Introduction to Launch Vehicle Analysis and Design

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Lecture - 33

Rectilinear Trajectories

Hello and welcome. So, this particular session is tutorial number 1 in which we will go through some of the problems in the context of a rectilinear trajectory. And we will also look at some of the important solution steps and aspects of the various formulations that we have done in this context with more details. So let us begin.

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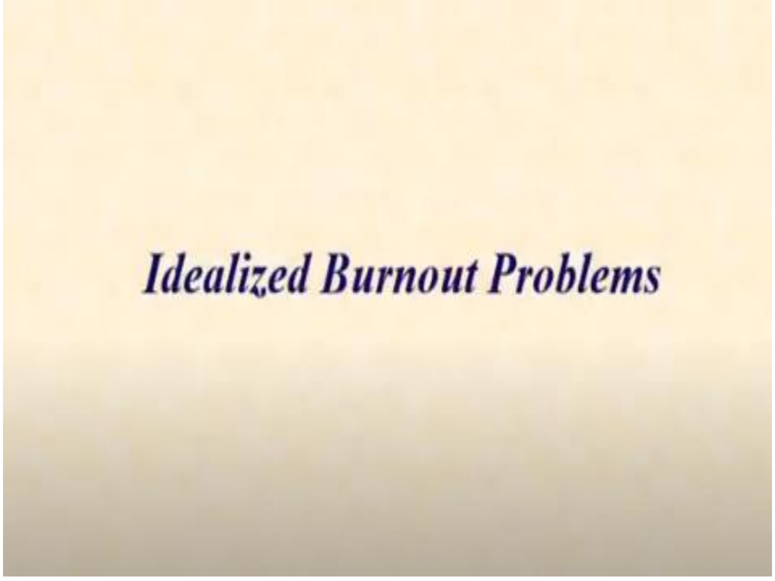

Let us first explore the idealized burnout solution that we had started off at the beginning of our discussion on launch vehicle trajectories.

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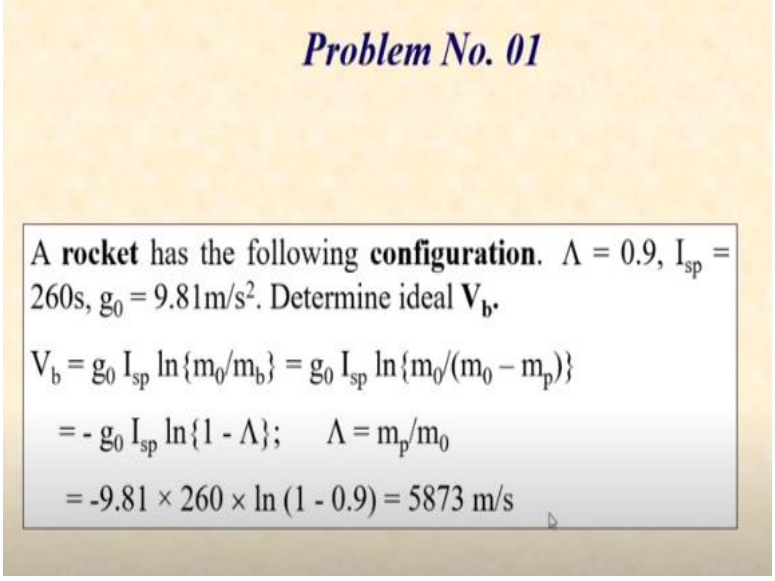

So let us take the problem. So, a rocket has the following configuration. That is its propellant loading is 0.9 that is $\frac{m_p}{m_0}$ at a specific impulse of 260 seconds. Let us try and determine the ideal burnout velocity. So, we recall the expression for the ideal burnout velocity as $g_0 I_{sp} \ln \frac{m_0}{m_b}$, where m_b is your burnout mass.

We know that this m_b is nothing but $m_0 - m_p$, where m_p is your propellant mass. So, what we can now do is we can bring the symbology that we have defined earlier and we can show that the expression for V_b in the context of ideal burnout is $-g_0 I_{sp} \ln\{1 - \lambda\}$ which is defined in the problem. So now we have all the parameters available for this expression and which now use to evaluate.

