Introduction to Aerospace Propulsion
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Module No. # 01
**Lecture No. # 07**
Tutorial

Hello and welcome to lecture 7 of the lecture series on introduction to aerospace propulsion. In today's lecture, as I mentioned in my previous lecture, we shall solve some problems basically related to work generated by a system. We shall mainly focus on the displacement work as I mentioned in my previous lecture that displacement work is of significant interest to engineers, because there are lots of engineering systems which operate on the displacement work mode. So, we shall try to solve some problems which are related to work generation through the displacement work mode.

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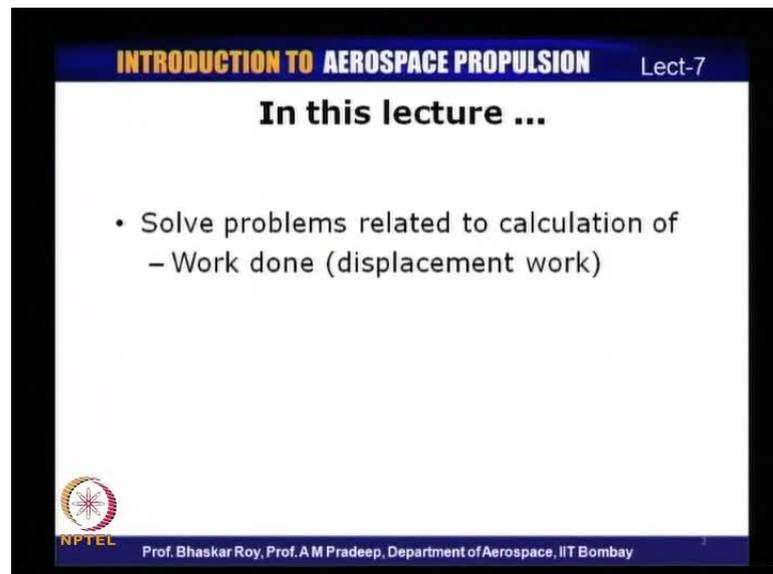
So, today's lecture will basically be on problem solving, numerical problem solving related to calculation of work done and with more focus on the displacement work done.
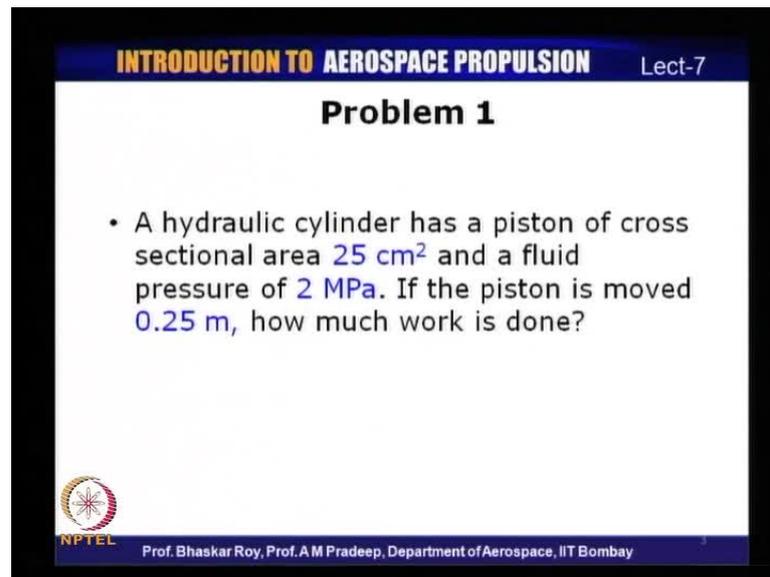
Let us look at the first problem that we have. Well, we have a problem here which states that hydraulic cylinder has a piston of a cross sectional area of 25 centimeter square and a pressure of 2 mega pascal. If the piston is moved by 0.25 meters, how much work is done? Here we have a question where in there is a piston cylinder arrangement, the area of the piston head is given and the pressure within the cylinder is also given to us. If the piston moves by a certain distance which is also specified, we need to find out how much is the work done during this particular process.

Before I start solving this problem what I would like to emphasize is a systematic approach to problem solving; so, it is very important that we adopt a certain systematic approach for any problem solving, whether it is thermodynamics or any other discipline for that matter. One of the important things is to first understand the problem definition very clearly and to be clear as to what is mentioned in the particular problem statement. The other important thing is to make a sketch of the problem that you have in hand.

In this example that we are looking, we have a piston cylinder arrangement and we have certain properties of the system which have been defined. So, it is important for us to mark these particular parameters, which have been defined in this problem on the drawing or the sketch of the problem definition. Then we also need to define the assumptions that we are going to make in a particular problem. In solving a particular problem, if required, also justify why these assumptions have been made. Some of the

assumptions are quite obvious like in most of the thermodynamics analysis that we are going to do; we are going to assume that the process is carried out quasi-statically. So, you may or may not mention this as an assumption, because it is an obvious assumption in almost all the thermodynamics analysis that the process is quasi-static.

If it is a non quasi-static process, then its analysis is entirely different, but we might as well mention the assumptions that we make besides the quasi-static assumption there could been other assumptions that are involved. So, it is also important to clearly explain the assumptions, which are made in the problem and then move towards solving understanding the problem itself and then solving the problem.

If we were in many of the problems that we will solve and some of them we will of course solve today. It would also be advisable to draw the process diagram in terms of PV or little later on in terms of temperature and entropy. So, the process illustration along with the process diagram in terms of PV or any other parameters would definitely make things a lot clear and easier to tackle than this directly trying to substitute numerical values in a formula and getting the answer, right? Well, basically it is trying to analyze a particular problem systematically. Also ensure that you do not make any error of judgment, when you try to understand a particular problem statement.

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So, with this in mind let us now look at what this particular problem is all about. In this problem we have a piston cylinder arrangement as is shown in this diagram. In this

piston and cylinder arrangement, we have a certain pressure which is within the system, within the cylinder volume, which is in this particular case given as 2 mega pascals.

There is a certain pressure, which is exerted on the piston by the gas which is host within the cylinder volumes. Now, the area of the piston is given as 25 centimeter square; during this process it could be it is basically an expansion process. The piston has moved by a certain distance which is specified as 0.25 meters arc or 25 centimeters.

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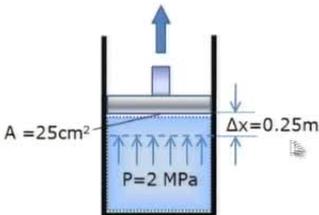
This is the problem and we are required to find what the work is done during this particular process? One of the assumptions as I already mentioned is of course, that the process is in quasi-equilibrium or it is a quasi-static process. It is possible for us to integrate PdV because it is a quasi-static process.

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To calculate the work done during this process as we have seen if the basic definition of work is that work is force with a displacement. In this case, we have F is equal to P times A pressure times the area. So, work is integral of F times dx that is force times the displacement. Since, force is equal to pressure times area we have work is equal to pressure into area into the distance traveled during this process. All these parameters are specified in this particular problem.

