

**Jet Aircraft Propulsion**  
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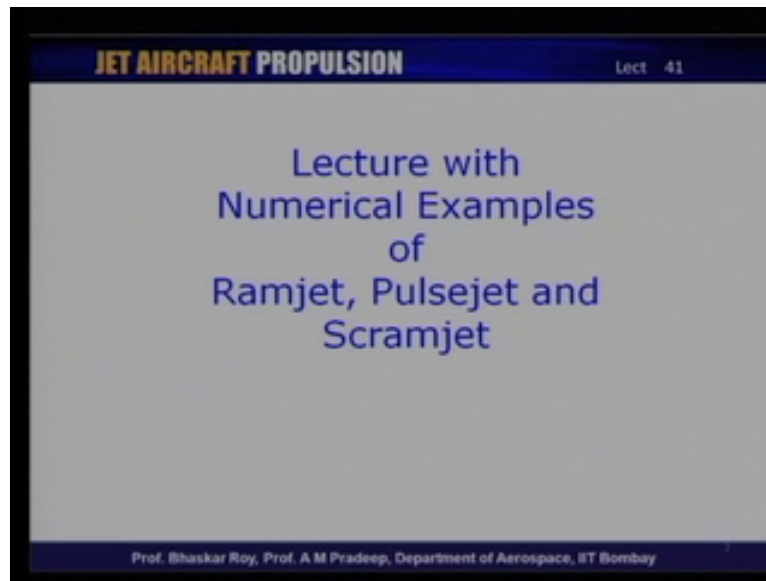
**Lecture - 41**

**Tutorial-7**

We have been talking about ramjets, pulsejets and scramjets over the last few lectures in this lecture series. Today, I will try to bring to you a few numerical examples of solution of typical problems related to ramjet, pulsejet and scramjet. I will take one problem for each of these jet engines and try to show you how simple calculations can be made to find out the performance of these engines. Now, these engines as you know are, fundamentally rotor less configurations which means, there are no rotating parts and as the result of which they are very simple engines.

However, when you get down to calculating the performance as we have done in the lectures, you would probably remember that large amount of flow in these engines are supersonic. Hence, you need to bring in the supersonic aerodynamic or gas dynamics, the shock theory and shock tables; and then you have the combustion where you need to bring in the Rayleigh tables or Rayleigh relations, or equations to find the solution to the flow through these engines. So, I will be essentially using those theories straight away. Assuming that you have already under gone those theories in other courses and as enumerated in lecture in this series; just couple of lectures back the analysis through various components would be taken up essentially with the use of those theories.

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So, we will get down to using the theories to find out the performance of ramjet, pulsejet and scramjets. These are numerical examples and through the numerical examples I will try to bring to you how this essentially the flow part, most part of which is supersonic can be assessed at various stations to get the overall performance of various kinds of jet engines. And at the end of it I will try to bring to you a comparative assessment of these three kinds of jet engines and how they compare with each other in performances. So, let us take a look at some of the performances of these jet engines through very simple numerical examples.

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**JET AIRCRAFT PROPULSION** Lect 41

**Problem-1 Ramjet**

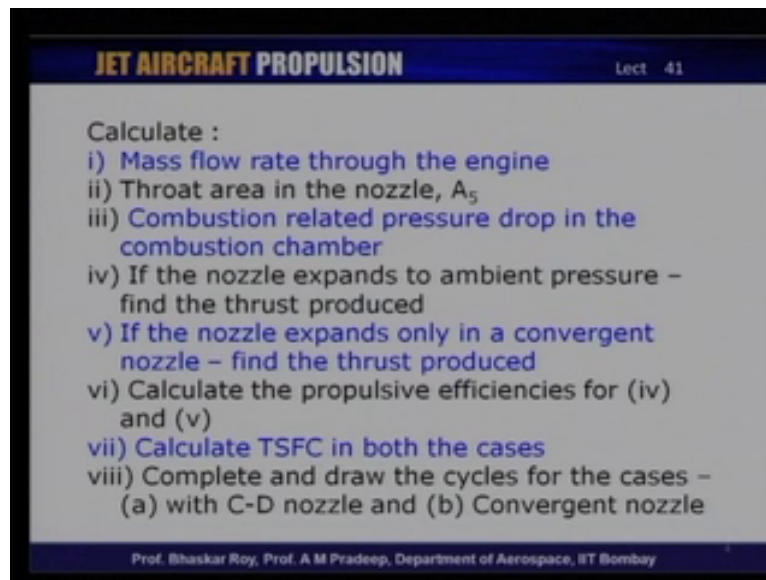
A ramjet is flying at Mach 1.818 at an altitude 16.750 km altitude ( $P_a = 9.122$  kPa,  $T_a = -56.5^\circ\text{C} = 216.5$  K., sonic speed,  $a = 295$  m/s). The flow is assumed to enter the intake of the ramjet through a normal shock standing at the intake face. No pre-entry loss or friction loss inside the engine is assumed to exist. Combustion delivery temperature is 1280 K. and the fuel -air ratio is 1:40. The area at the intake face is  $A_1 = 0.0929$  m<sup>2</sup> and at the Combustion chamber ,  $A_3 = 0.1858$  m<sup>2</sup>

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To begin with we will start with ramjet engine; will pose the problem first. A ramjet engine is flying at Mach 1.818 at an altitude of 16.750 kilo meters where the ambient pressure is 9.122 kilo Pascal and the ambient temperature is minus 56.5 degree centigrade which is 216.5 K; and the sonic speed is 295 meters per second. Now, the flow is assumed to enter the intake of the ramjet through a normal shock standing at the intake phase. So, the entire shock structure there is simplified into one single normal shock and analysis through that shock could suffice in solution of this problem. It is also prescribed that no pre-entry loss or friction loss inside the engine is assume to exists and hence that part is also simplified in this problem; that there is no pre-entry loss or there is no duct loss to be taken care of in the process of the solution.

