Introduction to Aerospace Propulsion

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Lecture No. # 32
Ideal cycles for Jet engines

Hello and welcome to lecture number 32 of this lecture series on introduction to aerospace propulsion. So, in today's lecture, what we are going to do is to understand and analyze the ideal cycle, which is ideal cycle, thermodynamic cycle behind or ideal thermodynamic cycle, based on which, all jet engines operate.

If you recall a few lectures earlier on, we had been discussing about some of the ideal cycles and one such cycle which I had mentioned at that time which is used for gas turbine engines is known as the Brayton cycle. So, we have already done some analysis of Brayton cycle and some variants of Brayton cycle like Brayton cycle with regeneration, reheating and so on.

So, today, we will take up Brayton cycle as applied for a gas turbine engine, which is what is used in most of the modern day aircraft. You already had some exposure to piston engines and the cycles behind piston engines in the last few lectures. In today's lecture, let us take a look at some of the thermodynamic principles in cycles behind the gas turbine engine that primarily the Brayton cycle.

So, we will see how we can analyze the Brayton cycle and how we can carry out what is known as an ideal cycle analysis of jet engines, and so, in today's lecture as well as the next lecture, we shall be carrying out ideal cycle analysis of jet engine cycles and their variants. So, there are different types of jet engines as you are perhaps aware that there are types of engines like turbojet and turbofan, turboprop, turbo shafts, ramjets and so on.

So, each of them though operate on the basic thermodynamic cycle that is the Brayton cycle, but they have some small differences in how the Brayton cycle is executed in each of these different engines. So, that we shall analyze in this lecture as well as the next. So, in today's section, we will take up the simple turbojet engine cycle and with afterburning, and so, let us take a look at what we shall be discussing in today's lecture.

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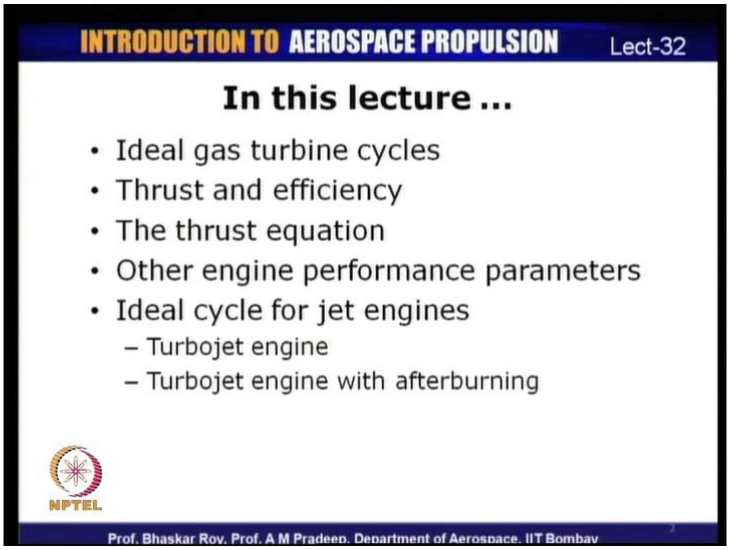

In today's lecture, we are going to be discussing about ideal gas turbine cycles. We shall derive equations for thrust and efficiency and other engine performance parameters like fuel consumption and so on. Then, we will spend considerable time on analyzing the ideal cycle for jet engines, the basic jet engine, that is the turbo jet engine. We shall then extend the analysis for a turbojet engine with after burning.So, these are some of the topics that we shall be discussing in today's lecture.

That is primarily to do with understanding the basic thermodynamic cycles and thermodynamic principles. So, I mentioned that gas turbine engines operate on the Brayton cycle and we are already are familiar with the different processes in Brayton cycle.

So, Brayton's cycle begins with the first process which is an isentropic compression process, and then, there is a heat addition which takes place at constant pressure. The third process is an isentropic expansion process and the last process is constant pressure heat rejection process.

So, ideally a Brayton cycle should be executed in this format which is a closed cycle format, but as you know, most of the jet engines, in fact, all the jet engines operate in an open cycle mode, that is, they do not have the same working fluid bridge continues to operate within the cycle, but as we have seen with air standard assumptions, we can assume that the exhaust which leaves the engine is equivalent or can be modeled as a heat reduction process, and therefore, it resembles an ideal Brayton cycle in that sense.

Now, when we say an ideal cycle, we primarily mean that there are no irreversibility's that are taking place in the system, that is, where it is a compression process or the heat addition processes, expansion process or heat rejection, all these processes are ideal, and therefore, they do not have any irreversibility's. Basically the first process that is the compression process and the third process which is the expansion process, they are isentropic in nature, that is, the entropy remains a constant during that process, and in the second and the fourth process, that is heat addition and heat rejection which take place at constant pressure. We are going to assume that there are no pressure losses which take place in the systems.