So, we have work is equal to pressure in this case it is 2 mega pascals which is equal to 2000 kilopascal into the area which is 25 centimeter square so, 25 into 10 power minus 4 meter square and delta x which is 0.25 meters. Another suggestion that I have when you try to solve a problem is to try and write the corresponding units of each of these parameters, so that you do not make a mistake while calculating by putting in the wrong units. For example, the area in this problem was given in centimeter square; it helps if you specify the units right next to the property. So that you know for sure that you have got the units.

So, you have pressure 2000 kilopascal area 25 into 10 raise to minus 4 meter square times delta x which is 0.25 meters. If you were to calculate this you would get one 1.25 kilo joules so, in this particular process the work done to move the piston was 1.25 kilo joules. This is a very simple problem where you were to require calculating the work done during a process.

If you have been specified the pressure, the area of the piston and the distance by which the piston moves, it is a simple problem where you can calculate work done by equating work is equal to pressure times area times delta x. In this case the work done to move the piston was calculated as 1.25 kilo joules.

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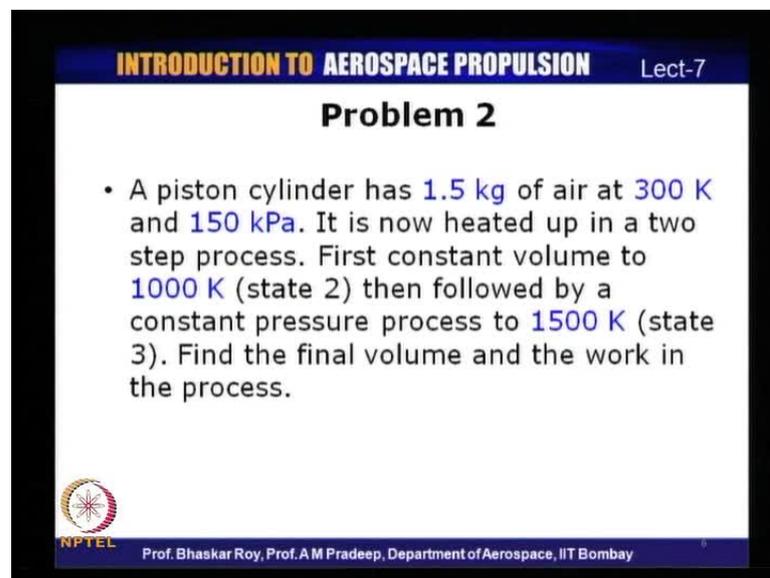
Now, let us look at the next problem that we have for discussion today, this again is a piston cylinder example. A piston cylinder has 1.5 kilograms of air at 300 kelvin and 150 kilopascals. It is now heated up in a two step process. First a constant volume to 1000 kelvin, which takes the process to state 2 as is mentioned in the brackets then followed by a constant pressure process 2500 kelvin state 3. Find the final volume and the work in the process.

In this case, we have a problem where we have a system which carries out a process in two steps; the first step we have a constant volume process which takes it from state 1 to state 2, where state 2 is 1000 kelvin, at state 1 the system was at 300 kelvin and 150 kilopascals after the first process at the end of which it reaches state 2.

The second process is a constant pressure process where in the system is taken to a temperature of 1500 kelvin, the pressure remaining the same. So, we are required to calculate the volume at the final state that is state 3 and the work done during this process.

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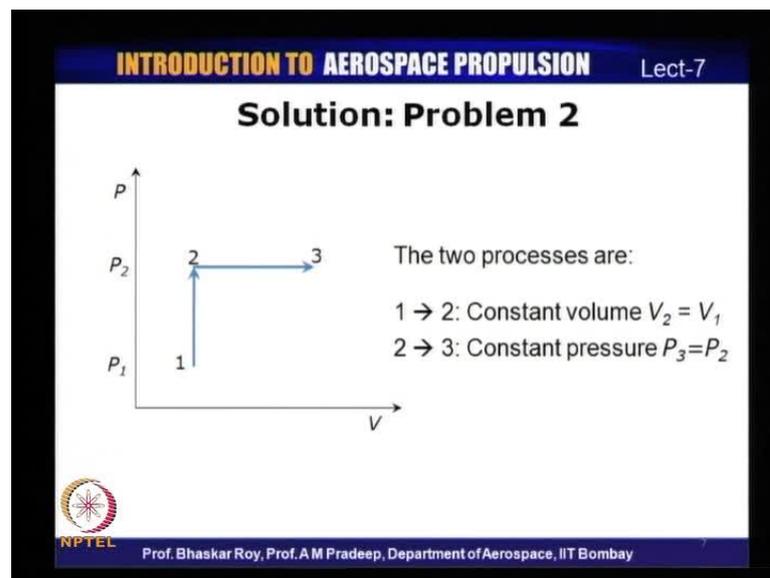
As I mentioned, one of the first things that we can do is to draw the process diagram that is the process path. In this example, we have PV diagram and the state was initially at the process or the system was initially at state 1 and state 1 to 2 is a constant volume process. Therefore, V 2 is equal to V 1 as you can see the pressures are not the same P 1 is not equal to P 2. The second process is a constant pressure process from state 2 to state 3. Since, pressure is constant we have P 3 is equal to P 2 and what we are required to find is the volume at 3 that is V 3 and the work done during this process.

So, if you recall from the previous lecture where we had discussed about work done during constant pressure processes and constant volume processes. If you recall that you can immediately see that work done during the first process will be 0, because it is a constant volume process and work done during the second process is the pressure times the difference in the volumes. The total net work done of this obviously would be equal to work done during the first process plus the work done during the second process.

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So, in this case besides the quasi-static assumption we shall also use the ideal gas approximation for air. In the medium of this particular process is air and we shall consider air to be in ideal, so that is the second assumption besides the quasi-static assumption.

At state 1 we know certain properties of the system some of them are temperature is known it is specified in the problem it is 300 kelvin, pressure is given as 150 kilopascals, then we know the mass of the system it is 1.5 kilograms and we also know the gas constant for air. So, gas constant for air is universal gas constant divided by the average molecular weight of air that will be equal to 8.314 kilo joules per kilogram kelvin divided by the molecular weight of air which is 29.

If we were to calculate that you would get 0.287 kilo joules for the gas constant for air. Since, some of these properties that is temperature pressure mass and gas constant are known and we have approximated air to be an ideal gas V 1 that is volume at state 1 will be equal to mRT 1 divided by P 1 this is from the state equation that is PV is equal to mRT therefore, V 1 is mRT 1 divided by P 1. So, all these parameters are known 1.5 is kilograms is mass R is the gas constant T 1 is temperature and P 1 is the pressure.

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If we were to substitute these values the volume at state 1 that is V 1 can be calculated as 0.861 meter cube, now we look at state 2. We have calculated all the parameters at state 1 we know the volume, we know the temperature; pressure all that is known at state 1, now proceed to state 2. We know that the first process which takes the system from state 1 to state 2 is a constant volume process. Therefore, V 2 is equal to V 1.