The combustion delivery temperature is prescribed as 1800 degrees; 1280 K and the fuel -air ratio is prescribed as one 1:40. The area at the intake phase is given as 0.0929 meter square and the combustion chamber area is given as 0.1858 meter square. Now, these are the prescription with which the ramjet engine has been created and we are expected to find the solution to the problem of how this engine would perform under the prescribe flow conditions at Mach 1.818 at an altitude of 16.750 kilo meters.

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Calculate :

- i) Mass flow rate through the engine
- ii) Throat area in the nozzle,  $A_5$
- iii) Combustion related pressure drop in the combustion chamber
- iv) If the nozzle expands to ambient pressure - find the thrust produced
- v) If the nozzle expands only in a convergent nozzle - find the thrust produced
- vi) Calculate the propulsive efficiencies for (iv) and (v)
- vii) Calculate TSFC in both the cases
- viii) Complete and draw the cycles for the cases -  
(a) with C-D nozzle and (b) Convergent nozzle

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What are required in terms of finding the solution or you find the mass flow rate through the engine which is the first thing of course, you would need to do. The throat area in the nozzle, the combustion related pressure drop in the combustion chamber; and if the nozzle expands to ambient pressure - find the thrust produced that is if full expansion is allowed to be talking place which means it could probably have C-D nozzle. If on the other hand, the nozzle expands only in a convergent nozzle then also you find the thrust and we would find that they are slightly different from each other. And calculate the propulsive efficiencies for these two cases; that is one in which sib nozzle, another in which you have a convergent nozzle. And then of course, calculate the thrust specific fuel combustion in both the cases; and then you see whether you can complete the cycles for these two cases, I will leave that to you to solve the problem of drawing the cycles. I will solve the rest of the problem, but I will leave it to you to complete drawing of the cycles of these two cases.

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**Solution :**  
Flight velocity=Intake velocity of air= $M_1 \cdot a = 536$  m/s  
From isentropic relations,  
Total temperature at entry,  $T_{0a} = 360$  K  
Total Pressure at entry,  $P_{0a} = 53.85$  kPa  
Ambient air density,  $\rho_1 = P_a / R \cdot T_a = 0.147$  kg/m<sup>3</sup>  
Mass flow through the Intake =  $\rho_1 \cdot V_a \cdot A_1 = 7.3$  kg/s  
Normal shock : From shock tables :  
At intake face, for  $M_1 = 1.82$ ,  $M_2 = 0.612$ ,  $T_2 = 334.8$  K  
 $P_{02} = 0.803 \cdot P_{01} = 43.25$  kPa,  $T_{02} = T_{01} = 360$  K  
Since the duct losses due to friction etc are zero,  
 $P_{03} = P_{02} = 43.25$  kPa,  $T_{03} = T_{02} = 360$  K

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Let us look at the solution that we can find prescribed is that the flight velocity is for that Mach number would be 536 meters per second; that is at Mach 1.818 at the prescribed altitude. And the total temperature at the entry would be  $T_{0a}$ , that will be 360k; and total pressure at entry  $P_{0a}$  would be 53.85 kilo Pascal.

The ambient air density which you can calculate from the ambient pressure and temperature that is prescribed would be 0.147 kilo grams per meter cube. Correspondingly, one can find the mass flow through the intake system or which is what the mass flow through the engine of the air would be  $\rho_1 V_a A_1$  and that could be 7.3 kilo grams per second. Now, if you enter the intake, it was prescribe that you have a normal shocks standing in front of the intake phase. So, the flow has to go through the normal shock to enter the intake and so, at the intake phase for a Mach number of 1.82; the corresponding Mach number behind the shock would be 0.612 and the corresponding temperature  $T_2$  would be 334.8 K.

$P_{02}$  across the normal shock would be 0.803 times  $P_{01}$ . So, that it would be 43.25 kilo Pascal and  $T_{02}$  correspondingly would remain same as  $T_{01}$ ; there is no change in total temperature across the shock and that remains 360 K. Now, it is prescribed that the duct losses are to be completely ignored. So, there is no losses there and as the result of which  $P_{03}$  at the entry to the combustion zone would be equal to  $P_{02}$ , and that would remain as 43.25 kilo Pascal. Temperature of course, continues remains same since no work has been done and there is no heat transaction either; hence the temperature remains same as 360K.

