Jet Aircraft Propulsion

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Module No. # 01

Lecture No. # 09

Tutorial-1

Hello and welcome to lecture number 9 of this lecture series on Jet Aircraft Propulsion. So, as promised in the last lecture, today we are going to solve some problems based on some of the discussions we have had in the last two three lectures. So, as you might recall during one of our early lectures, we started off with discussion on the Brayton cycle, we discussed about both the ideal and the Brayton real Brayton cycle, we also understood what are the different sources of loses, which lead to an actual Brayton cycle being different from that of an ideal cycle.

And basically the reason was that, the the presence of irreversibilities cause a Brayton cycle, an actual Brayton cycle to be quite different from that of an ideal Brayton cycle. And then subsequently, we have been discussing about application of the Brayton cycle in terms of the actual jet engine cycles. And then, we have discussed various types of the jet engine cycle like the turbojet, turbojet with afterburner, turbo fan and so on. So, these are different modes or types of jet engine cycles which we have discussed. And then, we have also looked at, what are the different components that constitute an a jet engine cycle, and how we can estimate the performance of these components.

So, what we are going to do today is to have some numerical problems with us and see how we can solve some of these problems based on what we have discussed. So, today we will be discussing about application, well numerical application of the various Brayton cycle, the real Brayton cycle and we will also be solving one or two problems based on component performance analysis.

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So, we will be having a tutorial today on ideal cycles, well real cycle as well and the component performance.

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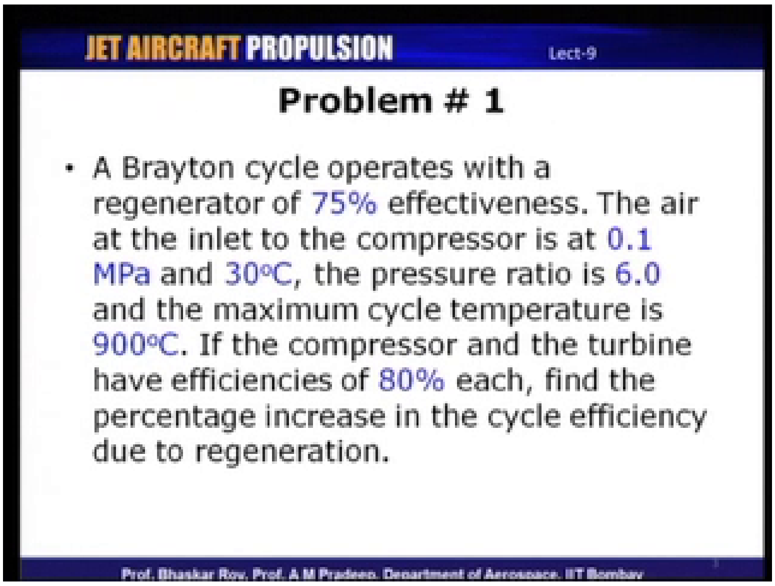

So, let us take a look at the first problem that we have. So, the first problem, the statement reads the following, it is a problem on a Brayton cycle. A Brayton cycle operates with regenerator of 75 percent effectiveness, the air at the inlet to the compressor is at 0.1 mega Pascals and 30 degree Celsius, the pressure ratio of the cycle is 6 and the maximum cycle temperature is 900 degree Celsius. If the compressor and the turbine have efficiencies of 80 percent each, find the percentage increase in cycle efficiency due to regeneration.

So, here we have a problem which is related to a Brayton cycle; obviously, it is not an ideal Brayton cycle, because we have effectiveness of the regenerator coming into picture, we have also been given the compressor and turbine efficiencies. So, put together it does not really constitute an ideal Brayton cycle. So, it is a real Brayton cycle analysis that we would need to do, we have been given some additional data like the cycle pressure ratio, the maximum cycle temperature and also the inlet conditions of air, like it is pressure and temperature. So, what we are required to find is the efficiency of the cycle with and without regeneration, it is basically to see how much improvement or if at all we get by using regeneration.

So, to begin solving this problem, as ahead probably indicated in one of my earlier lectures, and the earlier course that if you had a chance to go through it, I keep emphasizing the fact; that it is necessary for us to first understand the problem statement very carefully and preferably sketch a cycle diagram of the problem. Even though it may be a very trivial cycle, though you you might have a feeling that it is very trivial, it is does need to sketch a cycle diagram. I strongly recommend that you first sketch the cycle diagram on either PV or a TS diagrams, and then mark those points where the data is known, and those points where you need to find data.

So, this will greatly have reduce the effort of trying to figure out which data is given, which is not given and so on. And it is it is a much more organized way of solving a problem. So, taking q from that, what I have here is the cycle diagram for this problem.

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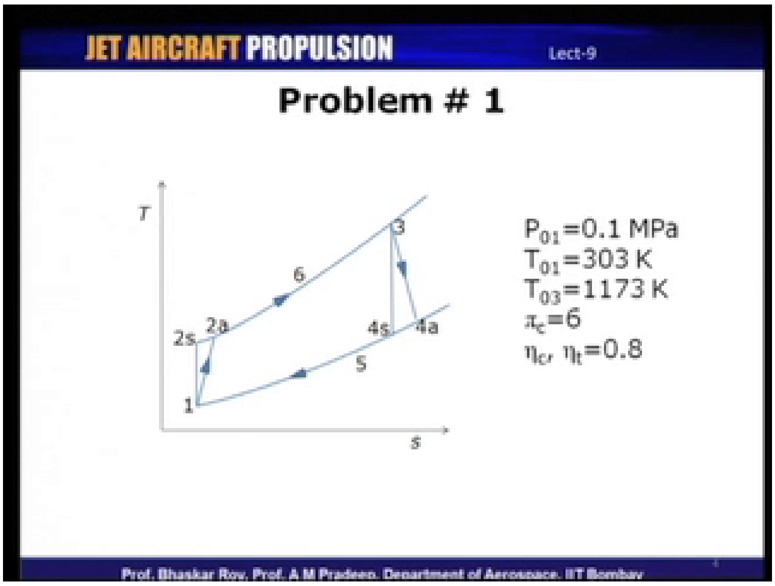

So, here we have a cycle diagram for a Brayton cycle, I have indicated both the actual and the ideal Brayton cycles. So, those points which are indicated with an s, that is 2 s indicates the isentropic point on that cycle, similarly 4 s is for an isentropic process.

Now, here in this problem, there is no mention about pressure loss occurring during either heat addition or heat rejection process. So, we can assume that there is no pressure loss occurring during these processes and so, it is a constant pressure process. But since, the compressor and turbine efficiencies are given; it means that those efficiencies, well those processes are not really isentropic.

So, if this is the Brayton cycle, Brayton cycle begins with the air at station 1, there is a compression which is non isentropic which which takes the air to straight to a. And and then there is a constant pressure heat addition between 2 a and 3; and that 3, we have an expansion process which is again non isentropic, and this process goes all the way up to 4 a.

So, between 4 a and 1, we have the heat rejection process and the, I have indicated two more point here 5 and 6 which could be somewhere in between these heat addition and heat rejection processes, these two points indicate the regeneration points. We will come back to regeneration little later, because that is the second half of this problem, we will first solve this problem, find the efficiency of this cycle without regeneration. Now, what is what are the data specified for us, we have the air inlet conditions P 0 1 which is 0.1 mega Pascals, T 0 1 that is the stagnation temperature at state 1, that is 303 Kelvin.

We also have the maximum cycle temperature which is T 0 3 that is 1173, 1173 Kelvin, these pressure ratio is 6, that is, so I have indicated pressure ratio as pi c, then we have the compressor and turbine efficiency, each of them equal to 0.8. So, this is the cycle diagram of Brayton cycle, both ideal and real Brayton cycles have been indicated there, something we had discussed in one of our earlier lectures.

So, in this cycle, what we need to do is, we know certain points, we know the pressure temperature, it is state 1, we know the pressure ratio for this compression and expansion processes, the efficiencies are known, and the maximum cycle temperature is known. And what for, what are we required to find? We are required to basically, find the efficiency of this cycle. Now, if you recall from your thermodynamics, if you have gone through the thermodynamic course, which I am sure you going to have, then you would recall that efficiency.

Thermal efficiency is basically defined as the net work output divided by the heat input, that is how much work output do you get from a cycle given a certain amount of heat, so that defines the thermal efficiency of a cycle. So, if we need to do that, we need to find w net, and in this case, the net work output as you know is the difference between the work generated by the turbine and the work consumed by the compressor.