If you again use the ideal gas state equation, then we get P 2 pressure at state 2 is equal to P 1 times T 2 by T 1 this is again from the state equation, because V 2 is equal to V 1. So, you get since P 1 is known as 150 kilopascals, temperature T 2 is given as 1000 kelvin, T 1 is 300 kelvin therefore, P 2 is equal to 500 kilopascals.

Now that you know pressure at state 2 we can proceed to state 3, because the second process is a constant pressure process. Therefore, P 2 is equal to P 3 so that is because the second process is a constant pressure process. From this we can all again use this state equation to calculate the volume at the end of the second process that is at state 3 so, V 3 is equal to V 2 times T 3 by T 2 again from the state equation.

We know volume at state 2, V 2 is known, T 3 is known and T 2 is also known. T 3 is specified as 1500 kelvin, T 2 is already known as 1000 kelvin, and V 2 we had already calculated from in the first step of this problem that is 0.861 meter cube. If you again substitute these values you get V 3 is equal to 1.2915 meter cube, so this is answered to

one part of the question, which was to find the final volume at state 3 or at the end of the process. So, the final volume that is V 3 is 1.2915 meter cube.

Now, the second part of the question is to find out the work done during the process. I already mentioned at the beginning that in this particular example, we have two processes for which we had calculated the work done in the previous lecture; one is a constant volume process and the second process is a constant pressure process. Net volume, net work done during this process will be work done during the first process plus work done during the second process.

We already know that work done during the constant volume process will be 0, because there is no change in volume, there is no displacement. Therefore, PdV work will be equal to 0 therefore, the net work during done during this process is equal to the work done during the second process which is a constant pressure process.

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Let us calculate now the work done during the second process. So, work done during this whole process that is denoted by W subscript 1 3 which denotes work done between states 1 and 3 is equal to W 1 2 plus W 2 3. Where the first one is work done during the first process between states 1 and 2 plus W 2 3 is work done during the second process between states 2 and 3. So, work done during process 1 2 as we know is 0 because it is a constant volume process.

Therefore, work done during 1 3 is equal to work done during the second process that is 2 3 is equal to P 3 times V 3 minus V 2, which is also is equal to P 2 times V 3 minus V 2 because P 3 is equal to P 2 it is a constant pressure process. Now, we know the pressure P 3 or P 2 which is 500 kilopascals, we also know V 3 and V 2 from the previous calculation. So, we just need to substitutes these values, so 500 multiplied by o1.2915 minus 0.861, this comes out to be 215.3 kilo joules.

So, the total work done during this process is 215.3 kilo joules, the net work done during the process is equal to the second process that is work done during the second process and that was equal to pressure times the difference in the volumes. Now, I also mentioned in the previous lecture when we were discussing about work that work net work done is essentially equal to area under the curve on a PV diagram.

If we now go back to the PV diagram, which we had plotted for this particular problem and then find they are under the curve, it will come out to be 215.3 kilo joules because it is basically equal to the pressure times the difference in the specific volumes difference in the volumes. So, it is P 3 or P 2 multiplied by V 3 minus V 2 so that is essentially the area under the constant pressure process that was straight line on the PV diagram horizontal line on the PV diagram.

That is essentially the area under the curve and that is equal to the net work done during this particular process. So that was problem number 2. Now, let us move on to another problem which is yet again a piston cylinder example.
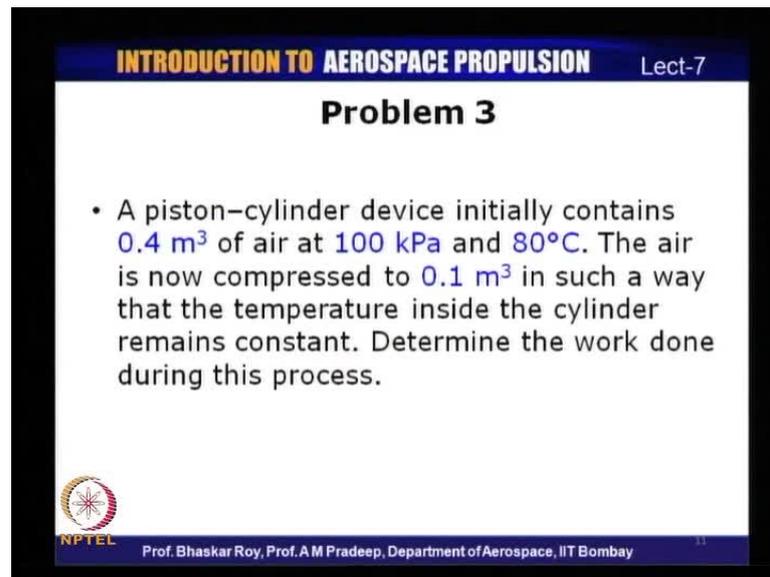
I think I mentioned in the first lecture like that was lecture 4 that we shall be discussing about piston cylinder examples very often during the thermodynamics course. Now, this example is about a piston cylinder device initially contains 0.4 meter cube of air at 100 kilopascals and 80 degree Celsius. The air is now compressed to 0.1 meter cube in such a way that temperature inside the cylinder remains constant. Determine the work done during this process.

So, in this piston cylinder example we have been specified some initial volume, initial temperature and pressure. Then the piston is compressed or the air with in the cylinder is compressed by the piston to its final volume which is 0.1 meter cube, but this compression is done in such a way that the temperature remains constant. So, it is a constant temperature that is an isothermal process and we are required to find the work done during this process.

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So, the first step to solving this problem is to draw the process path as well as the problem definition itself. Here we have a piston cylinder arrangement where in the initial volume is 0.4 meter cube, pressure is 100 kilopascals, temperature is 80 degree celsius which is in this problem a constant.

Now, this piston as it moves down compresses the air within the cylinder to a final volume which is 0.1 meter cube keeping the temperature same. So, on a PV diagram this process would look like this, the process is initially at state 1 which corresponds to a volume of 0.4 meter cube and a pressure of 100 kilopascals.

Now, following a constant temperature that is an isothermal process, the system is compressed which means that the volume has to decrease and the pressure has to increase, so this is an isothermal compression process. Final state of the system is at state 2 where the volume is 0.1 meter cube.

Again the assumptions as I mentioned need to be specifying the process in this case it is a compression process, it is again in quasi-equilibrium. At the specified conditions we will consider air to be an ideal gas and assuming that the temperatures and pressures are not close to the critical values for defining or for assume making the assumptions of an ideal gas for air. So, this is a problem which requires us to solve for an isothermal process. I think I discussed in the previous lecture also that this would be a process where the product PV will be equal to constant.

So, PV equal to constant if we were to assume the ideal gas assumption corresponds to an isothermal process. We will use PV equal to constant for this particular solving this particular problem. Now, let us look at how we can solve this problem.