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Using isentropic tables, at stn 2 behind the shock,  $M_2 = 0.6121$ , the area may be computed from  $A_2/A_1 = 1.16565$ ;

Then,  $A_{cc}/A_1 = (A_{cc}/A_2) \cdot (A_2/A_1) = 2 \times 1.16565 = 2.33$

Inside the combustion chamber, the Mach number may be computed from isentropic tables or relations as:  $M_3 = 0.26$

**Combustion chamber calculations :**  
Due to accompanying heat addition, Rayleigh flow tables or relations need to be utilized for calculation of parametric variations.

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Now, if you use the isentropic tables at station two behind the shock, we have found that the Mach number is 0.6121; the area then may be computed from the isentropic relations as  $A_2/A_1$  and  $A_1$  is of course, the intake area; and that would be 1.16565 as available from the isentropic tables. Then one can find the area at the combustion chamber and that would be given in terms of the area ratio across the duct, and the area ratio across the intake phase. And the product of these two comes out to be 2.33 that is area ratio between combustion chamber and the intake phase or one may call the intake throat where the normal shock is essentially standing. Now, inside the combustion chamber, the Mach number may be computed from isentropic tables that is before the combustion is initiated and before the combustion zone; and that comes out to be  $M_3$  would be equal to 0.26. Now, once we get into the combustion chamber, we have flow with heat addition and as a result the Rayleigh flow tables are relations need to be utilized for computing the variations of various parameters through combustion chamber. So, you need to get hold of Rayleigh flow tables or Rayleigh equations or relations.

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For a mach number of 0.26 inside the CC, at stn.2, with reference to the parameters at the Intake face,  $T_{03}/T_{01} = 0.275$  ;  $P_{03}/P_{01} = 1.214$ ,

Next with combustion delivery temperature given as  $T_{04} = 1280$  K,

$T_{04}/T_{01} = T_{04}/T_{03} \cdot T_{03}/T_{01} = (1280/360) \times 0.275 = 0.9759$

From Rayleigh Tables  $M_4 = 0.83$  and  $P_{04}/P_{01} = 1.014$

$P_{04}/P_{03} = P_{04}/P_{01} \cdot P_{01}/P_{03} = 1.014/1.214 = 0.8352$

Whence,  $P_{04} = 36.12$  kPa

CC pressure loss is  $P_{03} - P_{04} = 43.25 - 36.12 = 7.13$  kPa  
This amounts to 16.5% pressure loss – much larger than in a turbojet engine

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Now, for a Mach number of 0.26 inside the combustion chamber starting at station 2 with reference to the parameters of the intake phase, the temperature ratio from the intake phase could be found as the 0.275 and the pressure ratio with reference to the intake phase would be 1.214. Now, these are the total temperature and total pressure ratios the combustion delivery temperature as already been prescribed as 1280 K and hence the combustion chamber temperature  $T_{04}$  can be found from the temperature ratio  $T_{04}/T_{01}$  which would be the product of two temperature ratios  $T_{04}/T_{03}$  into  $T_{03}/T_{01}$ .

Now, the first one is 1280 by 360 and as a result of which we get the second one is 0.275; and we get ratio of 0.9759. Now, from the Rayleigh tables, we can see that the Mach number after the combustion would be 0.83 and the pressure ratio across the combustion zone would be 1.014. And as the result of which we can now find the combustion chamber pressure after the combustion that would be  $P_{04}$ ; that would be 36.12 kilo Pascal.

Now, this is what you get from the Rayleigh tables or Rayleigh relations and as a result of which one can say that the combustion chamber pressure loss that is  $P_{03}$  minus  $P_{04}$  would be 43.25 minus this 36.12 that we have found just now; and the pressure loss is 7.13 kilo Pascal. Now, this amounts to 16.5 percent pressure loss and as you were aware of find of pressure loss that is normally prescribed in turbo jet or turbo fan engines, this pressure loss is very high compared to those pressure losses where they are normally of the order of 3 to 5 percent.

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Flow in the C-D nozzle may be assessed as :  
Starting from Mach 0.83 (CC delivery) assuming isentropic flow,  $A_4/A_t=1.027$  in the convergent duct  
The exit area may be calculated from  
 $A_t=A_4/(A_4/A_t)=0.1858/1.02696=0.181 \text{ m}^2$   
As no duct loss is prescribed,  $P_{0t}=P_{04}$ ,  $P_t=0.25P_{04}$   
At the exit, after flow through the divergent duct,  
 $M_e=1.55$  ;  $A_e/A_t=1.211$ ,  $T_e/T_{0e}=0.675$   
 $A_e=1.211 \times 0.181=0.22 \text{ m}^2$ ,  $P_e = P_a$  as prescribed  
From  $T_{0t}=T_{0e}=T_{04}$ , as no heat / work is transacted  
Since,  $T_{0t}=T_{0e}=T_{04}=1280 \text{ K}$ , then  $T_e=864.5 \text{ K}$   
Jet Vel is now calculated  $V_e = M_e \sqrt{\gamma \cdot R \cdot T_e}=916.5 \text{ m/s}$   
For fuel-air ratio of 1:40 thrust is calculated as :