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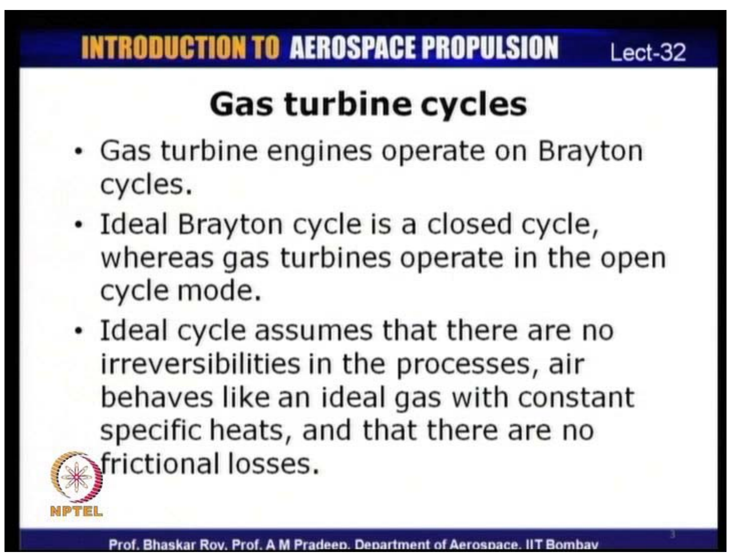

So, that is why we term these cycles as ideal cycle and it makes analysis a lot simpler, because we do not have to worry about efficiencies which are there in different components and how much is the pressure loss which takes place in the heat addition system and so on.

So, analysis of systems becomes simpler and the same time you get some idea of the performance of a cycle if it were to operate in an ideal mode, and therefore, as a starting point for designers, ideal cycle really helps in understanding of the thermodynamic cycle.

So, when we talk about the Brayton cycle, the ideal Brayton cycle is a closed cycle, but gas turbines operate in an open cycle mode, but they can be modeled using air standard assumptions and the ideal cycle assumes that there are no irreversibility's, and so, air behaves like an ideal gas with constant specific heats, and obviously, there are no frictional losses which is also part of the fact that there are no irreversibility's in the process.

So, these are some of the assumptions which we will be assuming in this analysis, and so, what we will do? Today to begin with, is to define and derive some expressions which basically tell us how efficient or how the performance of an engine is. One of the most important parameters which define so call, define an engine is the thrust which the engine develops. So, thrust developed by an engine is one of the basic parameters which basic performance parameters, which this, which basically gives some quality to an engine saying that this particular engine develops this thrust, because that is one of the main objectives of the engine that is to provide thrust for the aircraft.

So, we will first derive some expressions for, generalized expression for thrust generated by a get engine, and subsequently, we will also define some efficiency terms like now that an engine we know is developing some thrust. How efficiently is it converting fuel input into the thrust output?

So, there are different ways of defining efficiency of an engine. We will discuss these different efficiency terms as well. Some of the efficiencies you would have already been familiar with in the last few lectures, where you were exposed to propeller engines and different types of propeller engines and efficiencies associated with propeller engines.

So, today, we will be discussing about efficiencies which concern jet engines, and so, these are some of the efficiency definitions we will be discussing. We will also be talking about fuel efficiency, that is, how efficiently can an engine convert a given amount of fuel into thrust output. That is basically defined by the fuel efficiency or specific fuel consumption as it is called. So, these are some of the terms that we are going to discuss in the next few slides.

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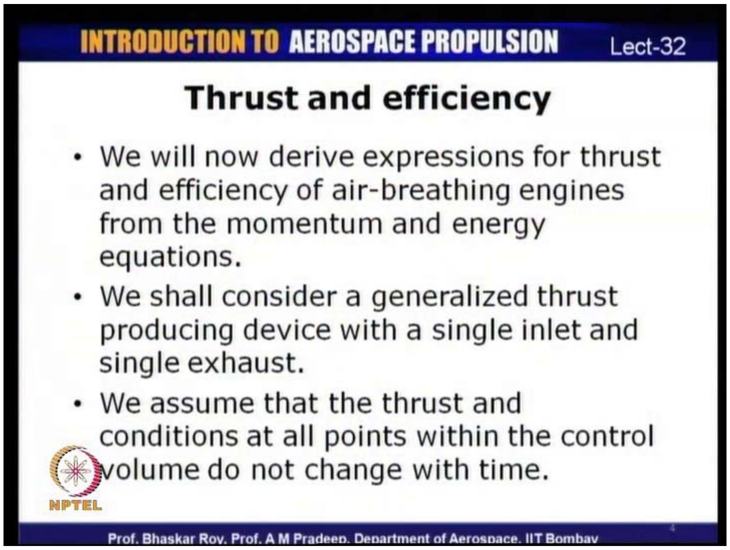

So, what we will do? First is that we will derive expressions for thrust and efficiency and which basically come from the momentum and energy equations and what we shall be doing is we will consider a generalized thrust producing device with a single inlet and single exhaust and there are engines as we will see in the next lecture which have multiple inlets and multiple exhausts, basically those which concern turbofan engines and so on, or in fact, even turboprops. So, these are engines which have multiple inlets and multiple outlets, and again, in this analysis, we will assume that the thrust and the conditions at all points within the control volume do not change with time, that is, we are going to assume that the thrust generated, thrust developed is not really a function of time, it is a steady state thrust which is being generated. So, that is another assumption which is going to be inherent in our analysis today.