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For an ideal gas at a constant temperature here T subscript 0, T naught is remaining constant here, it is an isothermal process. So, from the state equation product PV is equal to mRT naught or T 0 which is equal to C or a constant which means that P is equal to C by V where C is a constant.

So, to calculate work is integral 1 to 2 PdV and which is integral replacing P by C by V, we get C by V times dV or C times log natural log V 2 by V 1 which is P 1 V 1 times log V 2 by V 1. If you had done this exercise I think I mentioned towards end of the previous lecture that PV is equal to constant work done during PV equal to constant process need to be carried out by as an exercise for you.

If you had done that exercise, you would now recall that work done you would have calculated as equal to P 1 V 1 log V 2 by V 1. So, in this example we can replace P 1 V 1 by P 2 V 2 because it is a isothermal process, P 1 V 1 is equal to P 2 V 2 or since PV is equal to mRT 0 we can replace it by mRT 0. Also V 2 by V 1 can be replaced by P 1 by P 2, because P 1 V 1 is equal to P 2 V 2.

So, if we were to replace make these changes in the work equation which is P 1 V 1 log V 2 by V 1, we will get we will basically have to replace for P 1 V 1 in terms of mRT because mass is known gas constant for air is known and temperature is already known. So, this equation would reduce to work done is equal to mRT 0 log P 1 by P 2.

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If we were to replace those values, we get the pressure times the volume. This is P 1 times V 1 that is the initial volume times the pressure and the ratio of the either the volumes or the pressures in this case volumes are known so log of V 2 by V 1. So, if you were to calculate this what you will see is that you get minus 55.5 kilo joules so you may wonder what this minus means well this negative sign essentially indicates that work is done on the system so there is work input into the system and the during the last lecture we had discussed about sign conventions for heat and work.

Therefore, work done on a system is taken as negative and in this case it is a compression process during a compression process always work is done on the system and therefore, the work done in this particular example comes out to be negative minus 55.5 kilo joules and the negative sign basically indicates that work is done on the system and for any compression process work is always done on the system.

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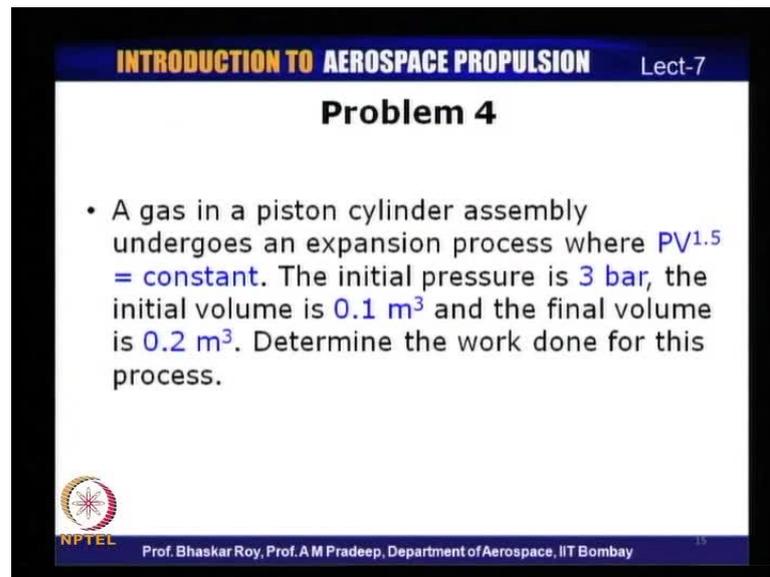
Now, let us look at our the 4th problem that we have for today's exercise in this example we have a gas in a piston cylinder assembly which undergoes an expansion process where PV raise to 1.5 is equal to constant the initial pressure is 3 bar and the initial volume is 0.1 meter cube and the final volume is 0.2 meter cube determine the work done for this process. So this is an example problem which we have where the process is taking a PV raise to n equal to constant where n is 1.5 in this example.

And we have the initial pressure and volume which have been specified we also have the final volume we need to find the work done during this process so PV raise to n if you recall from our discussion in the last lecture work done can be calculated by integrating PdV and because PV raise to n is constant you can replace for P is equal to C by V raise to n.
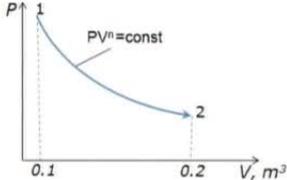
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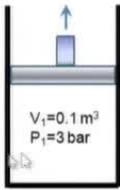
Therefore, determine the expression for calculating work done during this particular process now let us now look at the process in terms of the process diagram and the problem definition now we have P 1 and V 1 specified P 1 is 3 bar V 1 is 0.1 meter cube.

Now this is an expansion process and so the system was initially at state 1 which is 0.1 meter cube and by expansion following a process PV raise to 1.5 equal to constant it reaches state 2 which is at 0.2 meter cube well some of the assumptions besides the quasi-equilibrium assumption are that gas is obviously in a closed system it is a close system and the expansion mode is a polytrophic process which is basically qualified by PV raise to n is a constant and that PdV work is the only work mode well this is some of this assumption are valid also for the previous three examples which we solved for again PdV work processes.

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So to solve this particular problem we will need to find the expression for work done during PV raise to n is equal to constant process I would suggest that instead of memorizing the expressions for work done in terms of pressures and 1 minus n and so on it is always advisable that you derive the expression from the first principle it is just a two step process so if you know how to integrate PdV and for different processes which are PV raise to n equal to constant and so on.

You would not have to memorize any of these expression for work done so work done during this process is W 1 2 integral between 1 to 2 PdV and so we replace P in terms of C by V raise to n and we get C times V 2 raise to minus n plus 1 minus V 1 raise to minus n plus 1 divided by minus n plus 1 which is basically P 2 V 2 minus P 1 V 1 by 1 minus n. Now the pressure at state 2 we can find using the expression P 2 V 2 raise to n is equal to P 1 V 1 raise to n because we know. That this is a PV raise to n equal to constant process so we can find pressure at state 2 from this equation so P 2 in this case equal to P 1 times V 1 by V 2 raise to n.

So, this if we were substitute the values you get we have converted here bar into pascal's so 3 bar is 3 into 10 raise to 5 pascal's so 3 into 10 raise to 5 pascal's times V 1 which is 0.1 divided by V 2 which is 0.2 raise to 1.5 so from this we get pressure at state 2 P 2 which is 1.06 into 10 raise to 5 pascal's.

Therefore we substitute for P 2 V 2 and P 1 V 1 and n in the work equation so work as we now know is P 2 V 2 minus P 1 V 1 divided by 1 minus n therefore, W is 1.06 into 10 raise to 5 pascal's times 0.2 meter cube minus 3 into 10 raise to 5 pascal's times 0.1 meter cube divided by 1 minus 1.5 so if we were to calculate this you will get work done as plus 17.6 kilo joules so this is the work done during this particular expansion process which is following a PV raise to n that is where n is equal to 1.5 equal to constant process.

So work done during this process can be calculated because we know that it is PV raise to n that is a polytrophic process.