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Here, it is 16.5 percent. So, the pressure loss in typical ramjets could be somewhat of a higher order than the turbo jets and turbo fans that we have done before. Now, if you take a ram jet flow coming out through C-D nozzle as prescribed the first option was the C-D nozzle which promotes full expansion through the nozzle to ambient condition. Now, starting with Mach 0.83 which is the combustion chamber, but delivery Mach number; an isentropic flow is assumed no again is prescribed that duct losses is zero; no duct losses are prescribed and hence the area ratio would be 1.027 across the pipe and then the exit area can be now calculated from  $A_e$  that is using the areas that we have already found the area ratio across the duct and the area available before. And that comes out to be 0.181 meter square; that is the exit area of the nozzle or of the whole engine. Now, since the no duct losses prescribes the total pressure at the exit phase remains same as  $P_{04}$  that we found earlier from the Rayleigh relations and as a result the value of static pressure at exit is 0.25 times, that is of  $P_{04}$ . And at the exit then one can find the Mach number  $M_e$  and that would be 1.5.

The corresponding area across the exit phase at the nozzle exit to would be 1.211; the area ratio and the static to total temperature ratio is 0.675. And hence the throat area compared to the exit area is 0.22 meter square and using these areas one can now find the throat temperature. Now, since no heat or work is been transacted; the total temperature of course, remains constant and hence the total temperature is remains same as  $T_{04}$  that was prescribed as the combustion delivery temperature that is 1280 K.



And hence the static temperature at the exit now would be 864.5 K. Now, using this one can now find the jet velocity at the exit and that would be equivalent to  $V_e$  equal to  $M_e$  root over of  $\gamma R T_e$  at the exit phase and that would be 916.5 meters per second. So, that is the velocity with which the flow is going out of the engine and hence the velocity that contribute towards the making of thrust.

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Thrust is given by:  $F = \dot{m}[(1+f)V_e - V_1] + A_e(P_e - P_a)$

C-D nozzle Thrust is,  $F = 4090 \text{ N}$

Propulsive Efficiency,

$$\eta_p = \frac{F.V_a}{F.V_a + \frac{1}{2}\dot{m}[(1+f)V_e^2 - V_a^2]}$$

= 51.2 %

For fuel-air ratio prescribed as  $f = 1/40$

Sp. Fuel Consn.,  $TSFC = f/(F/\dot{m}) = 0.16 \text{ kg/N-hr}$

If the nozzle is only a convergent nozzle:  
**The exit face is the throat of the nozzle.**

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Now, the fuel air ratio that is been prescribed for this engine is 1:40 and the thrust with the help of this fuel-air ratio, the thrust can now be computed and that could be given by the thrust relation that we have done many times before in the lecture series, and that comes out to be thrust  $F$  equal to 4090 Newton's or 4.09 kilo Newton's. Using these values one can find the propulsive efficiency; using the propulsive efficiency relation that we have done before and that comes out to be 51.2 percent, and as a fuel-air ratio is prescribed as 1:40.

Using that we can now find that T S F C, the thrust specific fuel consumption and that now comes out to the 0.16 kilograms per Newton hour. Now, if you look at these values you will see that the propulsive efficiency of this kind of jet engine is somewhat lower than the propulsive efficiency that we have seen in case of turbojets and turbo fans. And the turbo propellers for example, efficiencies are of the order of 80 percent; the turbojets, turbo fans normally have propulsive efficiencies of the order of 60 to 65 percent. And hence this kind of ramjet engine would have a propulsive efficiency which is somewhat of a lower order.

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convergent nozzle

Pressure ratio necessary for choking : 1.893

Available pr. ratio across the nozzle =  $P_{04}/P_a = 3.96$

The nozzle is choked, and exit pressure,  $P_e = 19.1$  kPa

$T_e = T_t = T_{04}/[(\gamma+1)/2] = 1280/1.2 = 1067$  K

Exit jet velocity,  $V_e = V_t = \sqrt{\gamma \cdot R \cdot T_e} = 654$  m/s

From isentropic tables, exit (throat) area,  $A_e/A_t = 1.027$

Whence, exit area  $A_e = 0.1809$  m<sup>2</sup>

Thrust  $F = \dot{m}[(1+f)V_e - V_1] + A_e(P_e - P_a) = 3587$  N

Sp. Fuel Consn., TSFC =  $f/(F/\dot{m}) = 0.1834$  N

Prop Efficiency  $\eta_p = F \cdot V_e / F \cdot V_a + \frac{1}{2} \cdot \dot{m} \cdot [(1+f)V_e^2 - V_a^2] = 54.8\%$

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Now, if we take the ramjet with a convergent nozzle which means the exit phase is essentially also the throat of the nozzle. And if we take that kind of nozzle as an active nozzle for this ramjet engine, we can calculate the performance all over again. The pressure ratio necessary for choking that is the ideal pressure ratio which you can calculate from ideal theory and that will be 1.893. The available pressure ratio across the nozzle right now is 3.96 from the computed parameters of this particular engine. And hence we can say that the nozzle is indeed choked; and hence the exit pressure would be as per the critical pressure that is ideally available and that would be 19.1 kilo Pascal. Correspondingly, that exit temperature which would be same as the throat temperature exit being the throat itself and that would be 1067 K. Now, using this value of K we can now find the exit jet velocity which is at the choking condition which means the Mach number is one and hence  $V_e$  which will be equal to  $V_t$  and that will be  $\sqrt{\gamma R T_e}$ , and that would be 654 meters per second.