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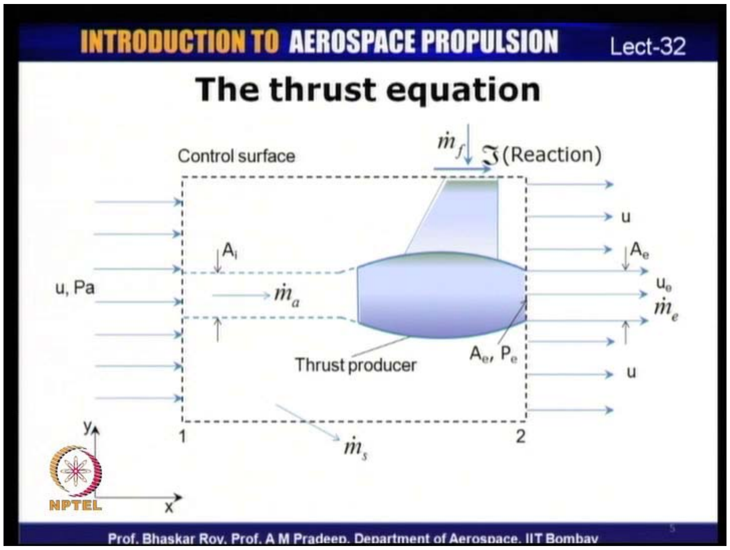

So, how are we going to start our analysis? We will basically consider an engine, a generalized engine which has a single entry and a single exit from the engine, and then, we will basically use the momentum, mass momentum and energy equations, and basically that will help us in identifying what is the force that is experienced by such an engine. As it consumes a certain amount of mass and expels certain amount of mass which also includes the fuel flow rate.

So, what is the thrust develop which is basically a force, which is acting on the control volume. So, let us take a look at the control volume and the control surface. So, the engine that I was talking about a generalized thrust producing device is shown here. This is the thrust producer which resembles the engine of a passenger aircraft. You might have noticed that in passenger aircraft, there are engines mounted beneath the wing, and so, let us say this is the structure or strut which is connecting the engine to the wing, and in almost all aircraft, the fuel is stored in the wings of the aircraft, and therefore, you can see there is a fuel flow rate which is coming in, let us say from the wing through this connecting device in to the engine.

So, let us take a closer look at what this control surface is all about. So, we have here a control surface, which is extending quite far upstream of the engine and it is terminating right at the exhaust point. You can see this is where the exhaust is and the control surface terminates right here and there is reason why it has to terminate right there. I will explain it a little later, and so, at the inlet, we have the velocity, ambient velocity which is u and it is at a static pressure of P subscript a, and then, we have this area which is given by A i which is the amount or a stream tube of air which goes in to the engine.

So, this stream tube area is basically referred to as the capture area, which is basically the amount of air which has been captured by the engine to generate the thrust, and the mass flow rate which is captured is denoted by m dot a. Then, at the exit, we have an exit velocity of u e mass flow rate of m dot e and the cross sectional area could be different from the entry, it is A e, and away from the engine, we have the ambient velocity which is u, and then, the exhaust area is A e and the static pressure right at the exit is P e.

Fuel flow rate is given by m dot f which is basically the fuel that is going in to the system and which means that m dot e is from mass continuity with m dot e should be equal to m dot f plus m dot a, and if there is some amount of mass flow rate which is escaping the control surface because of the presence of this obstacle, there could be some mass flow rate which escape from the control surface that we denote by m dot s.

And what is shown here is denoted by a symbol, let us say tau, capital tau and this is basically the reaction to the thrust which generated by the engine, and so, we also have a coordinate system indicated here. x is in the direction of the velocities or in the direction of thrust x, y is normal to that, and so, these are different salient components of the control surface and the engine which we are considering further analysis and what we are going to do is that in this particular engine which we shall now denote as a generalized thrust producing device. It has only a single entry and a single exhaust, but the thrust equation which we are going to derive can be extended for engines which have multiple entries and multiple exhausts. That we will take up in our analysis during the next lecture.

Now, in this particular engine, we have a certain amount of mass flow rate which is coming in, then fuel is added in to the gas turbine and there is an exhaust mass flow which is basically equal to the sum of the mass flow rate of air coming in plus the mass flow rate of the fuel, because mass continuity has to be satisfied.

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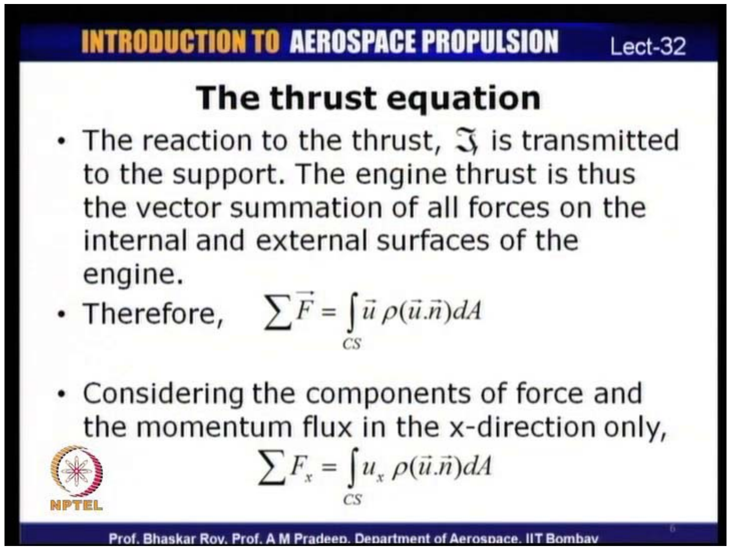

