Introduction to Launch Vehicle Analysis and Design

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Lecture - 06
Idealized Performance

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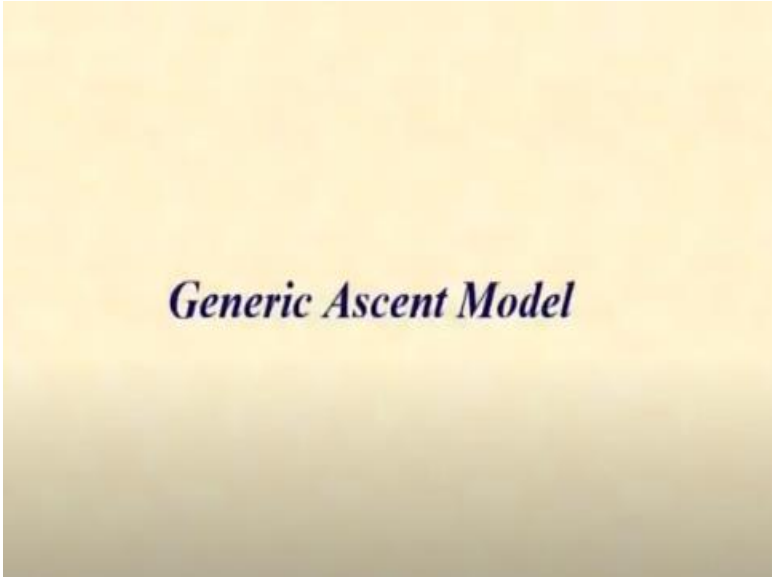

Hello and welcome. Now that we have established the various force models as per our Newton's law, which governs the motion of the launch vehicle, we now are in a position to synthesize those forces into a general form of the mathematical model that is applicable for the ascent mission.

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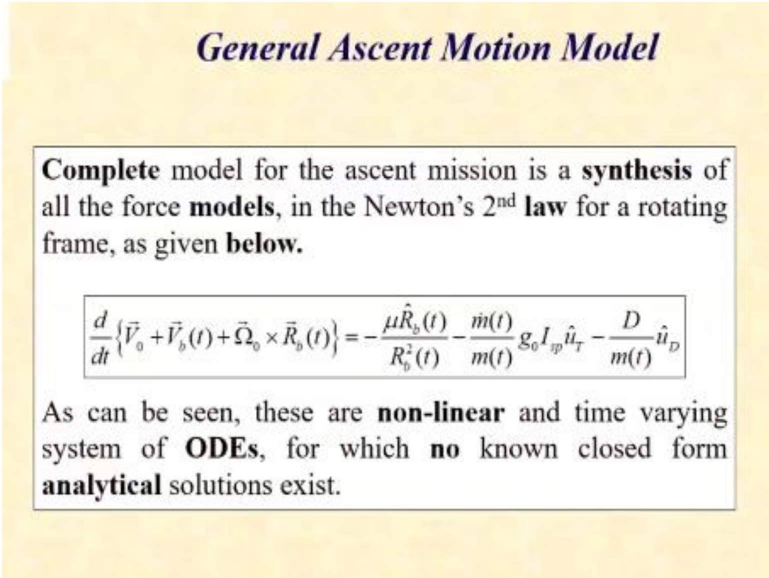

It is just a kind of a summation of all the forces on the right-hand side. The velocity expression that we have seen earlier and the Newton's law that they hypothesize assuming that earth's surface where we are going to define our coordinate system is a rotating frame of reference. And with that, we get the following equation. This is a first order vector differential equation in terms of the velocity and the forces.

So, as you can see, we have included the velocity expression in the most general case. And on the right-hand side now we have the three force terms that we have discussed just now. The first one is the force term coming because of gravitational, but expressed in a coordinate system which is along the radial direction. Then we have the thrust expression which is along the thrust direction, which is commonly the axis of the launch vehicle.

And the last one the drag term which is along the drag direction, which will be along the local velocity direction. So, as you can see the vectors involved in this expression are all in different directions and they will have to be described in a common coordinate system, Cartesian or polar or any other coordinate system, before we can actually proceed with the solution of this differential equation.

Of course, we also note that these equations are nonlinear in nature. They are also time varying because the mass is not constant. And there are no known closed form analytical solutions for the above vector differential equation. Of course, at some point these equations will have to be solved through numerical techniques to generate accurate solutions for the trajectory.