So, that is the exit jet velocity which is also the throat velocity of the convergent nozzle. Now, if you look at the isentropic tables, one can find that the exit area which is also a throat area; that area ratio from the jet pipe would be 1.027 and hence the exit area would be 0.1809 meters square. Now, using this values one can find the thrust; in this thrust the pressure is not been fully converted to ambient pressure. Hence there would be residual pressure thrust and if you plug in those values that we have calculated you would get a thrust of 3587 Newton's.

So, that is a thrust you get for convergent nozzle the corresponding specific fuel consumption TSFC would be 0.1834 Newton's and the propulsive efficiency using the same relation that we have used before would be 54.8 percent. Now, as you can see here with reference to the C-D nozzle, the thrust created is a little less. Correspondingly, it is a specific fuel computation is a little longer higher side, but the propulsive efficiency is a little longer higher side. One of the reasons the specific fuel the propulsive efficiency a little higher is because the even though the thrust created is less.

The exit jet velocity is much lower and as the result the exit jet wastage in terms of the jet velocity is somewhat lower. And as the result of that the propulsive efficiency is a little higher than what you got for C-D nozzle even though the thrust created is a little less. So, for a typical ramjet designer the question would be whether he would like to settle down for a lower thrust for a higher propulsive efficiency or whether you would like to go for a higher thrust and sacrifice a little bit of propulsive efficiency for his performance. So, these are the choices typically a ramjet engine designer would probably be looking at.

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**Problem 2 – Pulsejet**

An aircraft powered by a pulsejet engine is flying at 12 km altitude, at Mach 2. The engine parameters are given as : inlet area = 0.084 m<sup>2</sup>. Combustion chamber pressure development,  $P_{03}/P_{02} = 9.0$ , heating value of fuel,  $Q = 43,000$  kJ/kg, combustion efficiency = 0.96. Assuming ideal (no loss) flow through the intake, find:

(i) The air mass flow rate, (ii) Maximum Temp.  
(iii) fuel-air ratio,  $f$  (iv) Exit velocity,  $V_e$   
(v) Thrust of the engine (vi) TSFC

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Now, let us take a look at a pulsejet engine, we have done the theory; you have done the analysis and it is a very simple engine. And one of the advantages of pulsejet engine is that it operates at low speed, it can even operate from takeoff condition helping the aircraft even to takeoff. So, let us take a look at how a pulse jet engine typical would perform under flight conditions. So, a pulse jet engine problem can be posed in terms of an aircraft that is flying at Mach 2 at 12 kilo meter altitude and the engine parameters are given in terms of the inlet area which is 0.084 meter square.

The combustion chamber pressure development is given in terms of ratio. It is a pulsejet engine remember, it is enclosed combustion chamber and hence the pressure ratio needs to prescribe in one form or the other. And this prescription is pressure ratio is 9 that is  $P_{03}$  by  $P_{02}$  prescribed as 9; the heating value of fuel again is given  $Q$  is equal to 43,000 kilo joules per k g and the combustion efficiency prescribed is 0.96. Again assume that it is ideal flow; that means, there are no losses through the ducts or the intakes and assuming that kind of lossless flow situation; find the air mass flow, the maximum temperature, the fuel air ratio, the exit velocity, the thrust of the air engine and the TSFC the performance parameters.

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At 12 km  $P_{0a} = 18.75$  kPa ;  $T_{0a} = 216.65$  K  
The flight velocity is :  $V_a = M_a \sqrt{\gamma R T_a} = 590$  m/s  
The air mass flow rate is :  $\dot{m} = \rho_a \cdot V_a \cdot A_1 = 15$  kg/s  
Total temp across the intake diffuser remains constant,  $T_{01} = T_{0a} = T_a [1 + (\gamma - 1) M^2 / 2] = 390$  K =  $T_{02}$   
Total pressure,  $P_{01} = P_{0a} = P_a (T_{01} / T_{0a})^{\gamma / (\gamma - 1)} = 147$  kPa  
In the combustion chamber pressure rises from  $P_{02}$  to  $P_{03}$  by a prescribed ratio 9.0, and the temp ratio (temp change across the CC) is same by gas laws.  
The combustion delivery temp is :  $T_{03} = 3510$  K

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So, let us see whether we can find all those performance parameter using the prescribed values that is been posed here in this problem statement. What is stated here is at 12 kilo meters, it is flying at Mach 2 and hence the total pressure and temperature can be computed, and we get that has 18.75 kilo Pascal's; and temperature as 216.65 K. The flight velocity for the given Mach number would be 590 meters per second. The corresponding air mass flows using this flight velocity and given the air density at the prescribed altitude; and the air intake area would be 15 kilo grams per second. Now, the total temperature across the intake diffuser remains constant; there is no work been done; there is no transaction of heat or work and hence the total temperature is conserved; and that remains at 390 K with which it enters the combustion chamber; assuming there are no losses through the duct or vales that you have in a pulse jet. The corresponding total pressure is given in terms of  $P_{01}$  and that remains same as  $P_{0a}$ , and that is 147 kilo Pascal's. Now, in the combustion chamber it is prescribed that the pressure raises from  $P_{02}$  to  $P_{03}$  by a ratio of 9.