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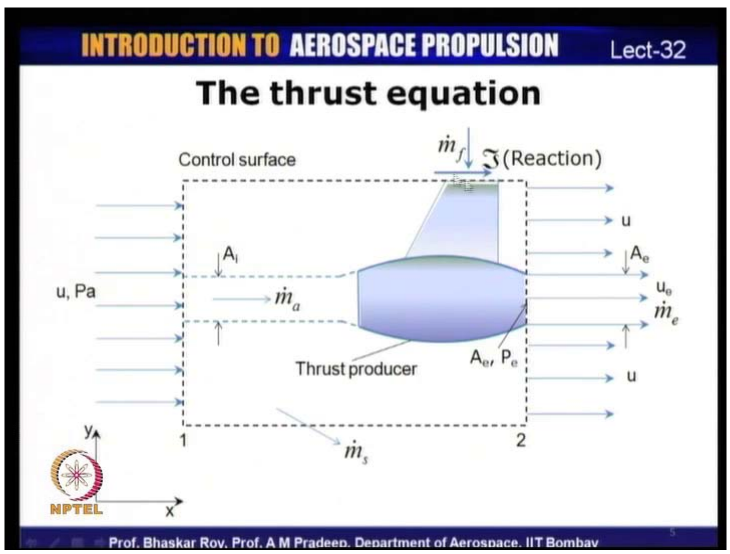

Similarly, we shall also use the energy conservation. At some point, we shall also use the momentum conservation to derive an equation for thrust. So, what basically we have here is that the reaction to the thrust which I had mentioned as tau capital T is basically transmitted to the support that is the support which holds the engine. So, right here, we have the support reaction is felt right there.

So, the engine thrust is basically a vector summation of all the forces which act internal or external to the engine, to it all the forces which act on the internal and external forces.

Some of them would be in one direction. The other forces could be in another direction, but if you have a vector summation of all these forces put together, then the net force that is felt is basically the engine thrust. Therefore, if we were to add up all those vector quantities, we have summation of vector force is equal to integral over the control surface vector u into product of rho into u dot n d a.

So, basically, this is mass flow rate times the velocity which is the change in momentum, which as per the Newton's second law is the force acting on the system. So, u multiplied by rho times u dot n which is the dot product of the vector velocity and the normal unit vector n multiplied by dA that is the area on which these velocities are taken so that basically gives us the mass flow rate multiplied by u gives us the momentum and change in momentum is basically the force.

Now, as I mentioned, we are going to consider only components of force and the momentum flux in the x direction. We are going to ignore the forces acting in any other direction, because that does not contribute to the thrust. Thrust is basically the force in the x direction. So, summation of F x, that is, forces in the x direction will be equal to integral control surface u x, which is velocity in the x direction multiplied by rho u dot n dA. So, this is basically considering the forces in the x direction.

So, thrust is something which is generated or which is required only in one direction and this is one aspect which is also utilized in some of the advanced military engines, which have what are known as thrust vectoring. That is the nozzle of the engine can be deflected in different ways to achieve thrust in different directions. You must have seen videos of aircraft which can take off without having to use a runway, which are known as vertical takeoff, and similarly, there are same aircraft can land without having to use a runway which are called as vertical landing.

So, there are aircraft which can do this or they can in flight do very extreme maneuvers by deflecting the nozzle, and so, that is generated basically by deflecting the nozzle, and therefore, the vector direction of the thrust is deflected in different ways to achieve thrust in different directions, and under normal circumstances like a passenger aircraft which you have seen perhaps flown is that the nozzle is stationary and fixed, and so, it always generates a thrust in one direction, that is, in the x direction let us say, and so, that is basically equal to the summation of all the forces which act in that particular direction, and so, that is how we have going to neglect the forces in other directions. There are of course be forces in other directions, but their magnitudes are going to be much small as compared to the force in the x direction.

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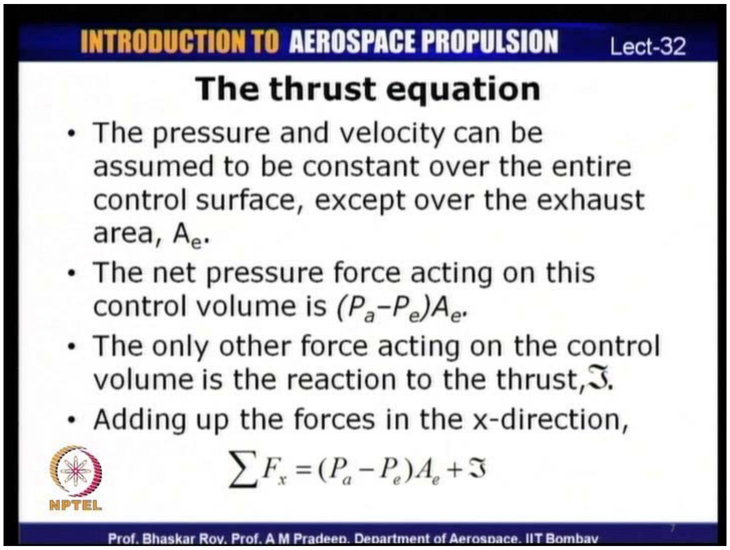

So, with this in mind, what we are going to do is also assume that the pressure and velocity is a constant over the entire control surface except over the control exhaust area, because we have seen at the exhaust, the velocity and the pressure can be different, and therefore, else where we can assume that the pressure and velocities are constant.