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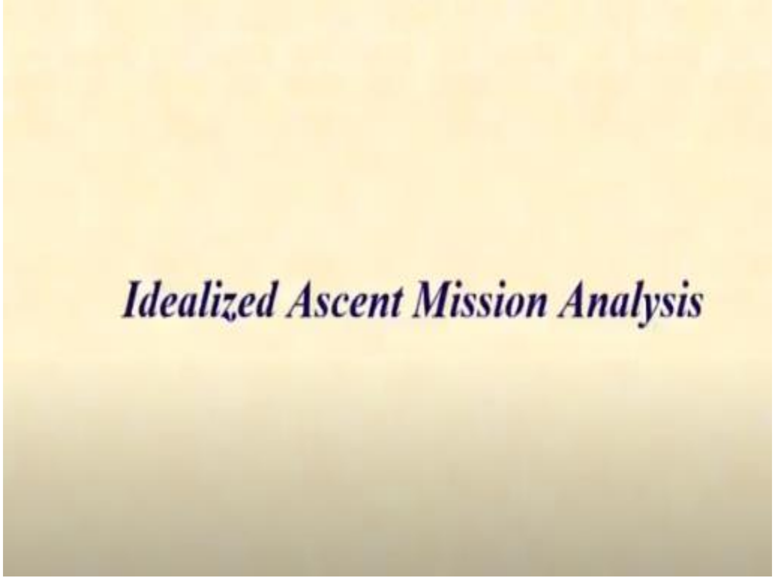

But to understand the overall implication of this equation, we can adopt a simplified approach through what we call an idealized ascent mission analysis, wherein we make certain assumptions, and in the process the equations are simplified so that we get a broad idea of what the performance is likely to be in a given context without large amount of computational effort. So let us begin our discussion on the idealized ascent mission analysis.

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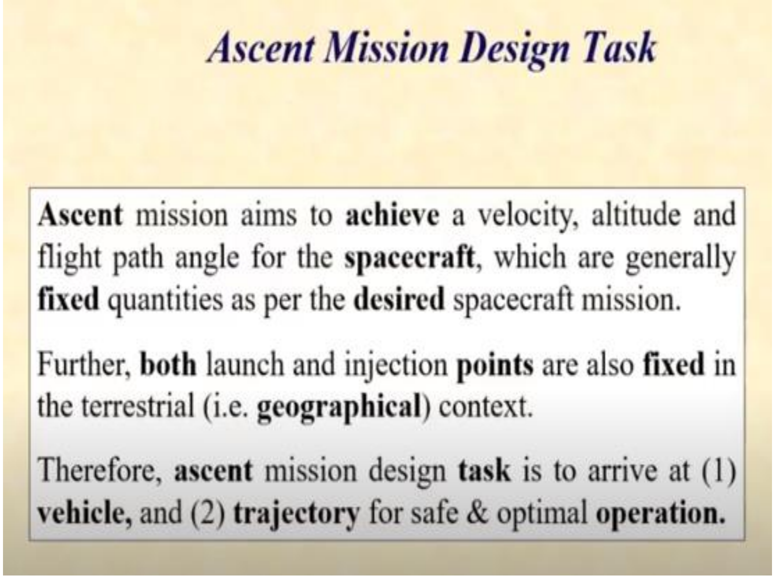

In this context, let us first establish the ascent mission design task that is of our main interest. The main aim of ascent mission is to achieve a velocity, an altitude and a flight path angle for the spacecraft or the space object that we are going to release at the end of the mission. And typically, these are fixed quantities as per the desired spacecraft mission. With this course, we are not directly dealing with the spacecraft per se.

So, we will assume that there are going to be different kinds of spacecraft missions, which will require different values for the velocity, the altitude and the flight path angle at the terminal point so that our ascent mission to some extent will now be designed or will have to be designed in the context of the terminal conditions. Of course, we also realize that the launch point is also fixed.

By and large at least for each country, where they have those launch stations, the launch point is known. So, in the geographical context, both launch point and the injection point are going to be known a priori before a launch vehicle ascent mission is being planned. And that brings us to the fundamental requirement on the ascent mission as two tasks.

The first one to arrive at a vehicle which will achieve this objective and a trajectory or a flight path that this vehicle should take which will result in a safe and an optimal operation. So, you will realize that our ascent mission analysis and design will involve these two tasks that the vehicle itself and the trajectory.