So, w t minus w c will give us the net work output, and what about heat input? Heat input is basically the enthalpy difference between states 3 and state 2. So, difference between the enthalpy is there, will give us the heat added per kilo gram of air, so ratio of this two will give us the efficiency. Now, the question is how do we find the net work output of the turbine, and the work consumed by the compressor, again I would suggest that we need to have a flashback, go back to thermodynamics, you would recall that, the compressor and turbine, both of them are steady flow processes. And for a steady flow process, you might recall that the net work output is, or work input required is the difference in the enthalpies.

So, for a compressor, net work output would be equal to h 0 3 minus h 4 a, or h 4 in this case, h 0 4. And so, that is in turn equal to C p times the temperature difference, similarly for a compressor it would be equal to h 0 3 minus h 0 2. And so, the difference between these two temperatures would give us the compressor work output.

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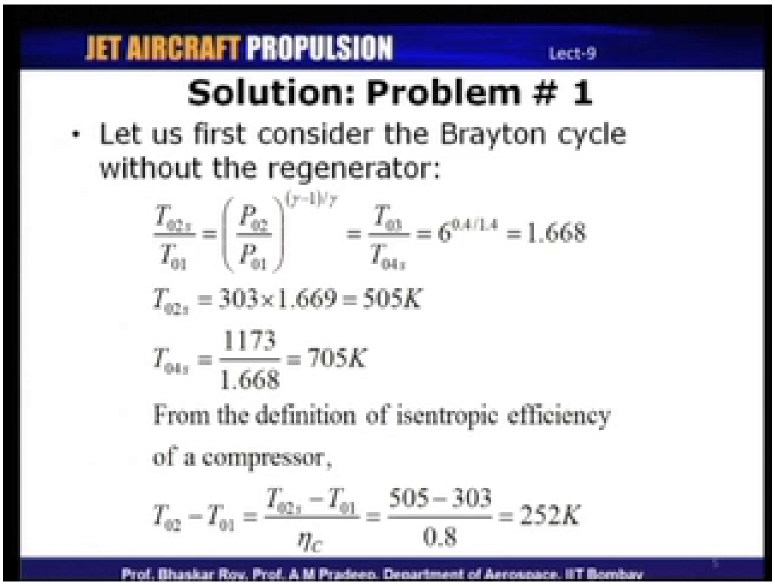

So, with this in mind, let us begin our problem solving here, we will first as I mentioned consider a Brayton cycle without any regeneration, so without regeneration, let us look at how the cycle behaves. Now, for the first process which is non isentropic, we know it is not isentropic, now if it were to be isentropic, then we know the temperature at the end of the first process would be T 0 2 s which is this stagnation temperature at the end of compression for an isentropic process.

So, this divided by T 0 1 should be equal to the pressure ratio raise to gamma minus 1 by gamma, this come from the isentropic analysis which we have discussed earlier. So, temperature ratios, stagnation temperature, or static temperature ratios can be related to the corresponding stagnation pressure or static pressure ratios, the raise to gamma minus 1 by gamma, this is from, it it follows from isentropic relations, this is also equal to T 0 3 by T 0 4 s. And what is that equal to? This is equal to cycle pressure ratio raise to gamma minus 1 by gamma, and what is gamma? Gamma is 1.4; of course it is not explicitly stated in the problem. So, what I will suggest is, in in a problem where gamma is not been explicitly stated, you can safely take gamma as 1.4 which is the specific ratio of specific heats for air. So, if gamma is equal to 1.4, then we have 6 raise to 1.4 minus 1 that is 0.4 divided by 1.4 which is 1.668.

So, we get T 0 6, the T 0 2 s which has the stagnation temperature at the end of compression for an isentropic process which is equal to T 0 1 multiplied by this (Refer Slide Time: 11:35), so we get T 0 3 into 1.668, so that is 505 Kelvin. Similarly, T 0 4 s can be determined by, because T 0 4 s will be equal to T 0 3 divided by 1.668, T 0 3 is already been specified in the problem as 1173 Kelvin. So, T 0 4 s is equal to 1173 divided by 1.668, so that is 705 Kelvin.

So, we now have two temperatures with us, each of them are the stagnation temperatures for the process, if it were to have been isentropic. So, for an isentropic process, for an the end of the compression, if the process was isentropic we would have a temperature of T 0 2 s and similarly, for an expansion process that is the turbine process, if it were to be isentropic we get a temperature of T 0 4 s.

So, we have these two temperature with us, so what do we do with these temperatures, we knew the actual temperatures, because that is what is the actual cycle about, it is not Isentropic temperatures that we are concerned about, the work required by the compressor will depend upon the actual temperatures and not the isentropic temperatures.

So, how do you find the actual temperatures, so for this, we have the efficiencies with us, the compressor and the turbine efficiencies have been specified. And from our efficiency definition if you recall, we can relate the isentropic temperatures to the actual temperatures and since the efficiency is known, and we have also calculated the isentropic temperatures, we now should be able to calculate the actual temperatures.

So, from the definition of isentropic efficiency, let us take up the compressor first now we know that eta c which is isentropic efficiency of a compressor should be equal to T 0 2 s minus T 0 1 divided by T 0 2 minus T 0 1 or T 0 2 minus T 0 1 is equal to T 0 2 s minus t 0 1 divided by efficiency we have these both these temperatures with us which is 505 minus 303 divided by 0.8 which is the efficiency of the compressor, so this temperature difference comes out to be 252 Kelvin.

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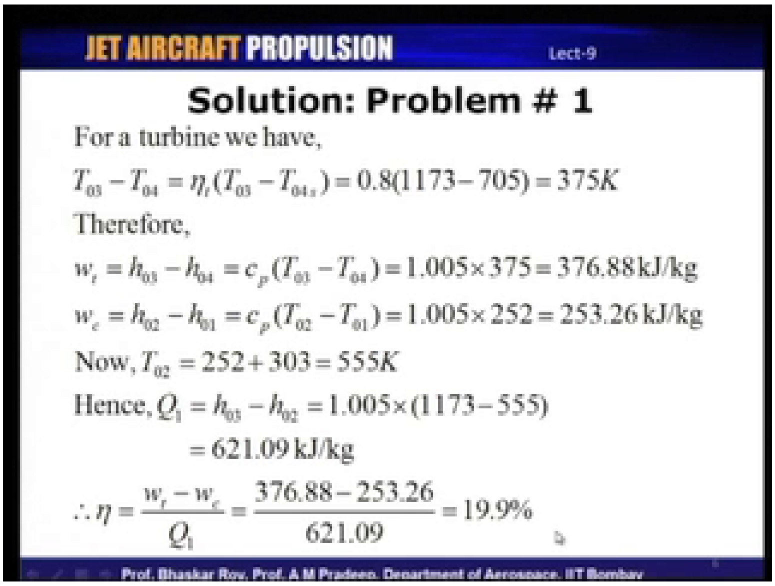

So, from the definition of isentropic efficiency, let us take up the compressor first. Now, we know that eta c which is isentropic efficiency of a compressor should be equal to T 0 2 s minus T 0 1 divided by T 0 2 minus T 0 1, or T 0 2 minus T 0 1 is equal to T 0 2 s minus T 0 1 divided by efficiency, we have these both these temperatures with us which is 505 minus 303 divided by 0.8 which is the efficiency of the compressor. So, this temperature difference comes out to be 252 Kelvin. Similarly, for the turbine we have, turbine efficiency is T 0 3 minus T 0 4 divided by T 0 3 minus T 0 4 s.

Therefore, T 0 3 minus T 0 4, which is the temperature difference, the actual temperature difference of the turbine that would be equal to efficiency times the isentropic temperature difference. So, that is equal to 0.8 multiplied by 1173 minus 705, this is 375 Kelvin. So, we now have the actual temperature difference for the compressor as well as for the turbine. So, we should now be able to find out the work developed by the turbine and work required by the compressor. So, w t is equal to h 0 3 minus h 0 4 which is equal to c P into T 0 3 minus T 0 4, that is again c P is not explicitly given in the problem as I mentioned; if it not given we assume it to be air and we we just take the specific heat at constant pressure for air which is 1.005 kilo joules per kilo gram Kelvin.

So, assuming c P to be 1.005 kilo joules per kilo gram Kelvin, this multiplied by 375 which is the difference T 0 3 minus T 0 4, this becomes 376.88 kilo joules per kilo gram. Similarly, we have w c which is work by the compressor is equal to t h 0 2 minus h 0 1 which is c P times T

0 2 minus T 0 1 which is again equal to 253.26 kilo joules per kilo gram. So, we can also now find out the T 0 2, because the difference is known and T 0 1 is known. So, T 0 2 is 252 plus 303 that is 555 Kelvin and why do we need T T 0 2? Well we need T 0 2 if we have to calculate the heat input.