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The fuel air ratio may be calculated  
 $f = \dot{m}_f / \dot{m}_a = (c_{p-gas} \cdot T_{03} - c_{p-air} \cdot T_{02}) / Q \cdot \eta_{cc} = 0.0975$

In the jet pipe:

The exhaust velocity is  $V_e = \sqrt{2 \cdot c_{p-gas} \cdot T_{03} \cdot [1 - 1/9^{0.25}]}$   
 $= 2297 \text{ m/s}$

Specific Thrust,  $F/\dot{m}_a = (1+f)V_e - V_a = 1931.5 \text{ N-s/kg}$

The thrust,  $F = 28950 \text{ N} = 28.95 \text{ kN}$

The TSFC =  $f/(F/\dot{m}_a) = 50 \text{ mg/N-s} = 0.180 \text{ kg/N-hr}$

Propulsive Efficiency  $\eta_p = F \cdot V_a / F \cdot V_e + \frac{1}{2} \cdot \dot{m} \cdot [(1+f)V_e^2 - V_a^2]$   
 $= 0.418 = 41.8\%$

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Now, that is prescribed and using the simple gas laws for static air or any gas in an enclosed volume, if you simply apply the gas laws the temperature ratio would be same as the pressure ratio and hence the combustion delivery temperature would now be  $T_{03}$  would be equal to 3510 K. So, if you do that you can now calculate what the fuel air ratio would be from the prescribed temperature values and if you put in what we have done before the fuel air ratio can be calculated from the heat that is generated by the fuel heating value that is prescribed; the combustion chamber efficiency that is prescribed and this comes out to be 0.0975.

Now, this takes us to the jet pipe that is the combustion is now over and the flow is now got into a long jet pipe of a typical jet engine, and comes out of that jet pipe with exhaust velocity which can be computed from the given parameters; and if you plug in those parameters you get exist exhaust velocity of two thousand 2297 meters per second. Now, using this you remember across the jet pipe the pressure ratio would be same as what is prescribed to you for the combustion chamber.

So, across the jet pipe the same pressure ratio is valid or active and using that you get a velocity which is 2297 meters per second. So, if you use that you can now find the specific thrust which would be one plus f into this  $V_e - V_a$  which was the entry velocity and that comes out to be 1931.5 Newton seconds per kg or Newton's per kg per second. Now, this is the specific thrust that you can compute correspondingly, if you multiply this specific

thrust with the mass flow that we have computed that comes out to be 28,950 Newton's or 28.95kilo Newton's.

The corresponding specific fuel consumption TSFC can also be found and this comes out to be of the order of 50 milli grams per Newton second, also can be written in terms of kilo grams per Newton hour. Now, if you again calculate the propulsive efficiency as we have done in the theory and this comes out to be 41.8 percent as you would see here, the propulsive efficiency of a pulse jet is somewhat lower those even a ramjet and hence wants to see whether a pulse jet is an efficient device fundamentally or not. In comparison we see that pulse jet efficiency somewhat lower than that of a typical ramjet. We will have a comparative look at these values at the end of solving of all the problems.

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**Problem 3 - Scramjet**

A scramjet powered aircraft flies at Mach 5 at 16.75 km where  $T_a=216.67$  K and  $P_a=9.122$  kPa. The intake has a shock structure of two oblique shocks with both deflection angles  $\delta = 10^\circ$ . By burning hydrogen fuel ( $Q=120,900$  kJ/kg), the temp is raised to 2000 K. The fuel air ratio  $=0.025$ . The nozzle expansion ratio is  $A_2/A_4 = 5.0$ . The inlet and the exit areas are  $A_1=A_5 = 0.2$  m<sup>2</sup>. If  $c_p = 1.51$  kJ/kg.K ;  $\eta_{cc} = 0.8$

Calculate :

- Mach number at combustion chamber inlet
- Exhaust jet velocity
- Overall efficiency

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Let us now take a look at a typical problem related to scramjets. If you look at a scramjet powered aircraft which flies at a Mach 5 typically, scramjet do fly at very high Mach numbers normally five or above at an altitude of 16.75 kilo meters where again the temperature and pressures are prescribed and the intake has a shock structure. Now, the intake shock structure is defined in terms of two oblique shocks both with deflection of 10 degree. So, this is now given with two oblique shock standing inside the intake or at the intake phase through which the flow has to get inside the intake and then inside the scramjet engine.



The fuel prescribed here is hydrogen which has heating values of 120,900 kilo joules per k g as you can see it is much higher than that of a hydro carbon fuels that were prescribed in the earlier to engines. The temperature is now raised to 2000 K; the fuel air ratio is 0.025; the nozzle expansion ratio is prescribed as 5 and the inlet to exit areas prescribed are  $A_1$  is equal to  $A_5$  or  $A_e$  that is 0.2 meters square. Now, if the  $c_p$  of this hydrogen fuel burnt gas is taken as 1.51 kilo joules per k g K and the combustion chamber efficiency is prescribed as 0.8 somewhat lower than the other jet engines.