Of course, we may not look at it in that order, but these are the two tasks that we will have to perform before we can say that we have an ascent mission, which ultimately is going to achieve the objective of a space mission.

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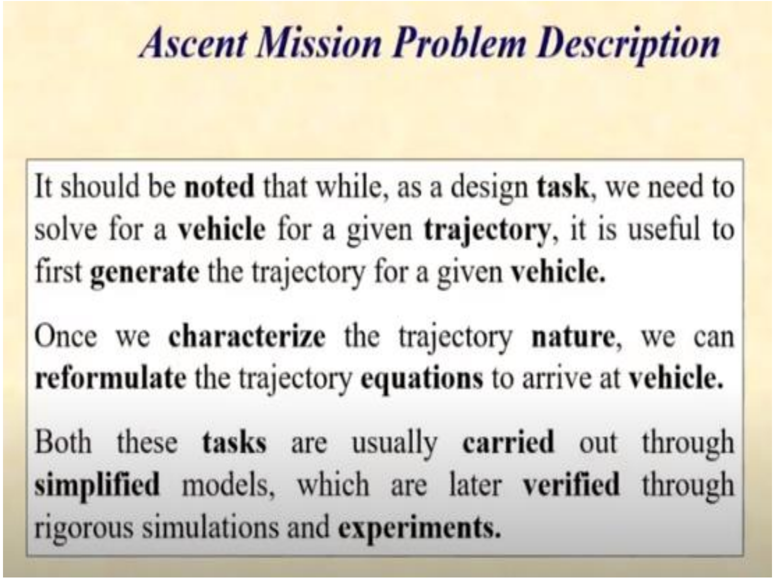

Of course, if we see it from a design perspective, we realize that we need to solve for a vehicle for a given flight path or a trajectory. The reason being that your starting point is fixed, the endpoint is fixed and there are only a few possibilities for the flight path that you can think of for making the mission optimal, safe and feasible.

So generally, the design task would be approached in the manner that a trajectory or a flight path would be specified and you would have to arrive at a vehicle which will achieve that flight path and trajectory. However, we are going to start our discussion with the reverse process that if I have a launch vehicle, what kind of flight path I will get.

And this becomes an analysis problem where for a given launch vehicle, we arrive at the flight path and the terminal parameters and see whether they meet the requirement, go back and tweak the rocket so that you achieve what you want. This way also one can actually arrive at the launch vehicle and the trajectory but through an iterative process.

Of course, once we understand the nitty gritty of this process, we can always invert this exercise and then we say okay, now can we use this information to specify a trajectory and then arrive at the rocket. We will actually do both of these, but we will first go through the process of arriving get this understanding through the analysis route. Of course, the next step is are we going to use the most general differential equation that we have seen just now?

Obviously not, because for this exercise, we need simplified models, which we can work with quickly and then we set up a process by which we can verify the result through rigorous simulations through both accurate modeling and even through experiments.

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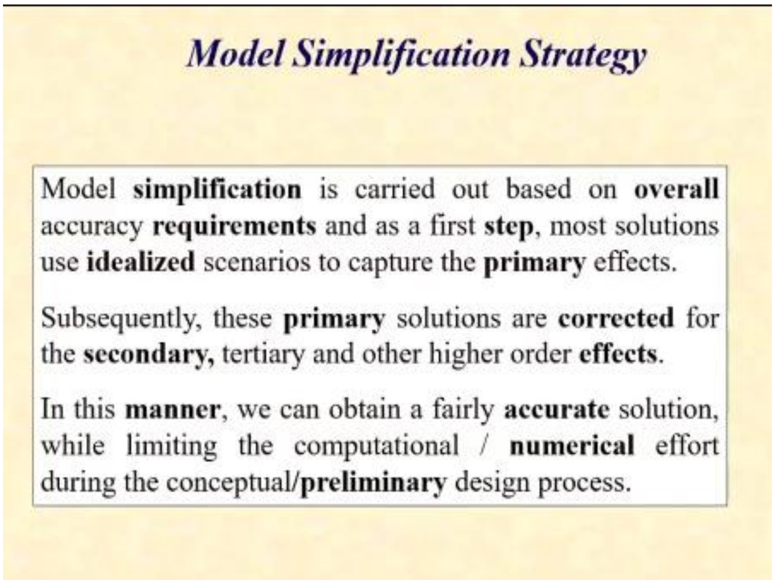

So how do we do this model simplification? So, the strategy for model simplification is based on primarily overall accuracy requirement of the solution, which means how accurate solution are you looking at? Are you looking at 1% accurate, 0.1% accurate or 5% accurate or even 10% accurate? Once you fix a number it is possible for you to ignore certain higher order effects which are necessary for achieving those accuracies.