So, it is a simple task to substitute these numbers. So, 9.81×260 that is our I_{sp}. And the $\ln(1 - 0.9)$ which is of 0.1. And this results in a number 5873 m/s. My suggestion is you can also independently carry out this task once you have gone through this particular session completely so that you also become familiar and comfortable with these expressions.

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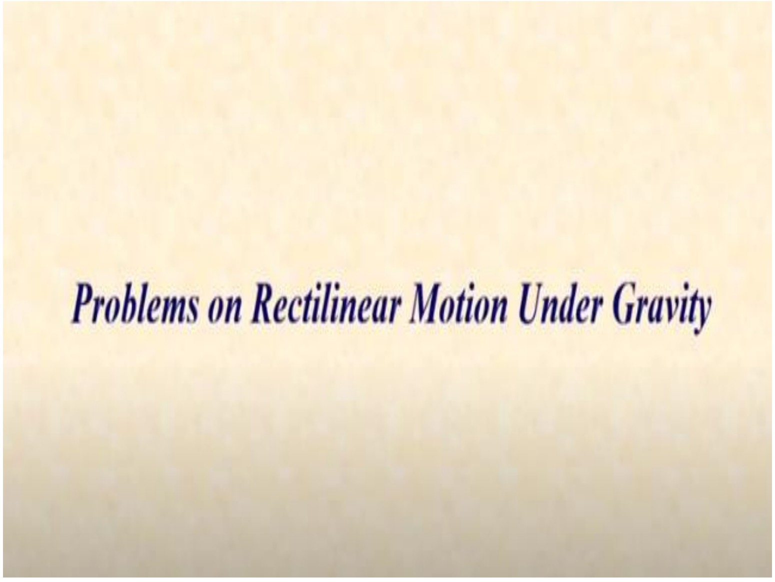

Let us now move over to the problems of rectilinear motion under gravity. Which means, let us now include the effect of gravity and let us look at the nature of problems which will get solved.

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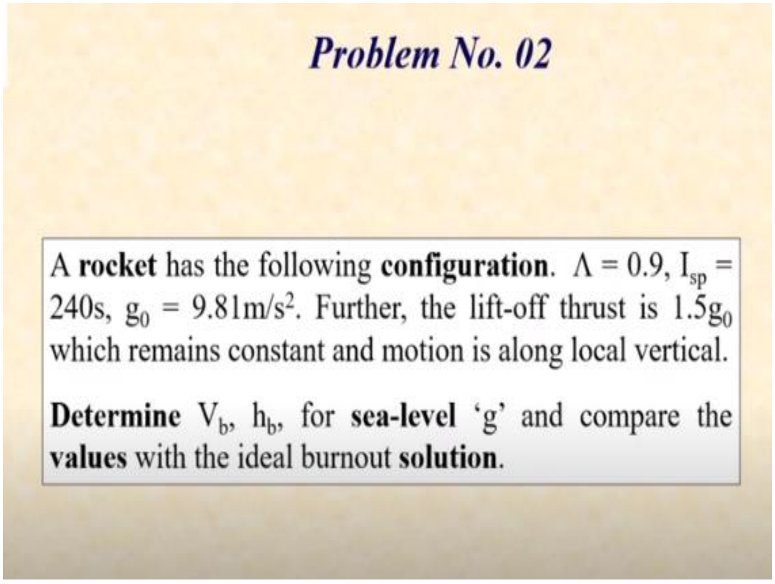

So let us take the same rocket that we defined earlier. That is, it has a propellant loading of 0.9, but I_{sp} is only 240 seconds and lift-off thrust is 1.5 times g_0 given in the form of thrust per unit mass. So, but we are saying that at the liftoff the thrust is 1.5 times g_0 and that remains constant and the motion is along a local vertical, which means it is a constant thrust case.