So, what we have now calculated is the work developed by the turbine which is basically the enthalpy difference across the turbine and so, for that we need the actual temperatures. How do we calculate actual temperatures? We calculate actual temperatures based on the efficiency definition and for that, we need the isentropic temperature, isentropic temperature comes from the pressure ratio raise to gamma minus 1 by gamma. So, the next parameter that we need to find is the heat input.

So, heat input is basically the enthalpy difference between states 3 and 2 and so, that would be equal to h 0 3 minus h 0 2 which is c P times T 0 3 minus T 0 2. And after we have calculated the heat input, we can now easily calculate the efficiency, efficiency will be equal to w t minus w c divided by Q 1 and which is basically net work output by heat input.

So, let us do that and see how how much efficiency do we get for this cycle (Refer Slide Time: 17:08). So, for this cycle the heat input comes out to be h 0 3 minus h 0 2 that is 1.005 into 1173 which is the max cycle temperature minus h 0 2, that is c P times 155. So, heat input is 620.09 kilo joules per kilo gram, therefore efficiency is equal to the net work output which is w t minus w c 376.88 minus 253.26, the whole thing divided by 621.09, so this is 19.9 percent. So, this Brayton cycle has an efficiency of 19.9 percent that is it can convert just about 19.9 percent of the heat input to net work output.

This solves one part of the problem, now the next part of the problem is to see what happens if we use a regenerator, do we get an increase in efficiency; if so, how much improvement we get. So, regenerator as you know is is is a mechanism by which we can transfer some amount of heat from one part of the cycle to another. And so, the obvious thing to do in such a cycle is to transfer the heat which is rejected, part of the heat which is rejected. Obviously, you know you cannot transfer the entire heat which is rejected, because that would make heat rejected equal to 0, and that it it violates the second law of thermodynamics.

So, that is not possible, but we can definitely transfer some part of the heat which is rejected back to the heat added process, thereby we can save on the head added, and we also save on heat rejected. So, that seems to be a clever idea of increasing the efficiency, because we are reducing Q 1 without effecting either w net w t or w c and therefore, net work output. So, net work output does not change, but what does change is the heat input, we can reduce heat input by using regeneration. So, let us see if using a regeneration regenerator of certain effectiveness, given as 75 percent, does that make a difference in the cycle efficiency, let us take a look at that.

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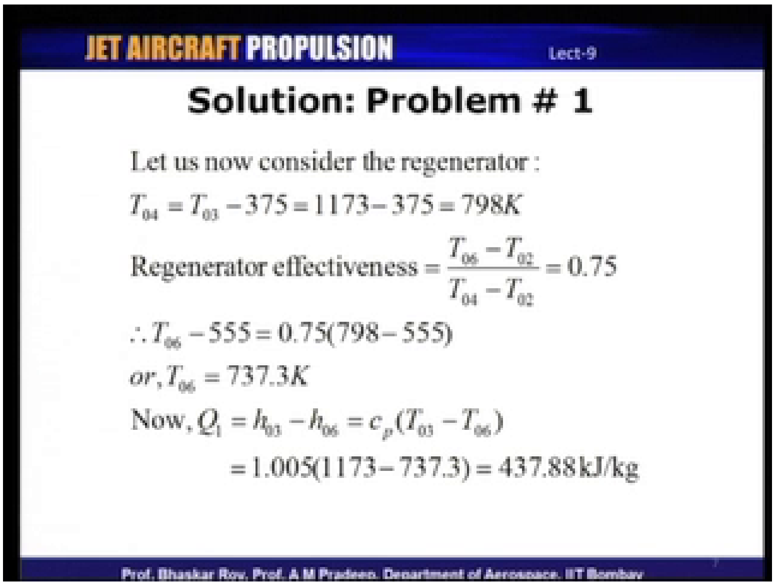

Now, if we consider a regenerator in in which case we will need to find a few more temperatures, we will need T 0 4, T 0 4 is T 0 3 minus 375 that is 1173 minus 375 equal to 798 Kelvin.

Regenerator effectiveness is given as 0.75 and this is defined as T 0 6 minus T 0 2 divided by T 0 4 minus T 0 2. And it means, it is basically, effectiveness of regenerator tells us that if effectiveness was equal to 1, then we will get T 0 4 is equal to T 0 6 is equal to T 0 4, that is we can heat the compressor outlet all the way up to a temperature which is equal to the turbine exhaust. So, that is a ideal process if the regenerator had no irreversibilities perhaps that would be possible, but you know it is not, because regenerator effectiveness in this case is 0.75. And so, T 0 6 is a temperature which is less than T 0 4 and so, we need to find how much it is.

So, from the definition of effectiveness of regenerator, we have T 0 6 minus T 0 4 divided by T 0 4 minus T 0 2, this is equal to 0.75. So, all these temperatures are known, T 0 2 is known, T 0 4 is known, effectiveness is known, therefore we can calculate T 0 6. So, this comes out to be 737.3 Kelvin as compared to T 0 4 which is 798 Kelvin. So, if the effectiveness was 1, then T 0 6 will be equal to T 0 4 which is 798 Kelvin, now the heat input will now be equal to h 0 3 minus h 0 6.

So, you can see immediately the difference that, you need to only had this much amount of heat here instead of h 0 3 minus h 0 2. So, this will be equal to c P times T 0 3 minus T 0 6 which comes out to be 437.88 kilo joules per kilo gram.

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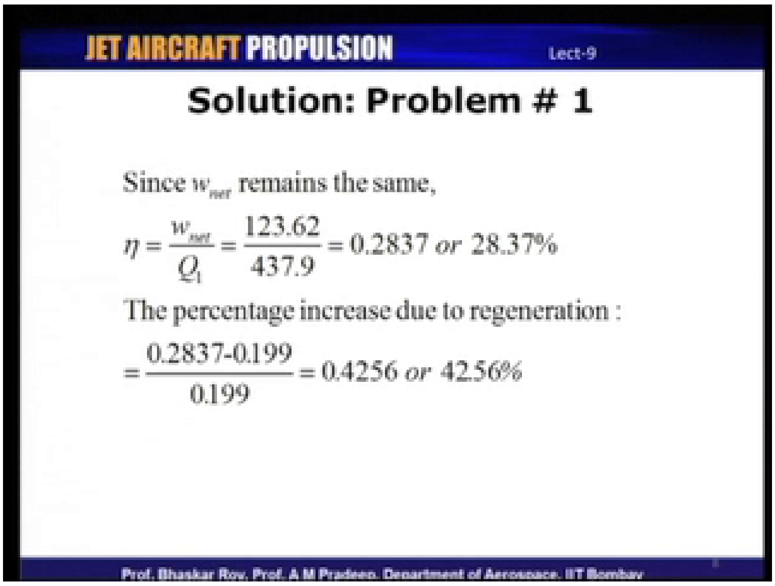

So, heat input has changed, net work output does not change, because we are not doing anything to the turbine or the compression process. Therefore, the efficiency of this cycle would now be equal to 123.62 divided by 437.9, this is equal to 0.2837 that is 28.37 percent, and the cycle efficiency without regeneration was 19.9. So, what is the percentage improvement increase due to regeneration, it s equal to the difference 0.2837 minus 0.199 divided by 0.199, so this is equal to 42.56 percent. So, what do you see here is that, using regeneration well how exactly regeneration is to be carried out is is another issue all together.

But if we were indeed able to carry out regeneration, even if the regenerator was not 100 percent effective, in this case it is 75, we still get an improvement in efficiency of the order of 43 percent as 42.56 percent which is the huge improvement in efficiency. So, the the moral of the story is that, regeneration is one of the ways of substantially improving the efficiency of a cycle, which is something you would have understood if you had gone through the thermodynamic course, that regeneration is definitely a way of increasing the efficiency.

In fact, if you recall there are two such cycles which have efficiencies as high as that of Carnot cycle, and what are those cycles? The Stirling and Ericsson cycles, both these cycles have two processes which are basically regeneration processes; one is at constant volume, the other is at constant pressure. So, regeneration, using regeneration Stirling and Ericsson cycles are able to achieve efficiencies which are as high or equal to that of a Carnot cycle, which means the cycles have the maximum efficiency possible for that temperature limits.