So, if you were to assume that, then the net pressure force which is acting on the control volume will basically be equal to the difference between the ambient pressure and exit pressure multiplied by area. So, P a minus P e into A e is basically the net pressure force which is acting on a control volume, because if there is a difference between the exit pressure P e and the ambient pressure P a, it will itself exert a force over an area A e, and therefore, that is equal to the net pressure force, and what is the other force that is acting on the control volume? There is only one more force which is acting, which is basically the reaction to the thrust which is indicated by t or tau.

So, if you add up these forces in the x direction, what we get is summation F x is equal to P a minus P e into A e which is the pressure force or the pressure thrust as we shall call it later on plus the reaction to the thrust which is tau. So, the net forces can be summed up to be equal to two parameters or two components. Net force is coming from two components - one is because of the difference in the pressure from, the inlet to the exit multiplied by the exit area. So, we have a certain pressure thrust and the other term is the thrust itself reaction to the thrust which is tau.

Now, our aim now should be to find out what is this sigma F x. In which case, if you can find sigma F x, then you know that you can find the thrust, because on the right hand side, you have the pressure thrust term and reaction to the thrust. So, if sigma F x is known, then that minus the pressure thrust term would be basically give us the reaction to the thrust.

So, how do we find out sigma F x? Now, to find out sigma F x, as I mentioned earlier, we shall now carry out a mass balance and momentum balance across the control surface, across the control surface and from the inlet to the outlet. So, if we do a mass and momentum balance, we should be able to find out an expression for the summation of forces in the x direction.

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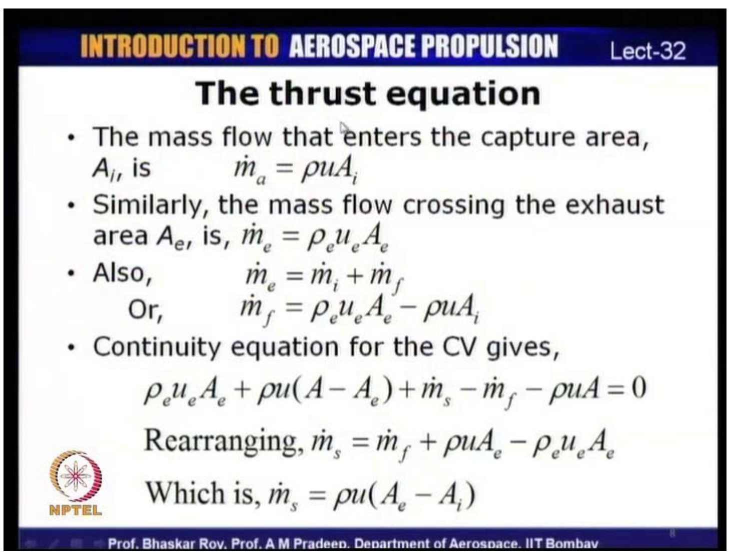

So, if we look at the different mass flow rates which are entering and exiting, the capture, the control volume. We have the mass flow which is entering the capture area. I mentioned capture area is the area of the stream tube which actually enters in to the engine, and so, mass flow rate through that is equal to m dot a which is density times velocity times the cross sectional area.

So, here, the density is rho; velocity is u and the area is A i. A i corresponds to the capture area which is the area through which m dot a is inducted into the engine. Similarly, mass flow rate crossing the exhaust area is m dot e which is equal to rho e u e times A e - where rho e is the density of the air or the combustion products at the exhaust; u e is velocity at the exhaust; A e is the area at the exhaust, but we also know that m dot e that is exhaust mass flow should be equal to m dot i plus m dot f.

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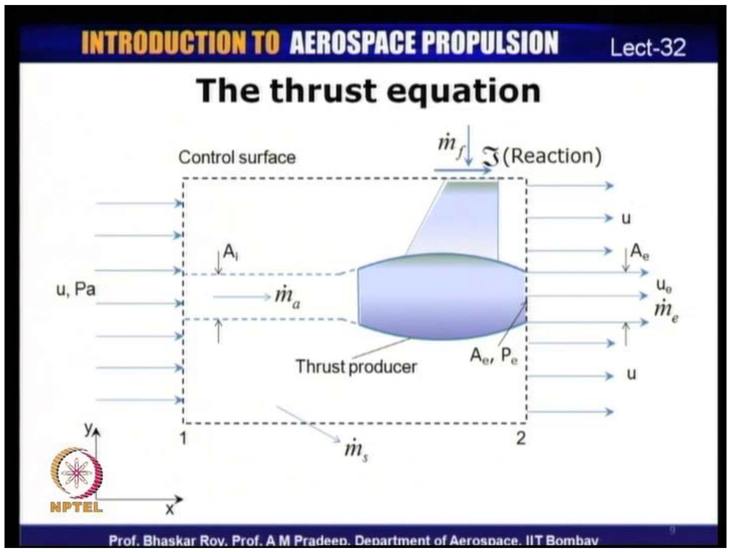