Please note a constant thrust case is same as a constant burn rate case. Because our thrust is $\dot{m} g_0 I_{sp}$. So, if thrust is constant, we already know g_0 and I_{sp} are constant for practical purposes. So obviously, that $\dot{m}$ has to be a constant. Let us try and determine the burnout velocity, the altitude first assuming the gravitational acceleration to be the sea level value, and let us compare these with the ideal burnout solution.

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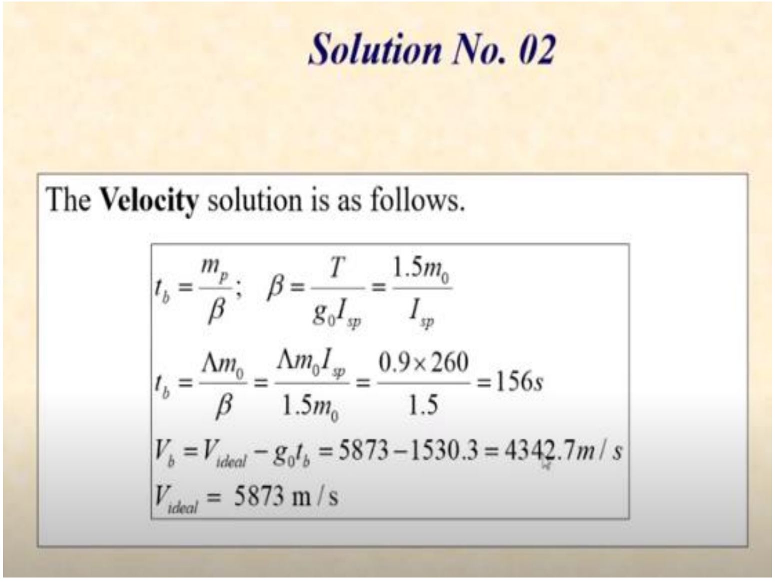

So, the velocity solution is as your burnout time is nothing but the ratio of $\frac{m_p}{\beta}$. And please note, we have not specified the β, what we have specified indirectly is the lift-off thrust which remains constant. So, which means we have specified thrust and from thrust we must discover the burn rate $\dot{m}$. So, we bring in that idea. So β is nothing but our $\dot{m}$ and that is nothing but thrust divided by $g_0 I_{sp}$.

And thrust is nothing but 1.5 times $m_0 \times g_0$ because $1.5 \times g_0$ was the acceleration given per unit liftoff mass. So, the total thrust would be $1.5m_0g_0$. Of course, we can cancel g_0 which I have done here. So, I am only writing $1.5m_0$. Divide that by I_{sp}. This is my β. So, what is t_b? It is $\frac{m_p}{\beta}$. Now what is m_p?

m_p is λm_0. This is our definition. So, I substitute that expression here, $\frac{\lambda \times m_0}{\beta}$ and now I get an expression for burnout time in terms of λI_{sp} and just the ratio of 1.5. And it directly tells me that it will take 156 seconds to complete the burning. Now with this t_b, I just go back to my V_b expression under gravity which is nothing but $V_{ideal} - g_0 t_b$.

And if I do this, I find that my velocity is going to be 4342.7 m/s as against 5873 for the ideal burnout case. Let us now move over to the altitudes.

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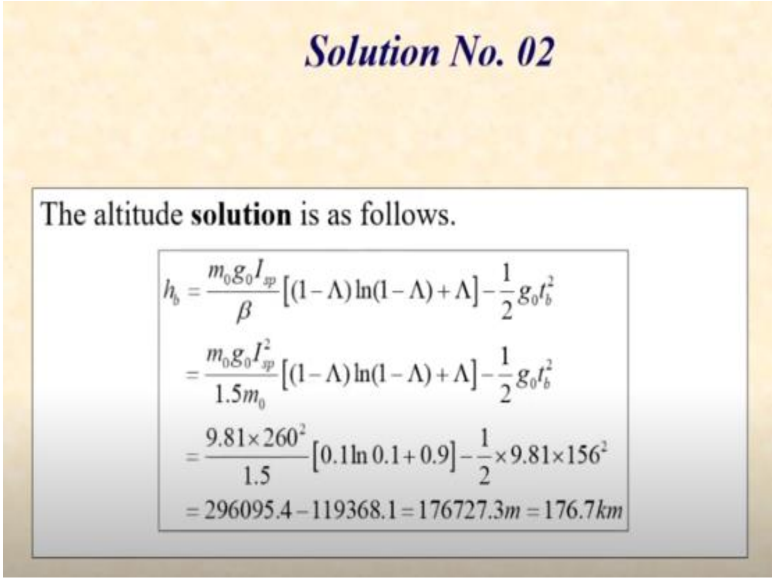