So, regeneration, of course again during the discussion on thermo dynamics, you may have come across description about Stirling and Ericsson cycles. Why why is that, these cycles are not used? They are not used, because the regeneration makes the whole cycle extremely bulky and complicated and so, it is not suitable for modern day applications. And so, they have not really been used in any application, even though they have efficiencies equal to Carnot cycle efficiency.

So, regeneration, so this example that we have just solved is one way of trying to see if regeneration makes a difference in the overall efficiency or the thermal efficiency of the cycle. And we have seen that, it does make a lot of difference, it makes about, and then this particular case we got about 43 percent improvements in the efficiency of a cycle with regeneration. So, this was one problem, the first problem that we solved today by using the Brayton cycle analysis for an real Brayton cycle with and without regeneration.

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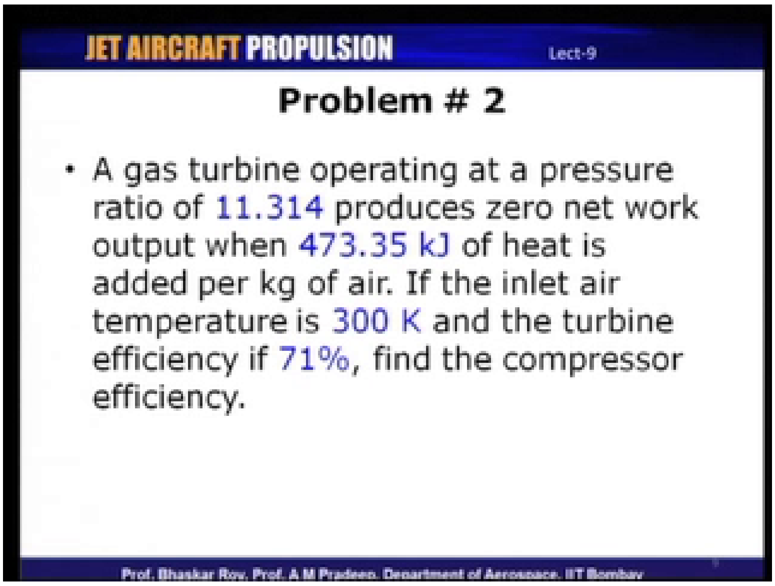

So, let us take a look at what is a second problem that we have for us today, second problem is again a gas turbine problem, again in some sense it is basically a Brayton cycle problem. So, here we have a gas turbine operating at a pressure ratio of 11.314 produces zero net work output when 473.35 kilo joule of heat is added per kilo gram of air. If the inlet air temperature is 300 Kelvin and the turbine efficiency is 71 percent, find the compressor efficiency.

So, here this is again a Brayton cycle problem, but we have slightly altered it for a gas turbine. So, we have the pressure ratio specified and it is given that this gas turbine produces no net work output, which means that the work output of the turbine should be equal to the work input required for the compressor. So, there is no net work output, the heat input is given, the efficiency of the turbine is given, we are required to find efficiency of the compressor.

So, in this particular problem, we have which is again a simple Brayton cycle analysis, the net work output is given as 0 which is true for most of the air craft engines. If if you recall, aircraft engines do not regenerate net work output, turbine in an aircraft exits only for driving the compressor and a few other accessories. So, in most of the jet engines, the there is no net work output from the jet engine, because turbine drives the compressor and that is it is. So, there is no net work output that way, but of course, there are shaft power outputs required for turbo props and turbo shafts and so on, but for pure jet engines the net work output will be 0.

So, in this case, we have the cycle pressure ratio, we have certain temperatures given to us and efficiency of the turbine; we need to find what is the efficiency of the compressor.

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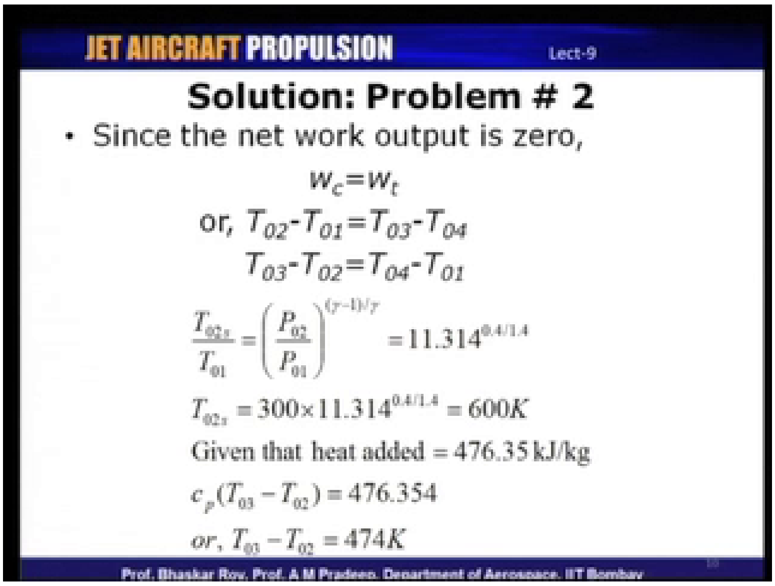

So, let us try to solve this problem, so here we have the net work output as zero which means that w c will be equal to w t, or T 0 2 minus T 0 1 which is the temperature difference across the compressor will be equal to T 0 3 minus T 0 4 which is the temperature drop across the turbine, this in turn is equal to T 0 3 minus T 0 2 is equal to T 0 4 minus T 0 1. Now, cycle pressure ratio is given as 11.314 and that should be equal to the isentropic stagnation temperature ratios, that is T 0 2 s divided by T 0 1 is equal to P 0 2 by P 0 1 raise to gamma minus 1 by gamma, this is equal to 11.314 raise to gamma minus 1 is 1.4 minus 1 that is 0.5 divided by 1.4.

Therefore, T 0 2 s that is this stagnation temperature isentropic will be equal to T 0 1 multiplied by 11.314 raise to gamma minus 1 by gamma. So, this comes to be 600 Kelvin, now heat added is given as 466.35 kilo joules per kilo gram. So, since heat added is the process between states 2 and 3, heat added occurs during that particular process. So, c P times T 0 3 minus T 0 2 is equal to 476.35, or T 0 3 minus T 0 2 is 474, because that is 476.354 divided by 1.005. So, we are assuming that specific heat at constant pressure is the same as that of air which is 1.005 kilo joules per kilo gram Kelvin.

So, we have calculated that is the difference that is T 0 3 minus T 0 2 which is 474 Kelvin. So, the next step in this is to calculate the corresponding temperature so that we can determine the efficiency of the compressor. Now, from our first equation that we wrote that was equating the work output of the turbine to that of the compressor, we can also write T 0 4 as equal to T 0 1 plus T 0 3 minus T 0 2, T 0 1 is known as 300 Kelvin it is given in the problem plus this difference T 0 3 minus T 0 2 we just calculated in the previous step that was 474, so we get a total temperature of 774 Kelvin.

Now, turbine efficiency is is given as 71 percent and the definition of turbine efficiency is T 0 3 minus T 0 4 divided by T 0 3 minus T 0 4 s. So, we can simplify that in a maneuvers shown here, T 0 3 multiplied by 1 minus T 0 4 by T 0 3 divided by T 0 4 as multiplied by T 0 3 by T 0 4 s minus 1, also we know that this ratio that is T 0 3 by T 0 4 s should be equal to the cycle pressure ratio raise to gamma minus 1 by gamma, because the net work output is anyway 0. So, the turbine pressure ratio will also be equal to the compressor pressure ratio, therefore we get this ratio from this expression (Refer Slide Time: 30:54).

So, here we have this ratio T 0 3 by T 0 4 s which we can determine from this expression and we also know the turbine efficiency. So, if you simplify that we get T 0 4 by T 0 3 is equal to 1 minus 0.71 by 2 which is 0.645, therefore T 0 3 is equal to 774 which is T 0 4 divided by 0.645 that is 1200 Kelvin. And therefore, T 0 2 will be equal to 1200 that is T 0 3 minus the difference between T 0 3 and T 0 2, that is 474, 726 Kelvin. So, we now have all the temperatures that are required for estimating the compressor efficiency, the compressor efficiency as we know it is defined as equal to the isentropic temperature difference T 0 2 s minus T 0 1 divided by T 0 2 minus T 0 1.So, all these temperatures we have already calculated.