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Flight velocity is :  $V_a = M_a \sqrt{\gamma \cdot R \cdot T_a} = 1475 \text{ m/s}$   
 Mass flow through the engine  $\dot{m}_a = \rho_a \cdot V_a \cdot A_1 = 43.3 \text{ kg/s}$   
 Inlet total temp  $T_{01} = T_{0a} = T_a [1 + (\gamma - 1) M^2 / 2] = 1300\text{K} = T_{02}$   
 From shock relations or tables,  
 Across the first shock, for  $M_1 = 5$  &  $\delta = 10^\circ$   
 Shock angle,  $\beta = 19.4^\circ$ .  $M_2 = 4.0$  and  $T_2 / T_1 = 1.429$   
 Across the second shock,  $M_1 = 4$  &  $\delta = 10^\circ$   
 Shock angle,  $\beta = 22.2^\circ$ .  $M_3 = 3.3$  and  $T_3 / T_2 = 1.33$   
 In the combustion chamber heat is added to air flow with supersonic speed

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Because of the supersonic combustion that is going on the efficiencies somewhat on the lower side; heat being added up at supersonic speeds. If that is so, the problem statement asked to calculate Mach number at combustion chamber inlet, the exhaust jet velocity and the efficiency of this engine. What we will do is we will find the exhaust velocity and then find the thrust, and the s f c; and we will find the propulsive efficiency as we have done before for comparison of the various jet engines. The flight velocity for this particular craft can be now computed from the given Mach number which is 5 and that comes out to be 1475 meters per second mass flow through this engine.

Again using the continuity as before we can find 43.3 kilo grams per second in view of the very high inlet velocity the mass flow going in is also very high; even though the density is of the same order and the area is of the same order. The inlet total temperature can now be computed from the isentropic relation that you are familiar with and this gives us say total



temperature of 1300 K which is conserved through the intake duct. From the shock relations or shock tables across the first shock for a Mach number of 5 and a deflection of 10 degree, the shock angle can be found at 19.4 degree and the Mach number just behind the first shock would be 4; and the temperature ratio across this shock would be 1.429. Now, with those values, the flow now goes into the second shock which again has a deflection of 10 degree. Now, the entry mach number to that is now 4; as the result of that with the help of shock relations or tables you can find the shock angle and that would be 22.2 degrees; the mach number across the second shock that is behind the second shock would be now 3.3 and the temperature ratio across the shock would be 1.33. Now, the flow goes into the combustion chamber for addition of heat by burning of the fuel and that is to be done at supersonic speed. So, as we have seen the combustion efficiency is going to be little on the lower side. If now, we get into the combustion chamber we know that we have to again use the Rayleigh flow relations or tables as we have done in the earlier two engines and those are to be diploid in supersonic flow conditions.

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Using Rayleigh Flow relations (or tables)  
 $M_4 = 1.26$ , and  $P_{04}/P_{01} = 1.033$ ,  $T_{04}/T_{01} = 0.966$   
 Combustion chamber pressure ratio,  $P_{04}/P_{03} = 0.228$   
 Fuel-air ratio,  $f = \dot{m}_f / \dot{m}_a = (c_{p-gas} \cdot T_{03} - c_{p-air} \cdot T_{02}) / Q \cdot \eta_{cc}$   
 $= 0.01093$

Nozzle Flow  
 For  $M_4 = 1.26$ ,  $T_{04}/T_4 = 1.317$ , critical area ratio = 1.05  
 Whence,  $A_0/A_t = (A_0/A_4) \cdot (A_4/A_t) = 5 \times 1.05 = 5.25$

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So, using those we can get the parameters across the combustion chamber. So, let us do that across the combustion chamber. So, the entry Mach number that we have found is from the shock relations most 3.3. The moment you have a burning of the fuel the Mach number reduces to 1.26 because the temperature is now very high and so, even if the flow velocity

remains same the Mach number would come down. So, the Mach number is reduced to 1.26; the pressure ratio is now 1.033 and the temperature ratio is 0.966.

So, the combustion chamber for pressure ratio across the combustion chamber only is 0.228 and the fuel air ratio is calculated with the help of the temperature that we have obtained and that comes out to be 0.01093. Now, that is a fuel-air ratio calculated from the various parameters that we have computed through the combustion chamber. Now, the flow gets into the nozzle. Now, in the nozzle the flow is getting in with Mach number that is 1.26 as a result of which it is already supersonic. The flow getting into the nozzle is already supersonic; flow through the combustion chamber was already supersonic. The temperature ratio this isentropic temperature ratio that is total to static at this station is 1.317; the critical area ratio that we get from this flow through the jet pipe and nozzle.