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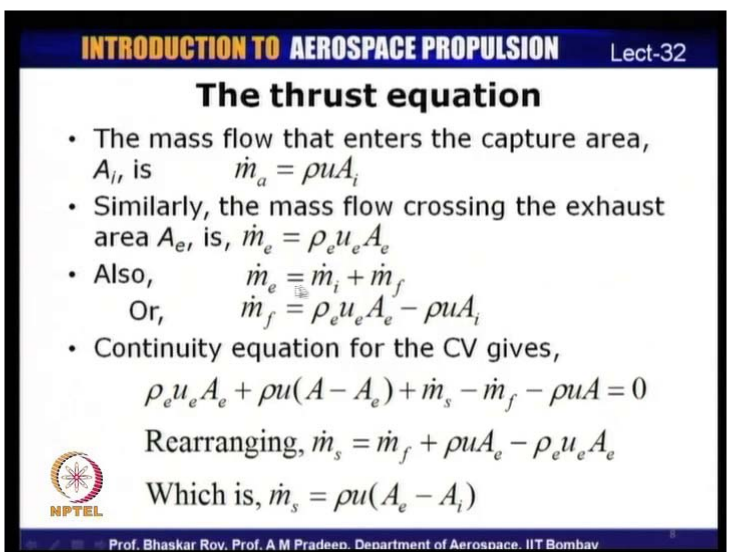

So, let me take a look at the thrust picture once again. So, here, we have m dot a which is the mass flow rate, which enters the engine and it is doing. So, at through a cross sectional area of A i, so, m dot a should be equal to rho times u times A i. Similarly, m dot e is the mass flow rate exiting the control surface which is rho e times u e times A e.

And if you look at mass balance which is entering and leaving the engine, we have m dot e which is equal to m dot a plus m dot f. So, we have m dot e that is exhaust mass flow is equal to inlet mass flow plus m dot f m dot a plus m dot f or we can write m dot f is equal to rho e u e A e which is equal to m dot e minus rho u A i, which is m dot a. So, now, let us use the continuity equation for the control volume. That is what is the mass flow entering the control volume. What is the mass flow leaving the control volume?

So, let us look at mass flow leaving the control volume in the first place. So, what are the mass flows leaving the control volume? We have a mass flow here that is m dot e, that is, rho e u e and A e and also you have the mass flow which is escaping the control surface from these areas, which should be equal to rho into u into A minus A e - where A is this cross sectional area, cross sectional area across the entire control surface. That is one of the mass flows leaving the control surface. The other mass flow is this m dot s.

So, there are three mass flow terms which are leaving the control surface - one is m dot e; the other is rho into u into A minus A e and the third term is m dot s, and what about mass flows entering the control surface? m dot a is one of them or in fact, let us take up the whole thing, that is, rho into u into A and m dot f.

So, if you look at the continuity equation, we have rho e u e A e which is m dot e plus rho into u into A minus A e plus m dot s minus m dot f minus rho times u times A. So, this should be equal to 0, because the positive term should be actually be equal to m dot f, that is, mass coming in should be equal to mass leaving out. So, the difference between the two should be equal to 0.

So, if you rearrange this, we have m dot s is equal to m dot f plus rho u A e minus rho e u e A e, but we have already derived a term or expression for m dot f which is equal to this, that is, rho e u e A e minus rho u A i. So, if you substitute for this m dot f in this equation, we get m dot s is equal to rho into u into A e minus A i. So, that is this term is that mass flow rate leaving the control surface is equal to this particular term, that is, rho into u into A e minus A i.

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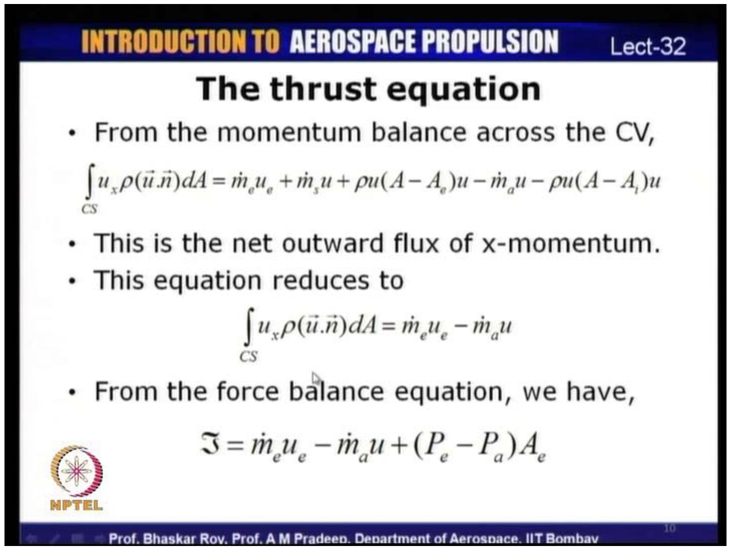

So, we have now carried out a mass balance which is basically the continuity equation across the control surface. What we shall do now is to use the momentum balance across the control surface, and therefore, we should be able to get a term or an expression for thrust which is developed by this particular thrust producing device. So, continuity equation is something which gives us the mass balance and the second balance which you are going to do is the momentum balance, which is going to give us the net force which is acting on the control surface.