So, we get 300 T 0 2 s which is 600 minus T 0 1 which is 300 divided by T 0 2 726 minus 300 which is equal to 0.704 or 70.4 percent. So, we have determined the efficiency of the compressor, because the efficiency of the turbine is given and how did we solve this problem, basically it is given that the net work output is 0. So, we can equate the work done by the turbine to that required by the compressor. And if we were to assume that, the specific heat is the same for both the compressor and the turbine, then we get, we can equate basically a temperature difference across the compressor and the turbine, we also know the pressure ratio of the cycle.

So, we can find out the actual isentropic temperatures and since turbine efficiency is known, if we substitute for the cycle pressure ratios and the isentropic temperatures, we can actually find all the temperatures which are needed to calculate the compressor efficiency. So, compressor efficiency in this case, we have calculated as 70.4 percent. So, this second problem that we have solved today here is again to do with the Brayton cycle as in the case of first cycle, first cycle of course we did Brayton cycle with and without regeneration. In this case, we have estimated the efficiency of one of the components which is the compressor, given some of the other cycle parameters. So, the next problem that we are going to solve here today would be on an intake. And we have already analyzed in our last lecture, an intake and its performance, we have seen that there are two distinct perform parameters; one is the total pressure ratio which is also known as the pressure recovery, and the second parameter is the diffuser efficiency, isentropic efficiency of a diffuser.

So, these are two parameters which which tell us something about the performance of air intake. So, in the next problem, we are going to talk about an air intake and how we can estimate some of the performance parameters of an air intake.

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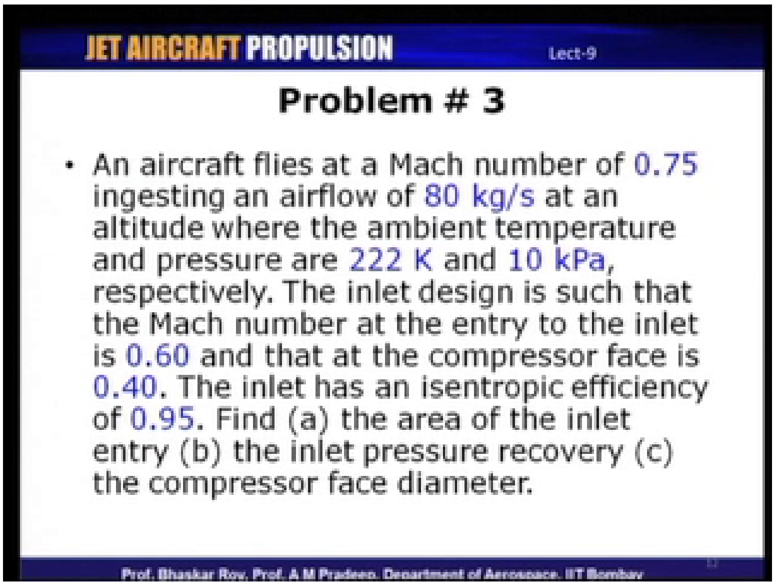

So, the problem number 3 that we have for today is an aircraft flies at a Mach number of 0.75 ingesting an air flow of 80 kilo grams per second at an altitude where the ambient temperature and pressure are 222 Kelvin and 10 kilo Pascals respectively. The inlet design is such that the Mach number at the entry to the inlet is 0.6 and that at the compressor phase is 0.4, the inlet has an isentropic efficiency of 0.95.

Find part a, the area of the inlet, entry part b, the inlet pressure recovery, and part c the compressor phase diameter, so in this particular problem, we have been given that an aircraft is flying at a certain Mach number at an altitude where the pressure and temperature is given. And then we have also been given what is a Mach number at the inlet entry and the compressor phase and also, of course the isentropic efficiency of this particular intake. So, given these parameters we have been asked to find the area of the inlet entry, and the pressure recovery and also the diameter of the compressor phase. So, based on some of these parameters which are known, we should be able to find out the required parameters.

So, first thing that we will do is to try to find out the properties at state 1, that is right at the intake entry that is state 1 and then based on that, we can find out the area of the inlet entry, because area is equal to the velocity divided by the mass flow rate, mass flow rate divided by velocity times density and then subsequently, we will move to station 2 which is the compressor phase.

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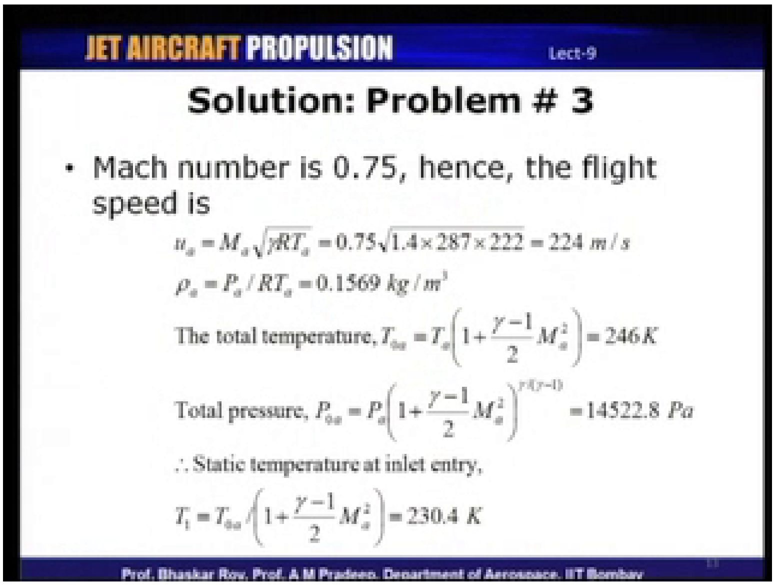

So, Mach number in this case is here given by is given as 0.75 and therefore, we can find these flight speed which is u a, u subscript a is the flight speed which is Mach number times square root of gamma R T a, this is in turn equal to 0.75 into square root of 1.4 into 287 into the temperature is given as 222.

So, the flight speed comes out to be 224 meters per second. And the density, the free stream density rho a will be equal to P a by R T a, which is from the state equation, this is equal to 0.156 kilo grams per meter cube. And so, once the Mach number is already known to us, the static temperature is known, T a is known to us. So, we can find out the total temperature which is T naught a, this is T a into 1 plus gamma minus 1 by 2 M square and that is basically from the energy equation. So, T a is given as 222, Mach number is 0.75, so we get T 0 a is equal to 246 Kelvin.

Similarly, we can find the total pressure, total pressure is P 0 a which is P a into 1 plus gamma minus 1 by 2 M square raise to gamma by gamma minus 1, the pressure is already given to us, P a is specified. Therefore, we can find P 0 a, this is 14522.8 Pascal or 114.52 kilo Pascals. Therefore, the static temperature at the inlet entry that is T 1 is equal to T 0 a divided by 1 plus gamma minus 1 by 2 M square.

And where does it comes from, this is basically, because there is no change in total temperature from the ambient that is at station a, all the way up to 0.1. And in fact, all the way up to called the compressor phase, there is no change in the total temperature, because firstly we assume that this compression process is adiabatic, there is no heat input or heat out rejected by this process. So, total temperature does not change, total pressure does change, because of frictional effects, but total temperature cannot change. So, T 1 is T 0 a divided by 1 plus gamma minus 1 by 2 M square. So, if you substitute for all these values, we get the static temperature at the inlet entry as 230.4 Kelvin.

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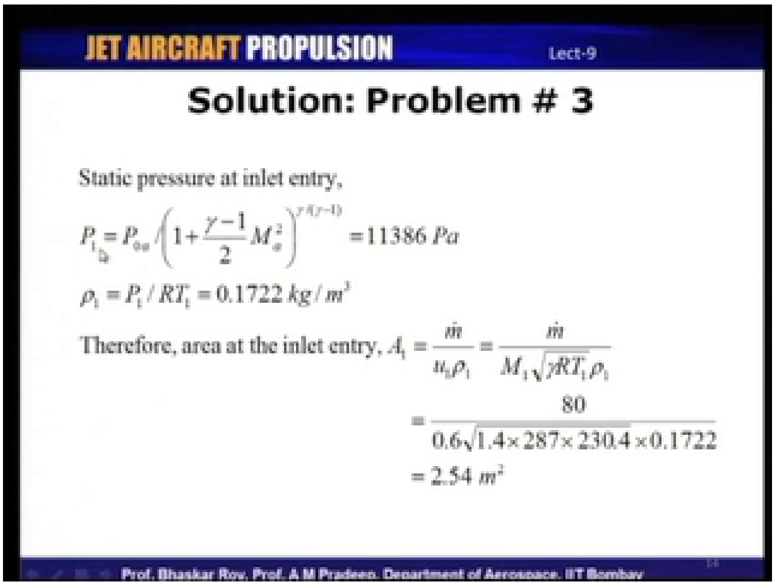

Having found the static temperature, we can find static pressure as well, P 1 which is P 0 8 divided by 1 plus gamma minus 1 by 2 M square raise to gamma by gamma minus 1.