And then, once you ignore the higher order effects, the equations automatically simplify to a form where simpler solution procedures can be used. So, we are going to use what is called an idealized scenario and to capture the primary effects. So, we are going to divide this whole game into three segments, the primary effects which is the most dominant physical effect, which is going to decide the performance.

Then the secondary which is the next level and the tertiary which are at further lower level. And then we say that we are first going to generate the solution for the primary effect and then we are going to correct it for the secondary effect. And then we will see whether the tertiary effects need to be taken into account or not.

Of course, even by including the secondary and the tertiary effects, we will make use of a simplified representation so that it still does not require too much of computational effort, but still gives us an order of magnitude understanding of what the secondary and the tertiary effects are going to be. In this manner, we can obtain a fairly accurate solution.

Please note the solution is not going to be very approximate once you include the secondary and the tertiary effects. But you are going to still limit the computation numerical effort, which is extremely useful during the conceptual or preliminary design process.

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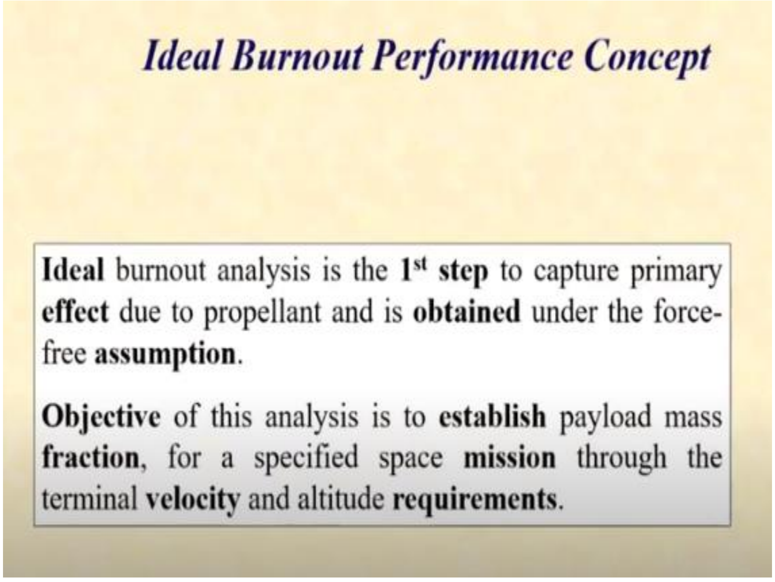

And in this context, I am going to talk about once the simplification and analysis, which goes by the name of ideal burnout performance, which is the first step to capture the primary effect due to the propellant. See, we have to realize that our primary objective is to launch a particular space object with a certain velocity. So obviously, propulsion is our primary focus.

We are going to burn a large amount of propellant. So obviously, we are going to generate large thrust and thrust is definitely going to be the major force. So initially, we can say that apart from thrust, there is no other force present or what is called the force free assumption. And we find that even with such a high level of simplification, we may be in a position to establish an important design parameter called the payload mass fraction.

For a specified space mission through prescription of terminal velocity and altitude requirements. And we find that this is a very important first step in sizing the ascent mission and the launch vehicle.

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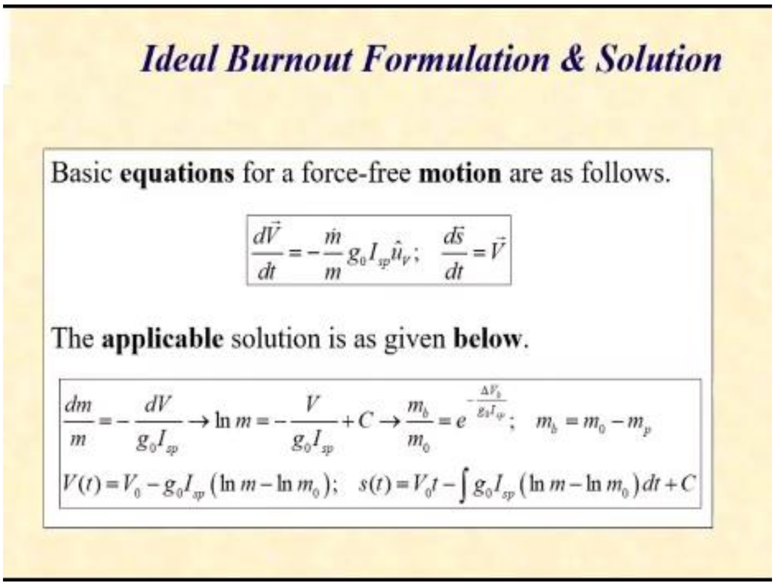