So, in this case, we recall the altitude expression which is $\frac{m_0 g_0 I_{sp}}{\beta}$ $[(1-\lambda)\ln(1-\lambda+\lambda] - \frac{1}{2}g_0 t_b^2$. Now as β is a function of I_{sp} and m_0, I substitute that $\frac{1.5m_0}{I_{sp}}$. m_0 will cancel. And what will be left will be $\frac{g_0 I_{sp}^2}{1.5}$. λ is already specified as 0.9. So, this will become $0.1\ln 0.1 + 0.9$.

And time is 156 seconds. So, t_b^2. I do this simple calculation and it shows that the altitude reached in this case will be 176.7 km. So, we now have the velocity which is go back 4,342.7 m/s and we have altitude which is 176.7 km in the present case.

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Problem No. 03

In **respect** of problem No. 02, obtain the **corrected** values of terminal parameters for the **'g'** applicable for '$\mathbf{h_b}$' and comment on the **result**.

$$g_b = \frac{g_0}{\left(1+\frac{h_b}{R_E}\right)^2} = \frac{9.81}{\left(1+\frac{176}{6371}\right)^2} = 9.290 m/s^2$$

$$\tilde{g} = \frac{9.81+9.290}{2} = 9.55 m/s^2; \quad V_b' = 4383 m/s; \quad h_b' = 179.9 km$$

There is an **increase** of ~40 m/s in 'V' and ~3 km in 'h'.

Now let us extend this problem, to problem number 3 where let us try and correct the g value applicable for that altitude. Please note, the altitude is almost 180 km. So, it is very large. So obviously, the sea level gravity value is no longer applicable. And we must now correct the gravitational value and recalculate the burnout parameters.

And let us see, what difference does it make if we correct the gravitational acceleration value because of the change in the altitude. So, we bring in our gravitational acceleration expression in terms of the altitude and radius of earth. And we find that at the burnout altitude, the gravitational acceleration will only be 9.29 m/s^2.

And now we bring in the simplification and the approximation that we have seen in the lecture that because this variation is small over a large altitude, an average value of gravitational acceleration can reasonably capture the effect of change in gravity. So, I define that $\tilde{g}$ as an average of the sea level value and the value of gravitational acceleration at 180 km altitude.

So that turns out to be 9.55 m/s^2. So now I go back and use this value of gravity to recalculate the velocity and the altitude. So, I find that the velocity is 4383 m/s, which is roughly about 40 m/s higher than what we have predicted from our conservative formulation based on sea level gravity. Whereas the altitude is higher by roughly about 3 km as compared to the 176 km altitude under the constant sea level gravitational assumption.

So, you find that even with a drastic change in the altitude of about 180 km, the effect of this on the terminal performance is still only marginal. For example, if you look at the correction due to velocity, it is typically of the order of 1%. If you look at the altitude correction, it is even less than that.

So, what it means is that for most cases of ascent mission which are going to end around 180 to 200 km, even if you do not correct for altitude, the conservative estimate is not very much off. And if we want a more realistic a simplified correction based on the average gravitational acceleration is more than adequate.

