} + \Delta T_{SHG}}{T_{SPH}}$$

instead of A it becomes S and what you can also do is you can basically leave the pressure ratios on one side and take the other terms on that side.

So I am just inverting the sides. And if we now continue to look at the ISA conditions or if we look at standard conditions, we have delta and theta as the ratios. So, when we put these values you can get a expression like this. What is the advantage of doing this?

The advantage of doing this is that you bring in  $T_0$  and  $P_0$  which are constants,  $T_0$  being 288.16 degrees Kelvin and  $P_0$  being 101325 newton per meter square. So, these two numbers are constants. So therefore their density ratio of the pressure altitude will be equal to two standard quantities their ratio times pressure at the ground under standard conditions, temperature at the ground and if any superheat is present times  $I_G$ . So, it simplifies.

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**H<sub>PH</sub> & I<sub>G</sub> Contd.**

$$\sigma_{SPH} = \frac{T_0}{P_0} \frac{P_{SG}}{T_{AG} + \Delta T_{shG}} I_G$$

Substituting constants and conversion factors, we get

$$\sigma_{SPH} = 0.284 \frac{P_{SG}}{(273 + T_{AG} + \Delta T_{shG})} I_G \quad (P_S \text{ in hPa (mB) and } T \text{ in } ^\circ\text{C})$$

We know that  $I_2 = \frac{\sigma_1}{\sigma_2} I_1$ ; (ignoring Superheat & Superpressure)

Thus,  $I_{GISA} = \frac{\sigma_{SPH}}{\sigma_{SG}} I_{PH}$ , Where

- $I_{GISA}$  = inflation fraction at ground level under ISA
- $\sigma_{SPH}$  &  $\sigma_{SG}$  = ISA relative density at PH and ground level

But  $I_{PH} = 1$  so  $I_{GISA} = \frac{\sigma_{SPH}}{\sigma_{SG}}$        $\sigma_{SPH} = I_{GISA} \sigma_{SG}$

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Now, the same expression I will just copy and paste here. So, this is the same thing which I am transferring to the next slide. Now, let us look at these numbers. So, I would like you to substitute for  $T_0$  and  $P_0$ ,  $P_0$  101325 and  $T_0$  288.16. I expect people to bring calculators in the class. So can you please do this? Find this ratio? So, all you can do in this is replace for  $T_0$  and  $P_0$  and what would the value be for  $T_{AG}$ .

If you assume G as the ground level or as the sea level, ambient temperature at ground or at ground zero altitude 288.16 same as  $T_0$  for sea level. So, what do you get? I want an expression which says  $\sigma_{SPH}$  which is the density ratio for a given pressure altitude in terms of the ambient

pressure on the ground, temperature on the ground, ground level inflation fraction and  $\Delta T_{SHG}$ . How much will it be? 0.284, correct.

So, if you now replace  $T_{AG}$  in centigrades, then it will be 273 degrees plus  $T_{AG}$ . So, by this simple expression you can get the value of the  $\sigma$ . So, how does it help you? How does this expression help you? Why are we doing this? Everything is in ground level. So, therefore what does it get you? What does it help you obtain? Let us say you are operating your airship from some place.

So, you know the pressure at that place  $P_{SG}$ , you also know the  $T_{AG}$  temperature at that place and you also know the superheat, temperature increase because of superheat. So, assume that I have a temperature sensor inside the envelope, I keep this airship on the ground for a long time and I find that because of the heating of the sun there is some temperature increase of the gas. So, when I know all these things, what do I get?

$\sigma_{SPH}$  that you get  $\sigma_{SPH}$  if you know the value  $I_G$ . Now think reverse, if you want to have a particular pressure altitude that means let us say you want to fly at least up to 5000 feet, then  $\sigma_{SPH}$  you can get from the tables. So with that we can get now the value of  $I_G$  that is what should be the inflation fraction on the ground, so that this airship from these operating conditions can go to a particular pressure altitude.

So, let us say you are planning a profile or a journey for airship from Pune to Mumbai or Mumbai to Pune. So, you need to know what will be the altitude which I should cover. So, we can back calculate. So, you can come to know how much should be the volume of air in the ballonnet as a percentage of the total gas for me to create the possibility of flying up to a pressure height because in normal circumstances we do not wish to exceed the pressure height.

Today, we will see what happens when you exceed the pressure height also but you would not plan for it. So, now we know that

$$I_2 = \frac{\sigma_1}{\sigma_2} I_1$$

for any operating condition if we ignore superheat or superpressure. So, therefore the inflation fraction under ISA conditions will be equal to the ratio of density altitude as pressure height upon the sigma in the ground into  $I_{PH}$ , right.

But  $I_{PH}$  will be 1, so therefore you can easily get the value of  $\sigma_{SPH}$ . So, the place where you are operating you also know the density of the air at that particular condition  $\frac{P}{RT}$ . So, let us say you know the density ratio  $\sigma$ . So, this will easily tell you either if you fix the inflation fraction at ground level, you will know how much higher you can go by getting the sigma value.

Please understand that  $\sigma_{SPH}$  is a number which from the atmospheric tables can give you the H or you can do the reverse if I want to get to that height, I should have the inflation fraction up to a particular minimum value.