Let us see how we can do this. So, with this assumption that apart from thrust, there are no other forces and that the vehicle is moving along at least to start with no gravitational forces, so moving along the radial line. We can simplify our equations of motion using the following equation. Here we have assumed that the rotation of earth and movement of earth surface has no impact on the velocity for the ascent mission.

This assumption essentially is driven by the fact that typical durations of any ascent mission are of the order of about 15 to 25 minutes, during which the changes in the position of earth in relation to the inertial frame of reference are not very large. So those kinematic effects we have ignored and we realize that we get an extremely simple representation.

This differential equation where we assume that the thrust is in direction of the velocity. Which means that we can now convert this into a scalar differential equation as $\hat{u}_v$ the unit vector on both sides is the same. And we can simply solve this equation as follows. So just applying the principles of our vector calculus and integral concepts, we can show that the solution of this differential equation appears in the form of this exponential expression where the mass is involved and the velocity is involved.

If you want to write the velocity expression, then it is in the logarithmic form. Of course, the kinematic relation as shown here in the above equation that is $\frac{d\vec{s}}{dt} = \vec{V}$ can also be integrated as expressed here. The question is what have we got? So let us explore this result a little further.

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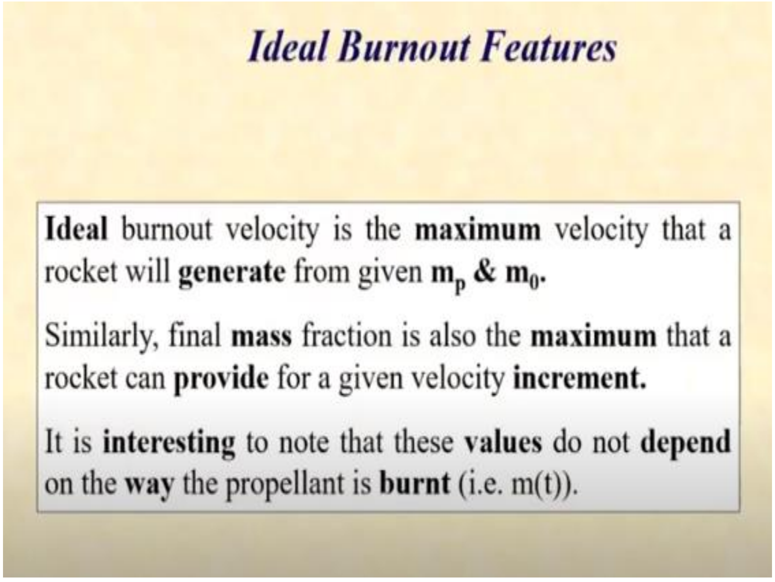

One thing that we note is that the ideal burnout velocity V_b which is mentioned in the expression earlier is the maximum velocity that the rocket will generate from given propellant and the liftoff mass. That is the maximum that you can take out of that rocket. Because we have ignored the effect of gravity, we have ignored the effect of drag, we have ignored all other effects.

So, this is the best possible performance that you can expect. And now we say that, because this is the maximum velocity that I can get, I will invert this relation to say that this is also the maximum mass fraction that the rocket is capable of giving for a specified velocity. And this is where the design wisdom is.

The design wisdom is that, if I have a velocity requirement, then that velocity when substituted in those simplified expressions will give me an $\frac{m_b}{m_0}$ or a final burnout mass fraction per unit liftoff mass. And it will tell me that for a given launch vehicle with a particular m_0 what is the amount of final burnout mass that I can have? And associated with that is the amount of propellant that I must carry and which should have what I_{sp}.

Of course, it is also interesting to note that these values do not depend on the way the propellant is burnt, which means is the best possible performance that one can think of.