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Nozzle Flow

Nozzle outlet Mach number,  $M_e = 3.23$ ,  
for which isentropic temp ratio  $T_{05}/T_5 = 3.11$

$$T_5 = \frac{T_3 T_{05} T_{04} T_4 T_3 T_2}{T_{03} T_{04} T_4 T_3 T_2 T_1} T_1 = 654.5 \text{ K}$$

The exhaust velocity,  $V_e = M_5 \sqrt{\gamma R T_e} = 1560 \text{ m/s}$

Specific Thrust,  $F/\dot{m}_a = (1+f)V_e - V_a = 102 \text{ N-s/kg}$

TSFC =  $107.15 \text{ mg/N-s} = 0.385 \text{ kg/N-hr}$

Thrust,  $F = 43.3 \times 102 = 4416 \text{ N}$

Propulsive Efficiency  $\eta_p = F V_a / F V_a + \frac{1}{2} \dot{m} [(1+f)V_e^2 - V_a^2]$   
 $= 0.527$  or  $52.7\%$

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And that would be 1.05 and then we can find the area ratio from the throat to the exit area, and that would be 5.25. So, the critical area ratio was the throat the area to the combustion chamber or the jet pipe that is the convergent part of the convergent divergent nozzle and then we get the area ratio from throat to the exit phase that is  $A_e$  and that comes out to be 5.25. So, on using these parameters we can find that the nozzle output Mach number would be  $M_e$  would be 3.23 that is clear supersonic Mach number at very high temperature.

And the isentropic temperature ratio at exit station and would be 3.11. So, the corresponding temperature at the exit can be found from the various temperature ratios that we have obtained from the various components and if we put them all together we get the temperature at the exit, and that would be 654.5 K. Now, using this temperature we can find the exhaust velocity  $V_e$  as before and using the Mach number that we are found just now that is 3.23, and that gives us a exit velocity of 1560 meters per second.

So, using this exhaust velocity we can now find first the specific thrust that is one plus  $f$  into  $V_e$  minus  $V_a$  and that comes out to be 102 Newton's per kilo grams. And using that we can find the thrust specific fuel consumption and that comes out to be 0.385 kilo grams per Newton hour. The thrust then now multiplied the mass flow with the specific thrust and that gives us a thrust of 4416 Newton's that is 4.41 kilo Newton's.

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	M	H km	$V_e$ m/s	F (kN)	TSFC Kg/n-hr	$\eta_{pr}$	F/m N/Kg/s
Ramjet (C-D noz)	1.8	16.7	916	4.09	0.16	51.2	560
Pulsejet	2	12	2297	28.95	0.18	41.8	1931
Scramjet	5	16.7	1560	4.41	0.38	52.7	102

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So, the specific thrust as we can see now are actually not very high in spite of the fact that the aircraft is flying at very high supersonic speeds. The propulsive efficiency that we get from these parameters can now be computed and it comes out to be 52.7 percent which is of the same order as ramjet engine that we are found earlier. So, the propulsive efficiency of scramjet engine is not much larger than that of a typical ramjet engine it is of the same order. So, the specific thrust that we see here is somewhat of a lower order and what we can do now is we can compare all the engines that we have computed all the parameters, and see what values we can get on comparative bases. For this engine, I will leave it to you to calculate the

overall efficiency; I suspect you will get the overall efficiency of a very low order of the order of ten or eleven percent now which says that the ramjet engine is likely to be not a very efficient device. So, let us take a look at the comparative values of the all the jet engines that we have done today. These are as I mentioned the rotor less engines; that means, there are no rotating parts, no compressors or turbines. So, these jet engines when there are compared to each other, they are all flying at supersonic mach numbers at very high altitudes and there exhaust velocities are very high. And as we can see here in a pulse jet engine because of the high pressurization that is achieved in the combustion chamber very high exhaust velocities are possible as a result of which very high thrust can be created.

One can see here that the propulsive efficiency of the pulsejet engine is somewhat of a lower order. The specific thrust of the scramjet engine is of the lowest and it shows in a thrust specific fuel consumption which is highest of the whole lot. So, in terms of comparative values the thrust creation capability of the pulse jet at lower Mach numbers is very high; however, it large difficult flying at very high Mach numbers because of the valves that it has the mechanical operation of the valves. It has a lower propulsive efficiency and correspondingly, it has a higher thrust making capability; however, the scramjet engine which flies at very high velocities it does create very high jet velocity.

But its specific thrust is very low and its thrust specific fuel consumption is high which means as an efficiency of the device; it is not a very efficient device for flying. So, on a comparative basis we can see that various rotor less jet engines compare with each other the propulsive efficiencies are lower than that turbo jet and the turbo fan engine that we have seen before; however, there are utility essentially is at very high flight mach numbers.

For pulsejet engines at Mach numbers of the order of two or below for ramjet engines for mach numbers of the order of one point five or above and for scramjet engines for mach numbers of the order of five and above; those are the mach numbers where which these engines would be of useful values and they would produce thrust of a necessary requirement. Scramjet engine has a little problem of producing reasonable amount of thrust and one has to compute very accurately to find out whether it is able to produce the positive thrust required for flying, it does produced its performance at somewhat low efficiency.

But that is the price you pay for flying at very high supersonic or hypersonic Mach numbers. So, that brings us to the end of this comparative assessment of jet engines which are rotor less

jet engines and that bring us to the end of the lectures on ramjets, pulsejet and scramjets. In the next class we will conclude this lecture series by looking at what all we had done and then we will be looking forward to various kinds of propulsive devices for flying of aircraft in future. So, in the next class we will conclude this lecture series and we will look forward to new kinds of devices that would make the aircraft fly in future.