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Now we shall make some interesting observations from this particular problem so work done as we have calculated from this process is coming out to be plus 17.6 kilo joules now again I leave it as an exercise for you to solve if n was equal to 1 instead of 1.5 if you had n equal to 1 which makes it an isothermal process that is PV equal to constant what is a net work done.

Well, if you if you actually calculate that you will get the net work done during that process as plus 20.70 kilo joules.

Now, if you substitute n equal to 0 then you can calculate the net work done which will be basically a constant pressure process work done you will be able to calculate as 30

kilo joules. So you can notice that as you change the value of n the work done during the same system initial state and final state remaining the same just that the process by which it is carried out is different.

You have different values of n you get different values of work done what you will immediately notice is that as you keep decreasing the value of n you get higher and higher work done so please recall from our previous lecture when we were discussing about the polytrophic process I had shown a PV diagram for different polytrophic processes for different values of n ranging from 3 2 1 and so on up to n equal to 0 so I will bring that slide here once again.
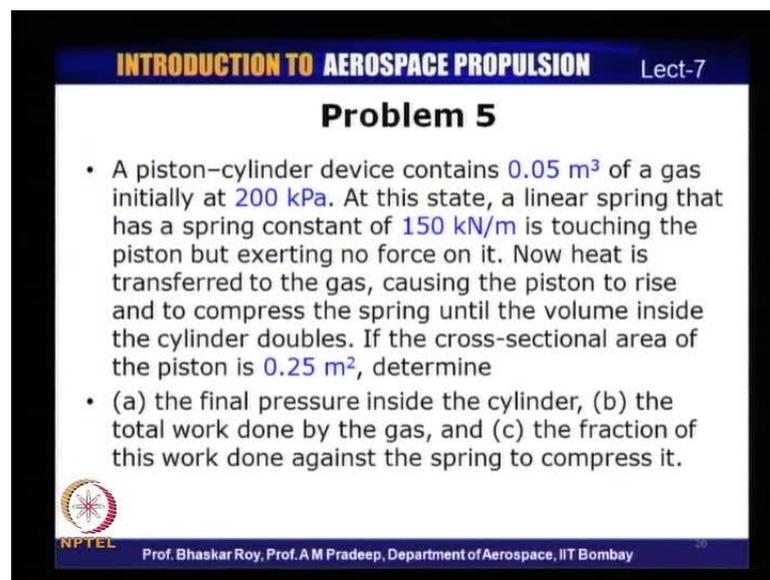
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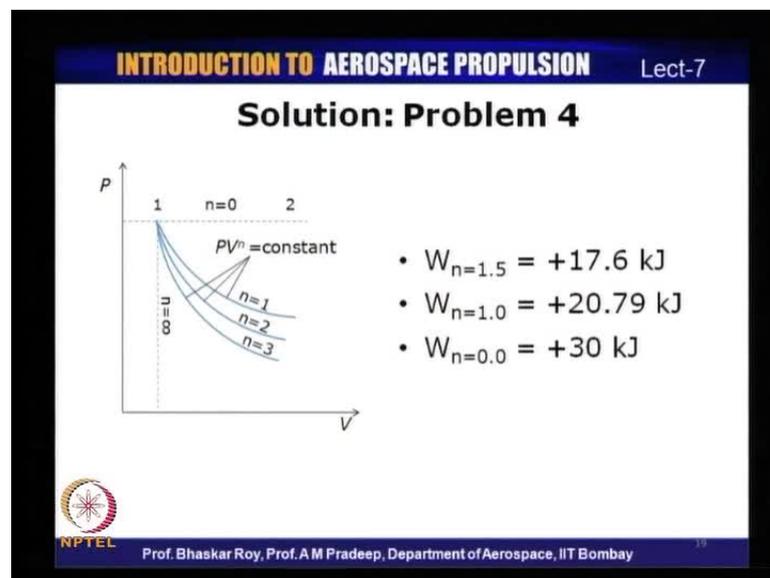
That we have here the PV diagram for the for different polytrophic processes so in this particular PV diagram you can see as you keep changing n where as you keep decreasing n n equal to 3 n equal to 2 1 and so on you can see that this area under this curve between 1 and 2 will increase so were for n equal to 1.5 that is W n equal to 1.5 17.6.

You would probably somewhere come somewhere here this would be the process somewhere in between n equal to 2 and 1 point and 1 n equal to 1 is here which is above n equal to 1.5 and therefore, you get a higher work output n equal to 0 is a constant pressure process you get the highest work output.

During the constant pressure process, so this is just to illustrate that as you keep increasing while decreasing the value of n the work done by the process will keep increasing so work done is obviously also a function of the polytrophic power that is equal to n so as you keep reducing n you get higher and higher work output the limit of that is n equal to 0 which is a constant pressure process were in you get the maximum work done during the particular process.

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Now let us look at the 5th problem we have for during for discussion during today's lecture this particular problem is about a piston cylinder device which contains 0.05 meter cube of gas initially at 200 kilopascals at this state a linear spring that has a spring constant of 150 kilonewtons per meter is touching the piston but, exerting no force on it now heat is transferred to the gas causing the piston to rise and to compress the spring until the volume inside the cylinder doubles if the cross sectional area of the piston is 0.25 meter square determine part a.

The final pressure inside the cylinder the total work done by the gas and part c the fraction of this work done against the spring to compress it so here we have what looks like a rather complicated problem but, as we understand the problem little better it will come that it is not as complicated as it looks like that though the question looks a little long.

Well here, we have a piston cylinder device were in the piston cylinder contains initially certain amount of gas which is at a pressure and a volume and then there is a spring which has a certain spring constant of 150 kilonewtons per meter it is just touching the piston but, a spring is not exerting any force on it now as you heat the gas in the cylinder the piston will start rising right because the specific volume of the gas will increase and as the gas rises that will compress the spring against the spring constant now so there is a certain work done by the gas against the atmospheric pressure and all that and also there is certain amount of work which the gas has to do against the spring to compress it.

So what we are required to find is the pressure inside the cylinder then we also need to find the work done by the gas that is total work done and also the fraction of this total work done which is the work done for compressing the spring.

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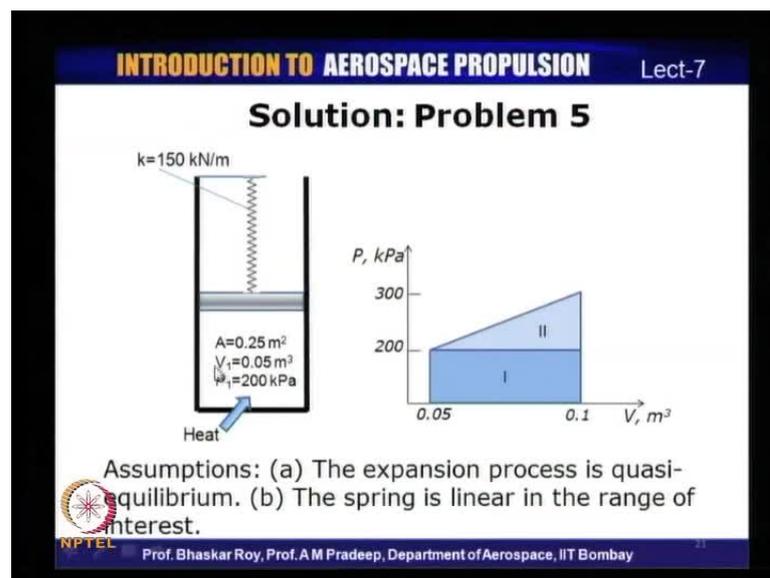
So let us now look at the process diagrams as well as the problem definition in terms of an illustration so here we have the piston cylinder arrangement now the piston is enclosing a certain gas at 0.05 meter cube volume it has a pressure of 200 kilopascals and area of the piston is also given as 0.25 meter square.