So, from the momentum balance across the controls volume or control surface, we have the momentum, the first term. If you remember this was equal to sigma F x which is equal to integral over the control surface u x into rho u dot n dA. So, what are the different momentum terms? One is the exit mass flow m dot e multiplied by the corresponding velocity u e plus m dot s into u plus rho into u into A minus A e multiplied by u.

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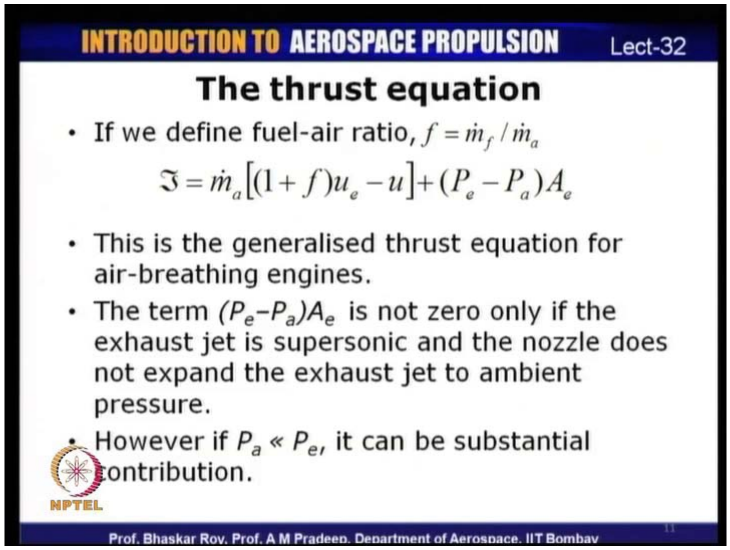

So, this is momentum at the exhaust or momentum leaving, and what is momentum entering the control volume? We have m dot a into u minus rho times u times A minus A i into u. So, this is the mass flow, which is these two terms corresponds to the momentum entering the control volume.

So, what we have here is the net outward flux of x momentum. So, if you substitute for m dot s and simplify, what we get is a very simple expression for the net momentum flux and that is basically equal to sigma F x is equal to m dot e u e minus m dot a into u. So, from the force balance equation, where we had sigma F x is equal to the P a minus P into A e plus the reaction to the thrust which was tau. If we substitute for sigma F x there, we have tau is equal to m dot e u e minus m dot a into u plus P minus P a into A e, which again we shall simplify.

Let us now define a fuel to air ratio, where fuel to air ratio is ratio of mass flow of fuel to mass flow of air. So, f is equal to m dot f by m dot a. If we substitute for that in the thrust equation, we have thrust is equal to m dot a into 1 plus f u e minus u plus P e minus P a into A e. So, this is the generalized thrust equation for air breathing engines.

So, you can see there are two components here for the thrust equation - the first term is because of the momentum; second term is because of the pressure. So, thrust is equal to m dot a into 1 plus f u e minus u which is the momentum or the ram thrust and the second term is the pressure thrust, which is difference between the exit pressure and the ambient pressure into A e and the second term is non 0 only if the exhaust jet is supersonic and the nozzle does not expand the exhaust jet to ambient pressure.

So, which means that in many of the cases, in most of the cases, we will have the contribution of the pressure thrust which is negligible or in fact even equal to 0, but if their differences are substantial, if the difference between the exit pressure and ambient pressure is very high, then there could be some contribution from the pressure thrust term, but majority of the thrust is due to the first term, that is the momentum difference generated because of the exit velocity being much different than the inlet velocity.

So, the thrust developed is a function of the mass flow rate of air as you have seen. Then it is also a function of the fuel flow rate or the fuel to air ratio. It is also a function of the exit velocity and the ambient velocity plus it is also a function of the net pressure thrust. That is the net pressure difference between the inlet and the exit multiplied by the corresponding area. So, thrust can compromise of these two terms, and in most of the applications, we will find that the pressure thrust contribution is very small. There is hardly any pressure thrust which is developed or generated by the jet engines. So, what we have defined here or derived here is an equation which is the generalized thrust equation applicable to all air breathing engines.

Now, air breathing engines is sometimes, something probably you have already been exposed to in some of the earlier lectures. It is basically those engines which use air as the oxidizer and fuel is added into the engine and combustion takes place in the combustion chamber. Unlike rocket engines which carry their own oxidizer and fuel and essentially rocket engines are not air breathing engines. So, all aircraft engines are basically air breathing engines.