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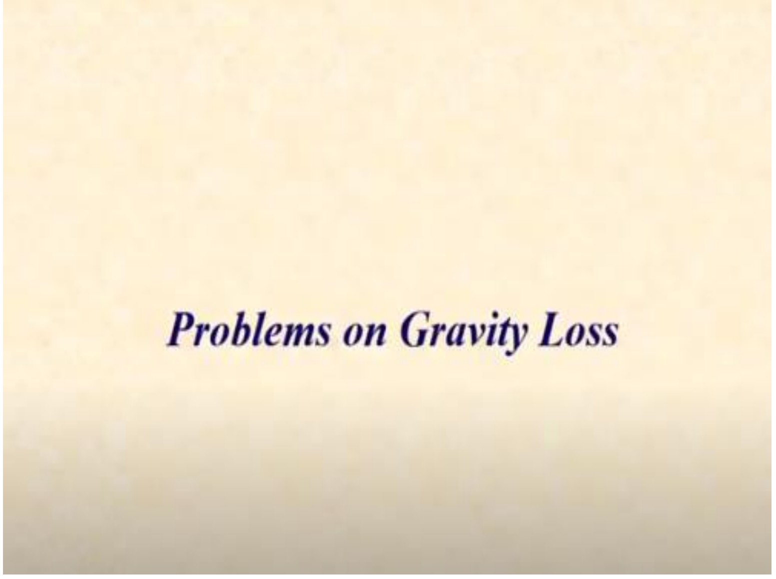

Let us now look at the gravity loss part because of the presence of gravitational acceleration.

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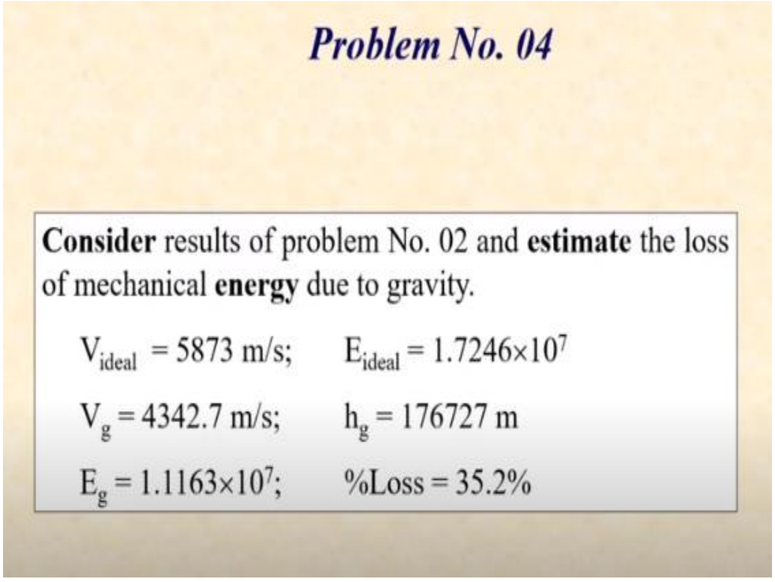

So let us take the problem 2 two that we have considered and estimate the loss of mechanical energy due to gravity that is as compared to ideal burnout. What is the loss of mechanical energy under the action of gravitational acceleration? So, we bring in the idea of energy per unit mass of burnout. We say that, the velocity under ideal condition is 5873.

So, energy of ideal burnout is 1.7246×10^7 in appropriate units. Now we have the velocity under gravity as 4342.7 m/s and altitude as 176727 m, consistent units. So, we calculate the energy as $\frac{1}{2}V_g^2 + gh_g$. That comes out to be 1.1163×10^7 in the same units. So as a percentage, the loss turns out to be 35.2%, quite large; very significant loss that we have incurred.

But then that is the problem that we have also seen earlier, that for a given burn rate, if it is low, you are going to incur a large loss of energy due to gravity. And if you want to reduce this, you must increase the burn rate.

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We will leave this problem at this point. And then we will go over to the next solution of straight-line trajectory and bring in the effect of aerodynamic drag and the loss that happens because of the aerodynamic drag.