So initially there is a spring here which does not exert any force on the piston spring constant is given to as now as you start transferring heat to the system the piston moves up and it moves till the volume inside the cylinder doubles that is the final volume would be equal to 2 into 0.05 that is 0.1 meter cube so which means that the piston does work

against the cylinder against the spring as well as work done during the expansion process assumptions obviously are that the expansion process is quasi-equilibrium and the spring is linear in the range of interest.

So the process path here is that initially we have the system at state 1 which is at V 1 is equal to 0.05 meter cube and P 1 is equal to 200 kilopascal and then there is an expansion process where in the volume increases to it is double the value initially and to a pressure which is 300 kilopascals and so this is how the process is what are indicated by 1 and 2 will be clear little later I shall explain what this means that is 1 and 2 in this diagram.

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So, the enclosed volume at the final state is twice the initial volume therefore, V 2 is equal to 2V 1 and therefore, it is 2 into 0.05 which is 0.1 meter cube.

Now, we need to calculate the displacement of the piston so how do we calculate the displacement of the piston we know the change in volume from the initial in the final states so delta V is known area of the piston is known so x which correspondence to the displacement of the piston will be equal to delta V by A which is 0.1 minus 0.05 by 0.25 so the net displacement we get as 0.2 meters.

Now, how do we find the force applied by this linear spring at the final state while force is the spring constant times the displacement so we know the displacement which we

have just calculated as equal to delta V by A and we also know the spring constant so spring constant k is given as 150 kilonewtons per meter.

So k times x will be the force which is been applied on the spring at the final state so k times x will be 150 kilonewtons per meter into 0.25 so that is 30 kilonewtons. So, this is the force exerted or force applied by the spring at the final state now we now need to find what is the final pressure at the state 2 that is, at the end of the process what is the final pressure.

Now, pressure as we know is force per unit area so we have just now calculated the force that the spring exerts at the final state so that force divided by the area will give you the pressure well that is the pressure which is basically the additional pressure because of the presence of the spring itself.

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Therefore, if you calculate P, P would be equal to the force that is F divided by A and so it is 30 kilonewtons divided by 0.25 that is 120 kilopascals; without the spring the pressure of the gas would remain a constant because you are allowing the gas to expand and therefore, the volume is changing but, the pressure will remain a constant so if the spring was not there the gas would remain at 200 kilopascal which is it is initial pressure while the piston keeps rising but, because of the presence of the spring the pressure rises linearly because it is a linear spring and it rises from 200 kilopascals to 200 plus 120 which is 120 is corresponding to the additional pressure which is applied by the spring

on the gas at this state. Therefore the final pressure will be equal to 200 pascal which is the initial pressure plus 120 which is the pressure due to the spring which is 320 kilopascals.

Therefore, the final pressure in the cylinder at the end of the process is 320 kilopascals so this is the answer to the first part of the question which was to find the final pressure at the end of the process at that is at state 2.

So, here the pressure corresponds to two different components one is because of the initial pressure itself which would have remained constant if they were no spring which was 200 kilopascals and because of the presence of the spring as the piston expands the spring exerts a pressure on the piston which we calculated as 120 kilopascals.

Therefore, the total pressure will be 200 plus 120 that is 320 kilopascals let us know calculate the work done required or work done during this process and as well as the work done against the spring that is the third part of the question.

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INTRODUCTION TO AEROSPACE PROPULSION — Lect-7

Solution: problem 5

- From the P-V diagram, it is clear that the work done during the process is the area under the process (a trapezoid in this case).

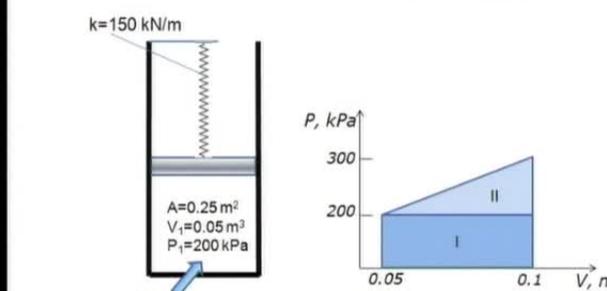$$W = \text{area} = \frac{(200+320)kPa}{2}(0.1-0.05)m^3 = 13\,kJ$$

- The total work done by the gas is 13 kJ

So, second part of the question is work done so how do we find the work done during this process if you go back to our first slide were we discussed about the process diagram and also if you recall our earlier discussions work done during this process will be work area under this curve.

You can clearly see that this is basically a trapezoid so if you calculate area under this curve because and you know the end points of the trapezoid so it is very easy to calculate area under this curve which is basically the work done during the process.

So, work done will be equal to area and how do you calculate the area because it is basically area of a trapezoid in this case the end points are pressures and volumes so it will basically be equal to 200 which is initial pressure plus 320 final pressure divided by 2 multiplied by the difference in the volumes 0.1 minus 0.05 therefore, if we were to calculate this the total work done during this process will be equal to 13 kilo joules well this is not the only way you can solve it you can all solve it in the same way as we solved the previous problems; that is, if you were to consider the process which has a pressure which is varying linearly you can calculate the work done during that particular process or you could also calculate separately work done if it was a constant pressure process and work done if due to the spring itself and then add up the two so there are different ways of calculating this easiest of them being just you calculate the area under the curve which corresponds to the particular process.

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Therefore, the total work that we have calculated comes out to be 13 kilo joules now so that is answer to the part 2 of the question part 3 of the question is to find the fraction of the work done against the spring to compress it.