So, this is 11386 Pascal, therefore density rho 1 is P 1 by R T 1 which is, now since we have calculated P one and T 1, we can find density that is 0.1722 kilo grams per meter cube. So, having found the density and the mass flow rate is already been given, we can now find out the area at the inlet entry. So, A 1 will be equal to mass flow rate divided by u 1 into rho 1, because mass flow rate is rho a v that is velocity times density times area. And how do you find u 1, u 1 is found from the Mach number, Mach number at inlet entry is given as 0.6. So, M 1 times square root of gamma R T 1 gives us u 1 that multiplied by rho 1. And so, if you substituted for all these values, we have mass flow rate as 80 k g s per second divided by M 1 which is 0.6 square root of gamma R T 1,1.4, R is 287 for air and T 1 is 230.4, just multiplied by density that is 0.1722, area at the inlet entry comes out to be 2.54 meter square.

So, this solves the first part of the problem where we are required to find out the area at the inlet entry, what we basically did was to find out the static temperature and pressure at the inlet of the diffuser, and that would help us in finding out the density. And since Mach number is given, we can find out the velocity and therefore, from from the mass flow rate relation, mass flow rate divided by u 1 times rho 1, we can find out the area at the inlet entry.

The second part of the problem is to find out the pressure recovery, now pressure recovery as you know it is the ratio of the stagnation pressures or at the outlet of the intake to the inlet of the intake. I already mentioned that stagnation temperature does not change all the way from inlet to outlet. So, T 0 a should be equal to T 0 1 should be equal to T 0 2, but that is not true for the pressure, stagnation pressure is not the same, there will be a stagnation pressure loss. So, we need to find out, how much is the pressure recovery.

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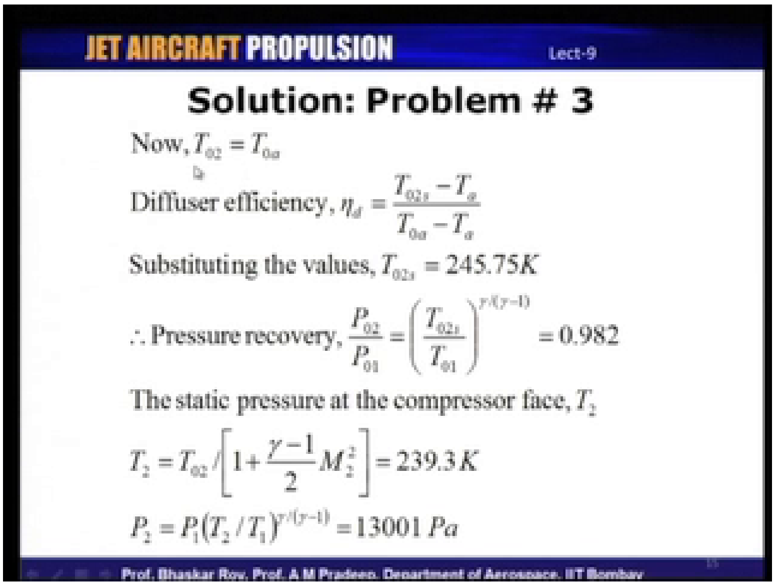

Now, in order to find out the pressure recovery, we will make use of the diffuser efficiency definition. Now, T 0 2 as we know it is equal to T 0 a, and the diffuser efficiency is defined as T 0 2 s minus T a divided by T0 a minus T a, diffuser efficiency is given as 0.95, T 0 a is known you already calculated that, T a is also known.

So, if you substitute for all these values, we get T 0 2 s. So, T 0 2 s comes out to be 245.75 Kelvin and therefore, pressure recovery is equal to P 0 2 by P 0 1 and that is equal to T 0 2 s divided by T 0 1 raise to gamma by gamma minus 1, pressure recovery of the diffuser is 0.982. This means that, from the inlet at station 1 to the outlet which is station 2 that is the compressor phase, there is a certain loss in total pressure that is ratio of these pressures, total pressures is 0.982. So, there is about 2 percent loss in total pressure as the flow moves from station 1 to 2.

Now, the next, the third part of problem is to find out the area at the compressor phase, you will use the same principle as we did for calculating the area at the inlet entry. We need static pressure and temperature at station 2, now static temperature can be easily found out, because T 0 2 is equal to T 0 a, which is already known. So, T 2 is equal to T 0 2 divided by 1 plus gamma minus 1 by 2 M 2 square, and M 2 is given as 0.4. So, if you substituted for all these values, we get T 2 is equal to 239.3 Kelvin. And what about P 2, P 2 is equal to P 1 into the static temperature ratios; T 2 by T 1 raise to gamma by gamma minus 1, T 1 has already been calculated in the previous part, so static pressure comes out to be 13001 Pascal.

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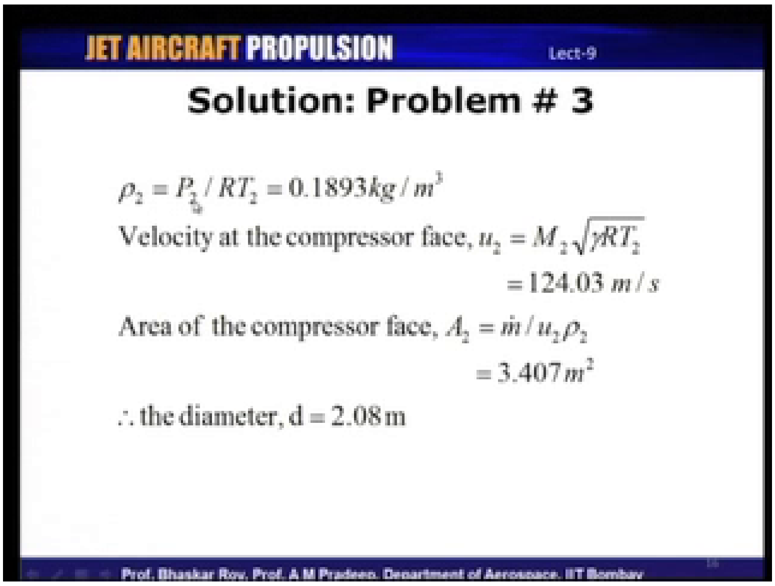

Therefore, density can be calculated, density is equal to P 2 by R T 2 which is 0.1893 k g per meter cube and velocity of the compressor phase is Mach number times square root of gamma R T 2. So, that comes to be 124.03 meters per second. Therefore, area at the compressor phase is mass flow rate is divided by u 2 times rho 2 that is 3.407 meter square. And once we know the area, we can also find out the ratio, the diameter which is pi d square by 4 is equal to the area and therefore, diameter can be calculated as 2.08 meters.

So, in this particular problem which was basically related to an air intake, we have been given some of the parameters like Mach number at various stations and also the diffuser efficiency, given those parameters we could find out the areas at different locations and also the pressure recovery, amount of losses, total pressure losses occurring in the diffuser which is basically specified by the pressure recovery. So, in this problem, we have basically tried to use the diffuser efficiency to calculate the pressure recovery across the diffuser.

Now, the last problem that we will be solving in today's tutorial section is something to do with in a nozzle and that is again component of a gas turbine engine which we were discussing in the last lecture. And nozzle as you know, it have been well basically have one of the parameters used to specify a nozzle is the nozzle efficiency. So, we will see how we can use the nozzle efficiency in assessing the performance of a nozzle.

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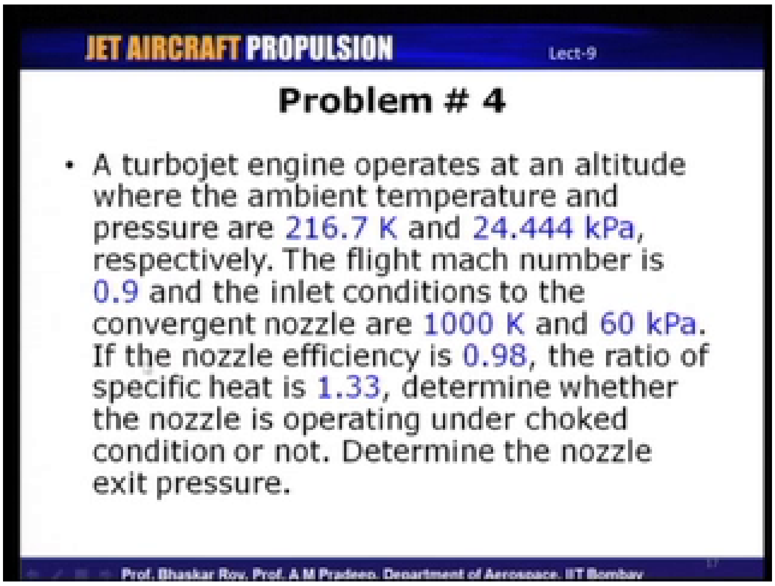

So, the problem statement number 4 is a turbo jet engine operates at an altitude where the ambient temperature and pressure are 216.7 Kelvin and 24.444 four kilo Pascals.