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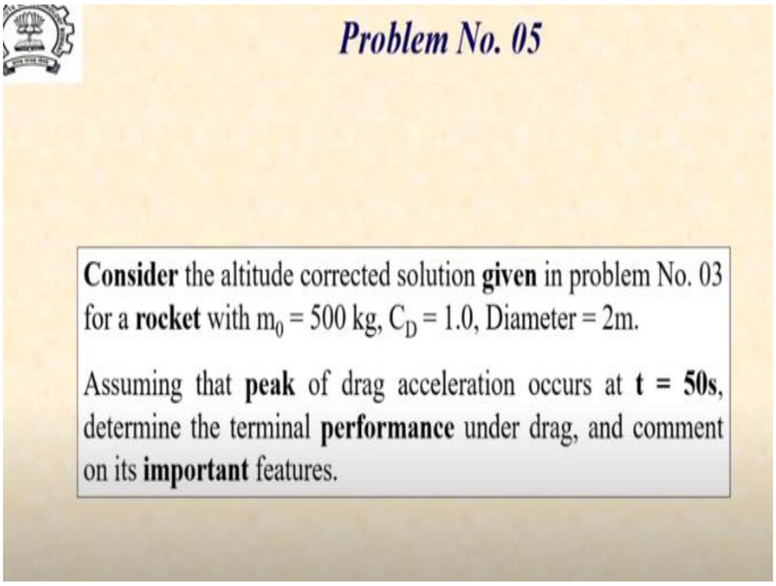

So let us consider the altitude corrected solution given in problem number 3. So, in problem number 3, we have corrected the altitude because of the change in gravity. And that is the new gravity value is what we have used for terminal velocity. So, we have a velocity of 4380 odd m/s and the altitude of almost 180 km that is 179 odd km.

And now we bring in the idea of a mass of rocket at liftoff of 500 kg. We say that it has a diameter of 2 m. And we are going to use the bluff body drag value of C_D with drag coefficient value of 1. Now let us assume that the peak of the drag acceleration is going to occur around 50 seconds.

And based on that we are going to bring in our simplified drag modeling and then we will try to find the impact of this on the terminal performance and find out in what way is the approximation valid or applicable and what is the overall order of magnitude of the impact of the drag.

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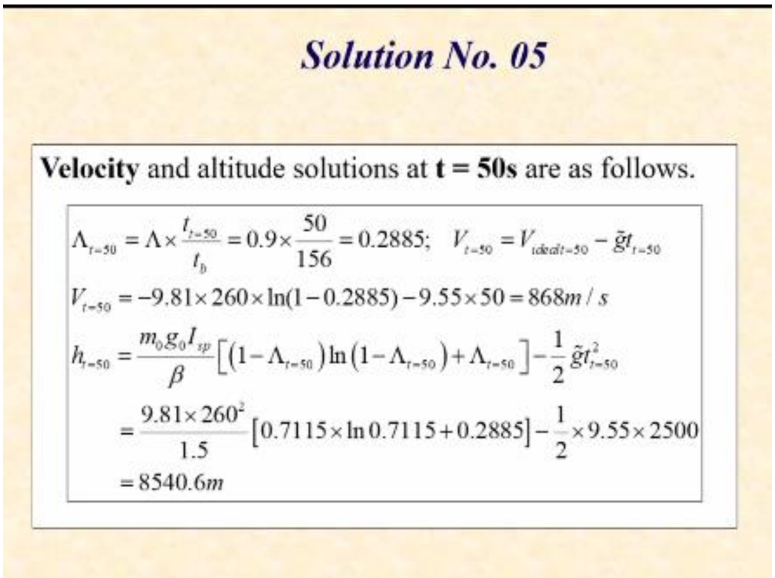

So let us first obtain the velocity at altitude solutions at $t = 50s$. So, at $t = 50s$ the amount of propellant that we are going to consume will be given by, because it is a constant burning rate, I know that I will burn all the propellant in 156 seconds. So, in 56 seconds, I would be burning only a small amount of propellant. So, propellant burned ratio becomes only 0.2885.