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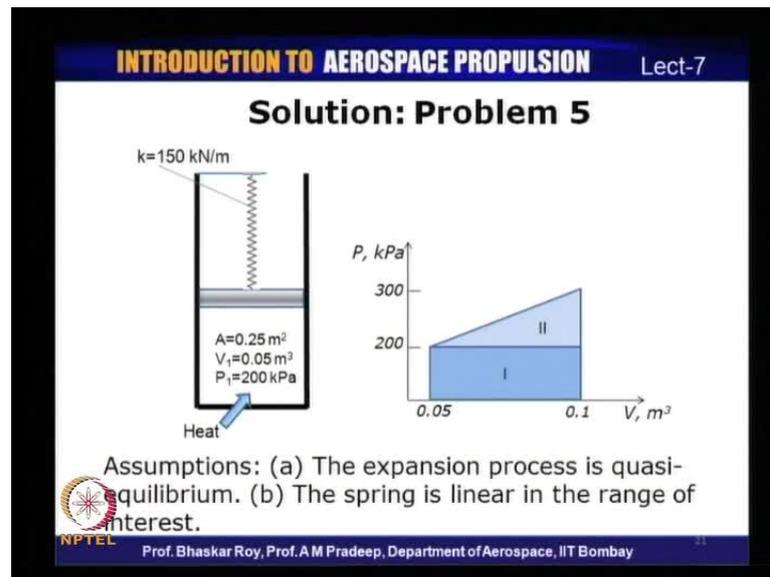
Now, so that, basically represents the region one here which corresponds to the amount of work that is required to work done against the spring. So, here in in the the p v diagram which I had shown, region one corresponds to the work done against the piston and atmosphere that is the work done if the work no spring and region two corresponds to the area, because of the presence of the spring itself. So, the work done by the spring is basically the work done under region two which is half of 320 minus 200 which is difference of pressures multiplied by the difference in the volumes that is 0.05 meter cube. So, if we calculate this, you will get 3 kilo joules

So, fraction of this work done against the spring, to compress it is 3 kilo joules. Well, you can also calculate the work done by the spring, using the spring constant which is given in this case already as 150. So, work done by a spring if you recall basic mechanics is half k times x 2 square minus x 1 square where x 1 and x 2 are initial and final positions of the spring. So, x 2 in this case is 0.2 meters x 1 is 0. So, if you calculate work during using this formula, you will also get the same answer that is 3 kilo joules. So, this solves our the fifth problem that we have solved today, which was of piston cylinder assembly and compressing a certain spring. So, the process consist of two different parts, one was to calculate the work done by the piston if there were no spring and also to calculate the work done because of the presence of the spring. And so we can actually add up the work done during these two separately to calculate the total work done during this particular process. And so we shall now discuss a few exercise problems

which we have, which I shall discuss with you and which you can solve later on at your leisure based on what we have discussed during the previous lectures as well as based on some of the problems which we have solved during today's lecture.

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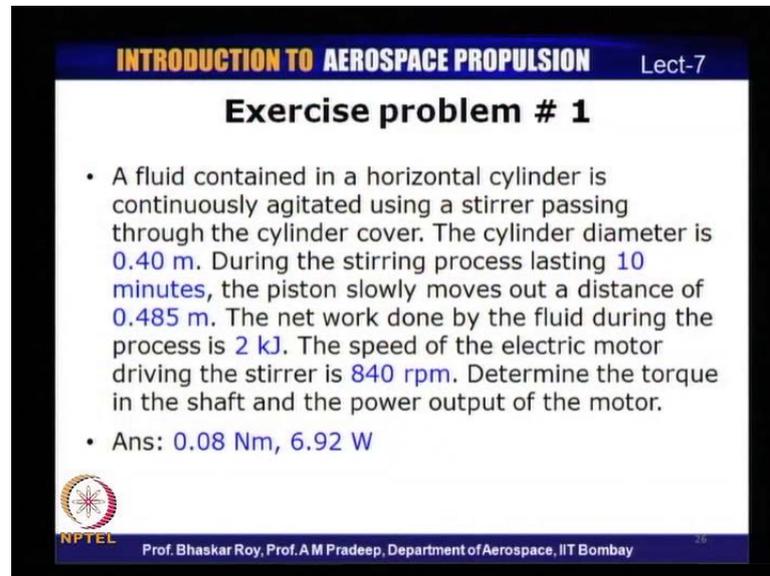
So, the first exercise problem which I have for you is on a fluid contained in a horizontal cylinder, is continuously agitated using a stirrer passing through the stirrer cylinder cover. The cylinder diameter is 0.4 meters during the stirring process which last for ten minutes the piston slowly moves out at distance of 0.485 meters. The net work done by the fluid during the process is 2 kilo joules. The speed of the electric motor driving the stirrer is 840 r p m. Determine the torque in the shaft and the work out well power output of the motor.

So this is an example of an exercise problem for you, were in, we have a a piston cylinder arrangement and through the head of the cylinder or the cylinder cover we have a stirrer which is rotated by an electric motor. And so, we are basically adding work into the system using a stirring process. And so, as you add energy to the system the piston moves because you are adding energy into the cylinder and therefore the cylinder, the piston expands and piston moves to a different state.

And so what we have been given is the net work done by the system during this process which is specified as 2 kilo joules and also we have been specified how much time this stirring process is done and ==during which== during this time, how much the piston has

moved. And so, you are required to find what is the torque in the shaft and the power output of the motor because work done is specified and the distances which have been moved have been given so, you can find the torque in the shaft and the power output.

So, I have given the answers here you can verify these answers by calculating it. You should be getting a torque in the shaft of 0.08 newton meters and power output of the motor as 6.92 watts.

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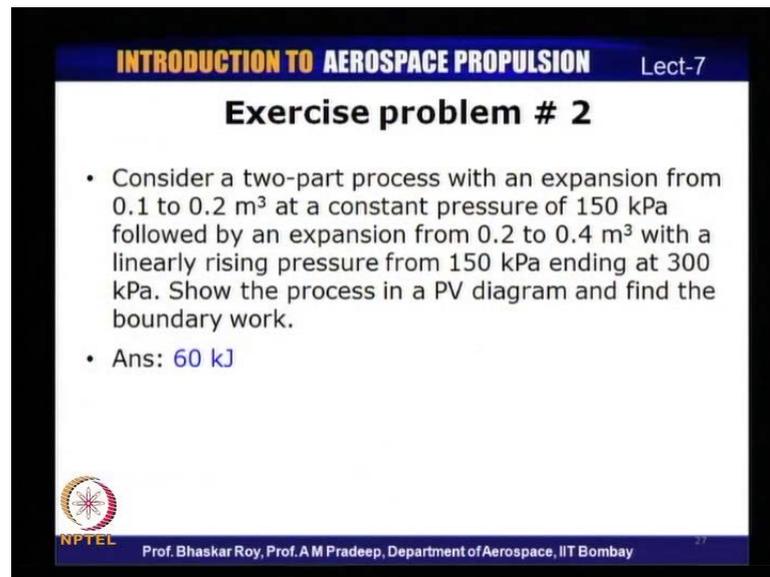
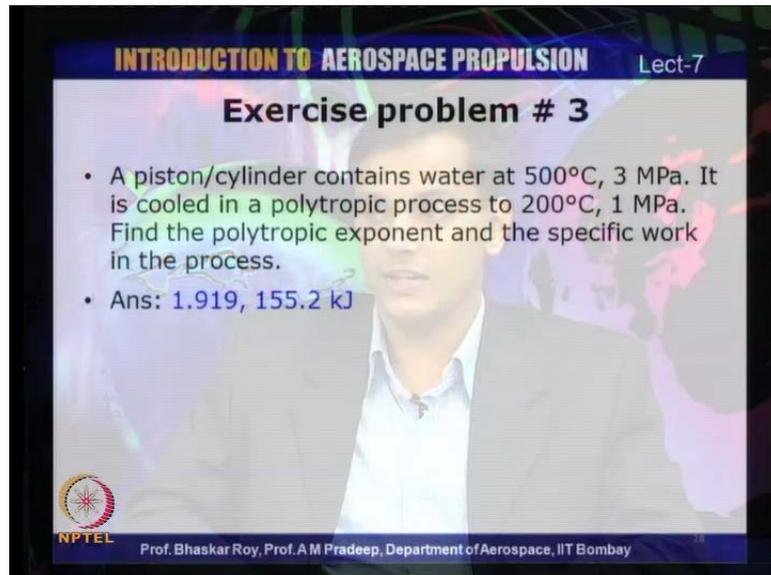INTRODUCTION TO AEROSPACE PROPULSION   Lect-7