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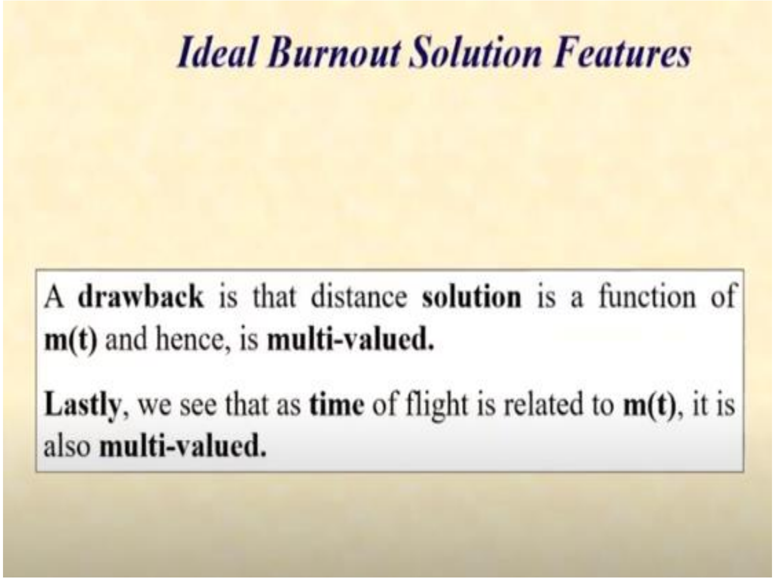

Of course, a drawback in this analysis is that the distance solution is multivalued. It will depend upon how I am burning. So different distance solutions. So obviously distance solution is not really useful in this context. But the velocity and the mass fraction are definitely useful. The same I can say about the time of flight, how long it will take to finish.

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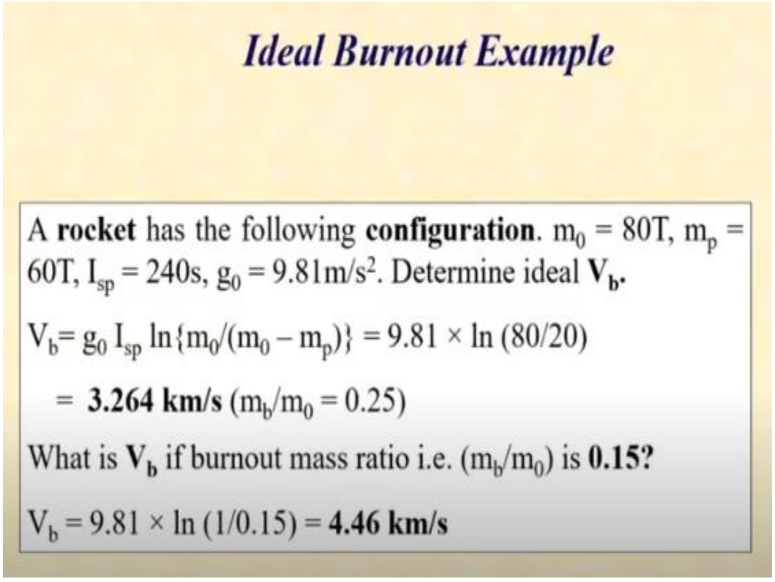

So let us go through a simple example just to understand what we talk about. So let us take a rocket that has the following configuration. That is, it has a liftoff mass of 80 tons, is carrying 60-ton propellant and that propellant has an I_{sp} of 240 seconds. Let us see what is the ideal burnout velocity that we are going to get.

So, what we do is, we use this expression $g_0 I_{sp} \ln \frac{m_0}{m_0 - m_p}$, the solution that we have just now generated. Substitute these quantities and this is what we get. So, we get a V_b of about 3.2 km/s or 3264 m/s with the provision that at the end of this ascent mission 20 tons of mass will be left which is not burnt. Which means my burnout mass fraction is 0.25.

Now, I want to explore this question that supposing I wanted to reduce my burnout mass ratio from 0.25 to 0.15. What is the V_b that I can get? Indirectly what I am saying is that, if I carry more propellant, what happens? When I say that my burnout mass fraction is 0.15, you get immediately guess how much of propellant I am carrying over above the 60 tons.

For this fraction of 0.15, we find that the burnout mass is going to be some percentage of, it is 15% of m_0 and 15% of 80 tons is 12 tons. So instead of 20 tons, we have only two 12 tons of burnout mass, which means we have now added 8 tons of additional propellant. And for that 8 tons of additional propellant, look at the change in the velocity. Instead of 3.26 km/s, we now have the velocity of 4.46 km/s.