So, the ambient temperature and pressure are given, the flight Mach number is 0.9, and the inlet conditions to the convergent nozzle are 1000 Kelvin and 60 kilo Pascals. If the nozzle efficiency is 0.98, the ratios of specific heats is 1.33, determine whether the nozzle is operating under chocked condition or not, also determined the nozzle exit pressure. So, in this case, we have the ambient conditions, the ambient temperature and pressure and then the nozzle entry conditions for the convergent nozzle. So, in this case the nozzle is convergent, and we have the inlet conditions for the convergent nozzle, the nozzle efficiency is also given, and based on this data we have to find out whether this nozzle is operating under chocked condition and also we need to find out the exit pressure of the nozzle.

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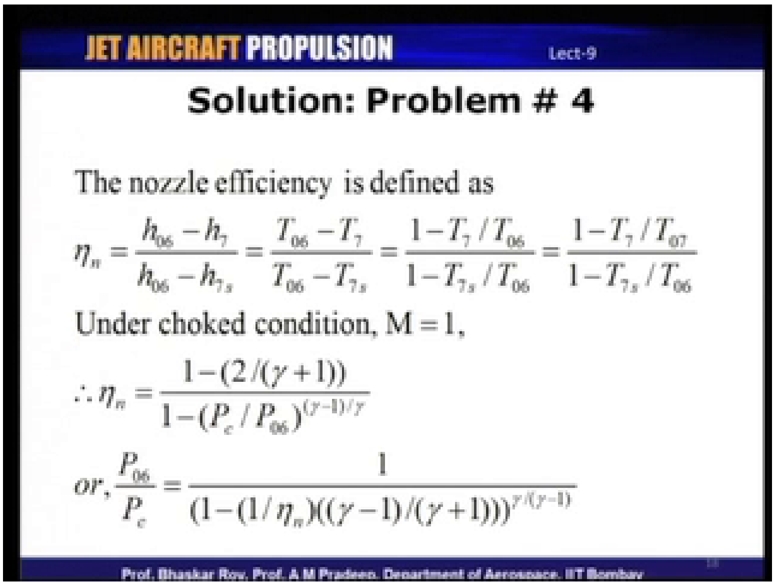

Now, let us go back to the definition of the nozzle, we have defined efficiency of a nozzle as efficiency of a nozzle is equal to h 0 6 which is the nozzle entry stagnation enthalpy minus h 7 which is nozzle exit static enthalpy divided by h 0 6 minus h 7 s where h 7 s is the static enthalpy for an isentropic process. This can be simplified as T 0 6 minus T 7 divided by T 0 6 minus T 7 s, which is in turn equal to 1 minus T 7 divided by T 0 6 divided by 1 minus T 7 s divided by T 0 6. Now, T 0 6 is also equal to T 0 7, because there is no change in stagnation temperature across the nozzle. So, we have 1 minus T 7 by T 0 7 divided by 1 minus T 7 s by T 0 6.

Now, under chocked condition, we know that the Mach number at the exit of the nozzle will become 1. So, if if we if we substitute for M is equal to 1, we know that T 0 7 by T 7 will be equal to 1 plus gamma minus 1 by 2 M 7 square. So, M 7 is equal to 0 as is equal to 1 and therefore, that expression reduces to 1 minus 2 by gamma plus 1. Similarly, the pressure under chocked condition is basically the critical pressure. And so, this temperature ratio that we have T 7 s by T 0 6 will become equal to the critical pressure divide by the P 0 6 that is total pressure, P c by P 0 6 raise to gamma minus 1 by gamma.

So, efficiency of the nozzle reduces to 1 minus 2 by gamma plus 1 divided by 1 minus P c by P 0 6 raise to gamma minus 1 by gamma, or this pressure ratio that we have P 0 6 by P c can be reduced as 1 by 1 minus 1 by eta and into gamma minus 1 by gamma plus 1 raise to gamma by gamma minus 1. So, this pressure ratio which is basically relating the nozzle entry total pressure to the critical pressure, critical pressure is the pressure which the nozzle attains when the Mach number at the exit is equal to 1 if the nozzle is operating under chocked condition. And if it is indeed operating under chocked condition, the the parameters are easily calculated, because Mach number is 1. So, if we know the ratio of specific heats and the nozzle efficiency, we can calculate the pressure ratio.

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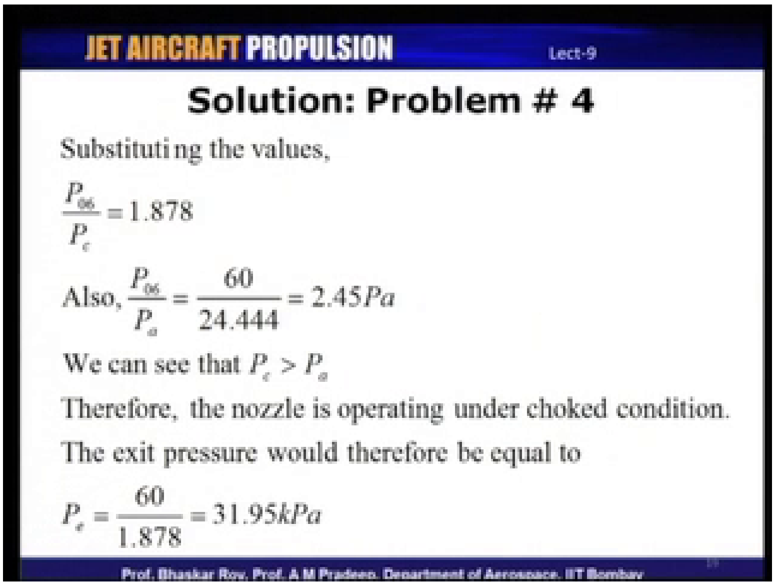

So, in this case we can now calculate P 0 6 by P c, because rest of the parameters are known, efficiency is known, the gamma is known. So, if we substitute for all those values, we have P 0 6 by P c is equal to 1.878, also we can calculate P 0 6 by P a as equal to 60 by 24.4 that is 2.45 kilo Pascal.

And therefore, we can see that from both these expressions, we can see that P c is greater than P a, this is critical pressure is greater than the static pressure ambient which means that the nozzle is operating under chocked condition. And so, that is if the nozzle exit pressure exceeds the ambient pressure, this means that the nozzle is indeed chocking that it the nozzle is operating under chocked condition, exit Mach number is equal to 1, the maximum mass flow rate that the nozzle can handle has already been basing through the nozzle. And once we can find out whether it is chocking or not, then the exit pressure is equal to the chocking pressure which will be equal to the inlet pressure divided by the pressure ratio 60 divided by 1.878 that is 31.95 kilo Pascals.

So, nozzle exit pressure can be calculated from this expression which is basically equal to the critical pressure that is P 0 6, 60 kilo Pascal divided by 1.878, 31.95 kilo Pascals. So, in this problem we have we have been, we we could use the nozzle efficiency definition and in cooperate that in the pressure ratio for calculating the critical pressure ratio to determine whether the nozzle is operating under chocked condition or not. And once it is chocked, then the exit pressure will be equal to the chocking pressure which again can be calculated from the critical pressure ratio.

So, what we are solved in in today's tutorial are four different problems, two of them related to Brayton cycle and two other problem related to the components, one to the intake and one to the nozzle. And of course, as I discussed in the last class, we will be talking up detailed analysis and discussion of all these individual components in different lectures, in few lectures from now. We will take up intake in detail, compressor and fan, combustion chamber, turbine, nozzle, all these components will be detailed discussed in detailed subsequently, their geometry instruction, etcetera also will be part of that discussion. So, I have few excises problem for you to solve based on our discussion and the tutorial that we had today.