Exercise problem # 2

- Consider a two-part process with an expansion from 0.1 to 0.2 m³ at a constant pressure of 150 kPa followed by an expansion from 0.2 to 0.4 m³ with a linearly rising pressure from 150 kPa ending at 300 kPa. Show the process in a PV diagram and find the boundary work.
- Ans: 60 kJ

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Well, the second exercise problem we have today is on again an expansion process p v diagram process and in this process it is a two part expansion process. One is a constant pressure process and consider a two part process with an expansion from 0.1 to 0.2 meter cube at constant pressure of 150 kilopascal's followed by an expansion from 0.2 to0.4 meter cube with a linearly raising pressure from 150 kilopascal ending at 300 kilopascal. So, show this process in a p v diagram and find the boundary work.

So, here you have a problem where the process is carried out in two parts first, is a constant pressure process where, the volume changes from0.1 to 0.2 meter cube and then there is an expansion process which takes it from 0.2 to 0.4 and this expansion process consists of a linear variation of pressure from 150 kilopascal's to 300 kilopascal's. So, you have to find the process. Well, work done during this process as well as obviously the to show the process on a p v diagram. So, the answer for this particular question would be the work done should be equal to 60 kilo joules.

And the third problem we have as an exercise problem for you is a piston cylinder arrangement which contains water at 500 degree celsius 300 mega pascal's. It is cooled in a polytrophic process so we have process which cools water from 500 degree celsius 3 mega pascal's to 200 degree celsius 1 mega pascal. We need to find the polytrophic exponent and the specific work in this process that is you need to find p v raise to n where n you have to have to find as well as the work done during this process. So, in this case you will find that the polytrophic exponent will come out to be 1.919 and work done during this process is 155.2 kilo joules.
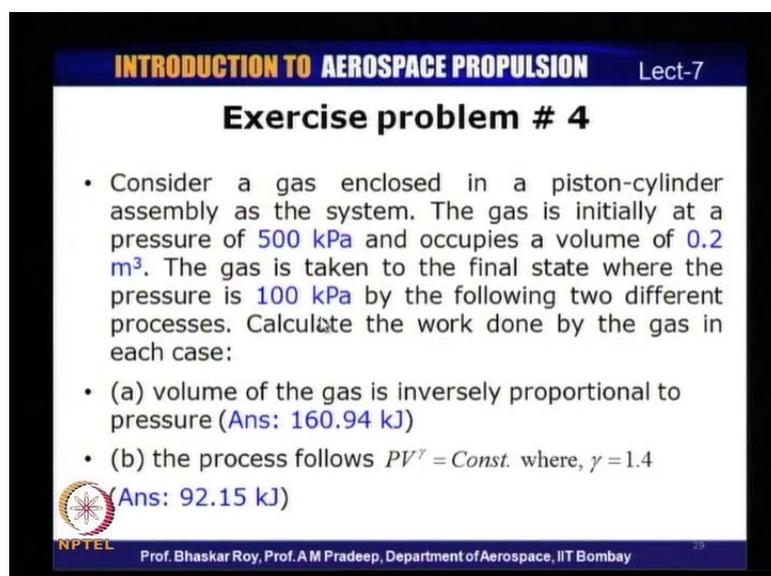
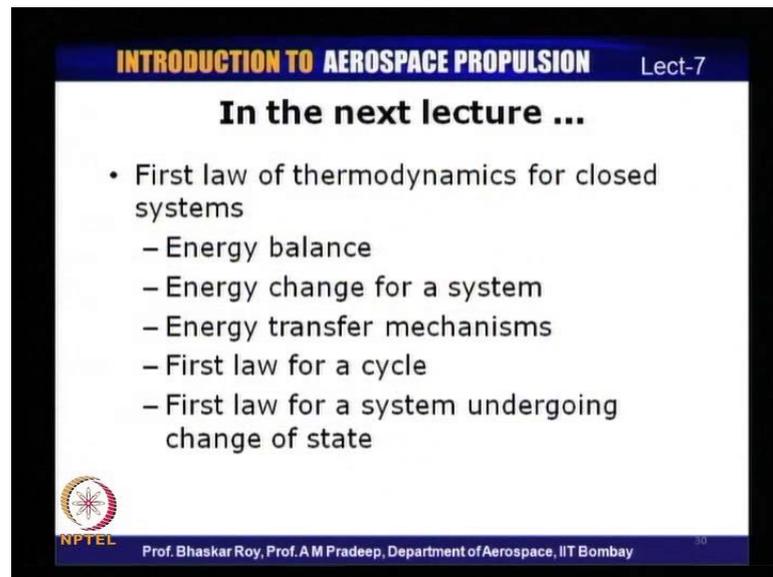And the last <mark>problem</mark> exercise problem for you is, consider a gas enclosed in a piston cylinder assembly as a system the gas is initially at a pressure of 500 kilopascal and occupies a volume of 0.2 meter cubes. The gas is taken to the final state where the pressure is 100 kilopascal by the following two different processes. Calculate the work done by the gas in each case. So the two processes are one is, volume of a gas is inversely proportional to pressure that is v is inversely proportional to p or in other words p v is a constant it is a isothermal process.

Second process is, process follows p v raise to gamma is a constant where gamma is equal to 1.4. So for this polytrophic processes you need to find the work done, and so for the first part is 160.94 kilo joules. For the second part that is p v raise to gamma question is 92.15 kilo joules. So, that brings us to the end of this lecture, where we disused about problem solving for work done during different processes and we have discussed some examples where we calculated work done during different types of processes. So, what we are going to do is in the next lecture is to discuss about the first law of thermo dynamics as applied to closed system.

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So, during this discussion that is during the next lecture we shall discuss about first law applied to closed systems, and we shall discuss about energy balance, we shall discuss about energy change for a system, we shall discuss about energy transfer mechanisms, during for closed systems. And we shall apply first law for a cycle as well as first law for a system undergoing a change of state . So, these are the things that we shall be discussing during the next lecture that would be lecture number eight.