Now with this I can go and calculate the velocity which is nothing but the ideal velocity at t equal to 50 seconds minus the gravity adjusted velocity. So, this turns out to be 868 m/s. So, at $t = 50s$, the rocket would have acquired a velocity of 868 m/s. Let us do the same thing for the altitude. So again, substitute the same expression of $\lambda_{t=50}$ into this.

The time is already specified as 50 seconds and by doing that, we show that it will reach an altitude of roughly about 8.5 km. So, these are the performance parameters under the corrected gravity value of 9.55 m/s^2 at $t = 50s$ where we have assumed that the peak of the dynamic pressure or the acceleration would occur.

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Solution No. 02

The **solution** for terminal performance under **drag** is as follows.

$$\rho_{t=50} = 0.442 kg/m^3; \quad D_{t=50} = \frac{1}{2} \times 0.442 \times 868^2 \times \pi \times 1.0 = 523.1N$$

$$m_{t=50} = m_0 \left(1 - \Lambda_{t=50}\right) = 0.7115 m_0 = 355.7kg$$

$$a_{Dt=50} = \frac{D_{t=50}}{m_{t=30}} = \frac{523.1}{355.7} = 1.47 m/s^2; \quad a_{Davg} = 0.735 m/s^2$$

$$V_D = V_g - 0.735 \times 156 = 4268.5 m/s \text{ (> 4243 m/s)}$$

$$h_D = h_g - \frac{1}{2} \times 0.735 \times 156^2 = 170956 m = 170.9 km$$

Once that happens, let us now use this information to find out the value of the drag, the drag acceleration and the average drag acceleration which is going to be used to calculate the modified terminal performance. So first we get the density from our atmospheric tables. At this altitude of 8.5 km, the density is 0.442 kg/m^3.

Using this density, the velocity and the surface area because of the diameter of 2 m that is radius of 1 m and $C_D = 1$, the total drag in this case is 523.1 N. Now the mass at this point is $m_0(1 - \lambda_{t=50})$. So that is $0.7115 \times m_0$ which is 500 kg. So, this turns out to be 355.7 kg.

Now the acceleration due to drag is nothing but the value of drag divided by this mass. So, turns out to be 1.47 m/s^2 that is the peak value. And from our rectangular approximation that we have been using, the average value of drag which is a constant is 0.735 m/s^2.

Now with this acceleration value, when we recalculate our velocity due to drag, it is velocity due to gravitational part -0.735 into the total trajectory duration which is 156 seconds and what we get is 4268.5. Now I want to point out an interesting observation. If we had not corrected for the gravity and drag, suppose we had neglected both the effects, which means we had not corrected the gravity for altitude.

Which means we had used the sea level value of gravity and we had not included the drag; this is the velocity that you have predicted, 4243. But, if we correct the gravity value for altitude, it is a plus for us because performance improves. And then if we include the drag the deterioration of performance in terms of velocity is still not sufficient and that we get a velocity which is still higher than what you would get if you did not consider the correction because of altitude and drag.

So, you will realize that, even if we do not consider the impact of correction to gravity because of altitude and if we ignore the drag in the first initial sizing of the rocket, we would get a fairly good idea of the rocket terminal performance from a design perspective. The change in altitude is from 179 to 170 km. The actual altitude that we have seen is of the order of about 176 km.

So, we find that if we were to use these values of 4268 and 170.9, this would be more or less the performance that would be close to the performance which you are going to get because of only sea level gravity modelling and nothing else. Hi, so in this lecture or our tutorial, we have gone through a set of five cases starting from the ideal burnout to the impact of drag.

And we have noted how the terminal performance gets influenced as we introduce the various corrections such as gravitational correction, correction to gravitational acceleration due to altitude and the correction in terminal performance due to the presence of atmospheric drag.

And we find that in the initial stages if you are looking at a gross estimate of the terminal performance, just by introducing the sea level gravitational model along a straight-line motion, the performance that we generate is fairly representative of what we are going to get under various realistic scenarios. So, with that, we come to the end of this tutorial. Bye, and see you in our next tutorial and thank you.