And that is approximately 1.2 km/s higher or close to an increase of around 33%. A very large increase for an additional 8 tons of propellant. The reason why I am giving this example is to sensitize you to this idea of what that idealized performance equation is giving us. It is giving you an important mechanism through which you can start then looking at different options.

How much propellant you should carry, what should be the I_{sp} of the propellant, how much of the burnout mass you can get, what is the performance coming from the spacecraft requirement? All that you can do with this simplified expression.

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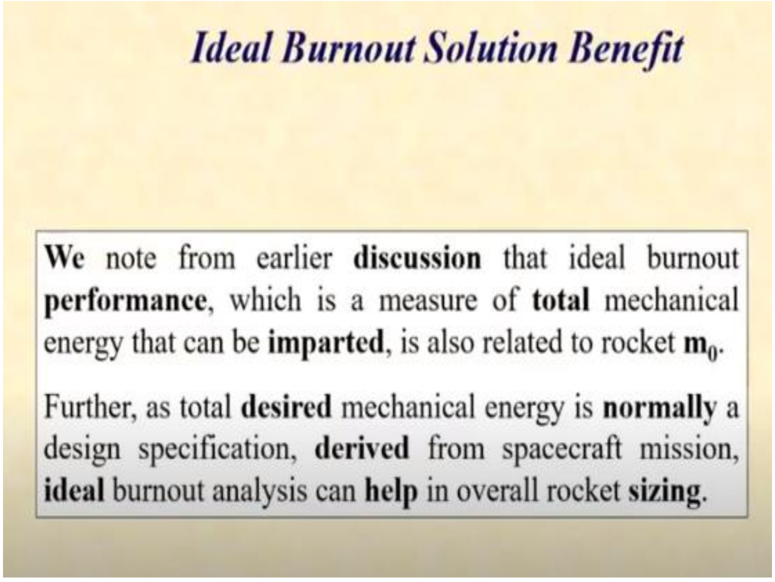

Of course, you also probably would have realized that this particular ideal burnout velocity represents the maximum amount of mechanical energy that can be imparted to that burnout mass and is of course related to the liftoff mass. So, there is another aspect that we can think of that if you are given the velocity, and if you are given the burnout mass, can you come up with what rocket will make it happen, which means can you come up with the value of m_0 ?

And you immediately realize that the same expression for known values of V, I_{sp} and m_b, we can find out what should be m naught. Which means, it will tell you which rocket will do the job or whether there is such a rocket which can do this job. Which means, you can also talk about feasibilities of the design requirements.

Suppose, you put a requirement which is not really practical or feasible, it will be possible for you to immediately check without any computational effort at all. And then we realize that the simple analysis that we had done can help us in overall rocket sizing without too much of computational effort, and it is a reasonably good solution, you have to understand.

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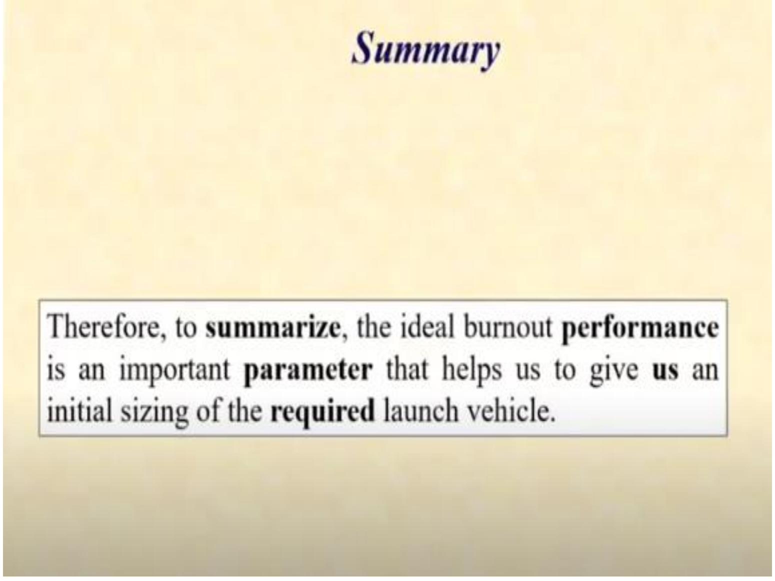

So, to summarize, the ideal burnout performance is an important parameter that helps us to give an initial sizing of the required launch vehicle. Of course, we will now make it more realistic, bring in other effects later in subsequent lectures. So, bye and see you in the next lecture and thank you.