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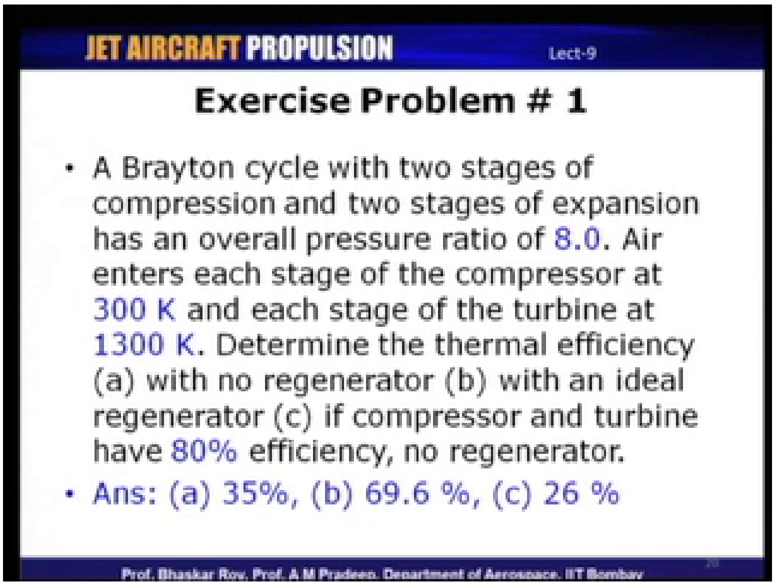

So, you have four different problems as excises a problem, the first one is for a Brayton cycle. So, a Brayton cycle with two stages of compression, when two stages of expansion has an overall pressure ratio of 8.0, air enters each stage of compressor at 300 Kelvin and each stage of the turbine at 1300 Kelvin.

Determine the thermal efficiency, part a with no regenerator, part b with an ideal regenerator, part c if the compressor and turbine have 80 percent efficiency each with no regenerator. So, answer to part a is 35 percent, part b is 69.6 percent, and part c is if compressors and turbine, both are efficiency of 80 percent and no regeneration, then the efficiency is 26 percent.

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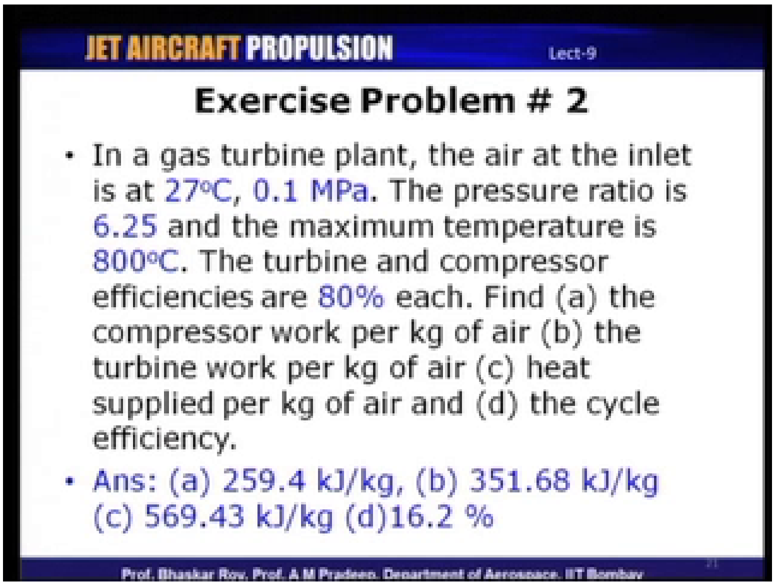

The second problem is in a gas turbine plant, air at the inlet is at 27 degree Celsius and 0.1 MPa, the pressure ratio is 6.25, the maximum temperature is 800 degree Celsius. Compressor and turbine efficiencies are in 80 percent each, find part a, compressor work per kilo gram of air, part b turbine work per k g of air, part c heat supplied per k g of air, and part d the cycle efficiency.

Answer to this is part a 259.4 kilo joule per kilo gram, part b 351.68 kilo joules per kilo gram, part c 569.43 kilo joules per kilo gram, and part d 16.2 percent.

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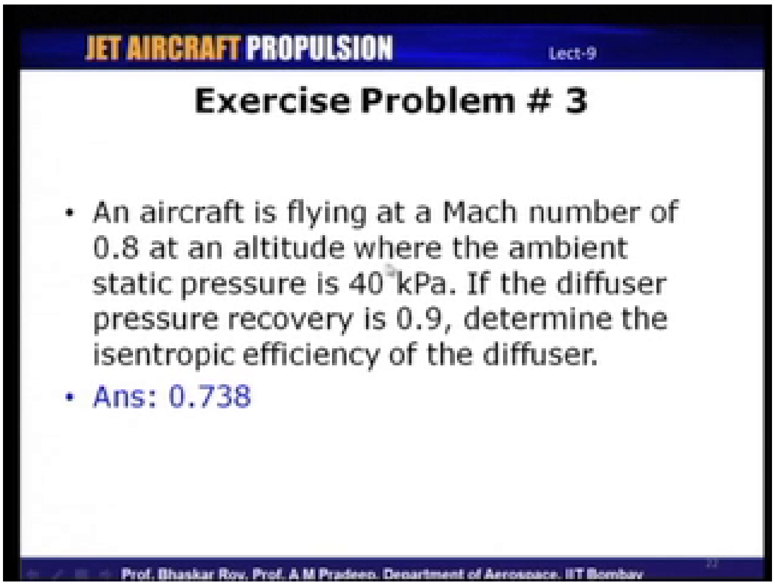

The third problem is to do with an intake, an aircraft is flying at an Mach number of 0.8 at an altitude where the ambient static pressure is 40 kilo Pascal. If the diffuser pressure recovery is 0.9, determine the isentropic efficiency of the diffuser, in this case the isentropic efficiency comes out to be 0.738. So, here we have a Mach number, flight Mach number, flight the static pressure is given and the diffuser pressure recovery is specified, we need to find isentropic efficiency, answer is 0.738.

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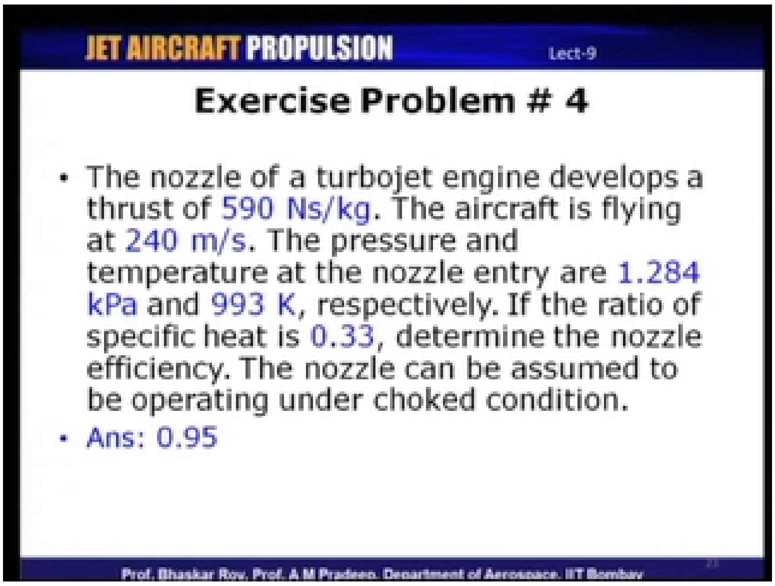

And the last problem is a nozzle of a turbo jet engine, it develops a thrust of 519 Newton's second per kilo gram, air craft is flying at 240 meters per second, the pressure and temperature at the nozzle entry are 1.284 kilo Pascals and 993 Kelvin respectively. If the ratio of specific heats is 0.33, determine the nozzle efficiency, the nozzle can be assumed to be operating under chocked condition. So, here its specified at the nozzle is operating under chocked condition and which means that, exit pressure is the critical pressure and so on, we need to find the nozzle efficiency. If the thrust is given, the flight velocities are given and nozzle entry conditions are also specified, so the nozzle efficiency is a 0.95.

So, here we have four different exercise problems and based on this particular tutorial, we had discussed today, I am sure you would be able to find out the, solve these problems and be able to find out the answers which have been given in these exercise problems. So, that brings us to the end of today's tutorial section, we will continue our discussion on cycle analysis of jet engines in the next class, we will basically be taking up real cycle analysis of different types of jet engine. And subsequently, we also have another tutorial section on the real cycle analysis of jet engines.