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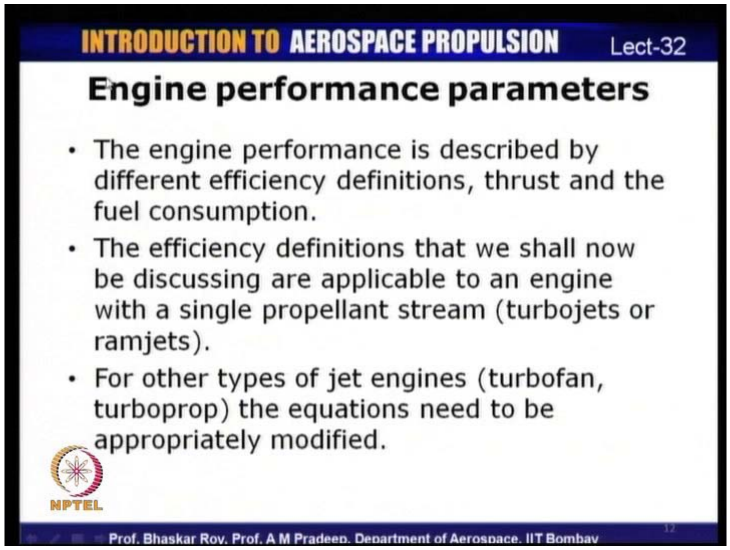

So, the generalized thrust expression that we have derived here. Let us take a closer look at that and we have the generalized thrust equation and what are the other different terms which define the performance of the engine. Besides the thrust, we also have efficiency terms. Each engine can be defined or described by a set of efficiency definitions.

We will take a look at those efficiency terms. Then, thrust is the other parameter and also the fuel consumption and we shall now discuss about efficiency definitions which are primarily applicable to engines with a single exhaust stream like turbojets and ramjets and for other types of jet engines like turbo fans and turboprops. Then the equation gets slightly modified, but the basic equation still remains the same, definition still remains the same.

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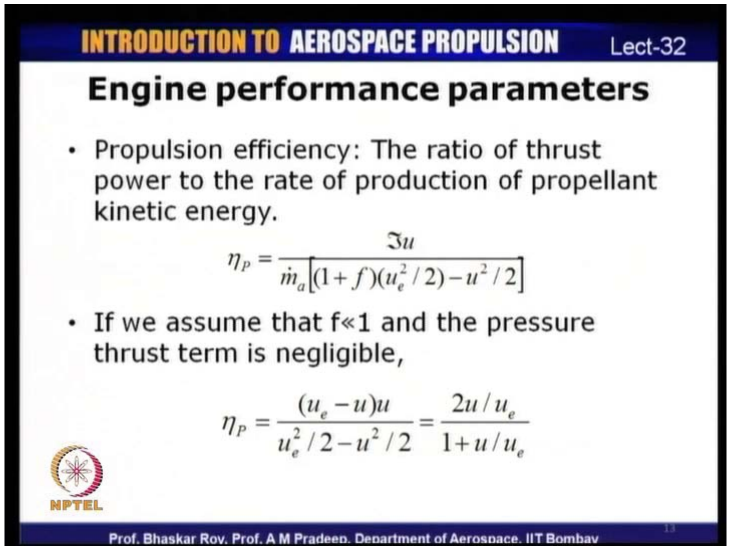

So, the first definition of efficiency that we are going to talk about today is the propulsion efficiency. So, what do we mean by propulsion efficiency? So, propulsion efficiency by definition is the ratio of the thrust power to the rate of production of propellant kinetic energy.

So, how do we define the thrust power and the rate of propellant kinetic energy? So, thrust power is primarily the product of the thrust and the ambient velocity or the flight speed that is u and the rate of production of propellant. Kinetic energy is the difference between the kinetic energy of the exhaust and the kinetic energy at the inlet which would be equal into m dot a into 1 plus f u e square minus u square u e square by 2 minus u square by 2. So, on the denominator that is basically the rate of production of propellant kinetic energy, we have mass flow rate of air m dot a into 1 plus f minus u e square by 2 minus u square by 2.

So, basically what we have is this term, that is, m dot a into u square by 2 is the inlet kinetic energy and m dot a into f is m dot e that is exit mass flow into u e square by 2 is the exit kinetic energy. So, difference between those two is the rate of production of propellant kinetic energy. So, this ratio is basically known as the propulsion efficiency thrust power to the rate of production of propellant kinetic energy.

Now, let us simplify this and see what happens. If we, let assume that the fuel flow rate f is much less than 1, which is true for air breathing engines. Usually the fuel to air flow ratio is very small, and so, if we assume that and also that the pressure thrust term can be neglected. That is, in the thrust equation, we have the second term which is equal to 0. Then, if we substitute for thrust equation on the numerator which is m dot a into u e minus u, then simplify it. What we get is the propulsion efficiency can be simplified as u e minus u by u square by 2 minus u square by 2 which is in turn equal to two times u by u e divided by 1 plus u by u e. That is propulsion efficiency is in some sense, function of the ratio of the velocities, the two velocities which are involved here - one is the flight speed or the inlet velocity and the exit velocity that is u e.

Which means that if u is equal to u e, we will have the numerator and the denominator is equal to 2 and then propulsion efficiency becomes 1, but in that case, if you look at the thrust equation, if the exhaust velocity and the inlet velocity are same, there is no net momentum and the thrust generated will become 0. So, it means that if you try to maximize the propulsion efficiency which happens when u is equal to u e at the same time means that you have a thrust which can become 0. So, it, the maximizing propulsion efficiency is probably not something that one would try to do, because, in, in that sense, you would also try to make the thrust more or less equal to 0, that is, with the assumption that you can assume f is much less than 1 and pressure thrust can be neglected which is probably true in many cases.

So, this was the first definition for efficiency that we have the propulsion efficiency. The second efficiency term that we shall be discussing is the thermal efficiency, which basically refers to or which is an indication of how much amount of energy that is input into the engine can be converted to thrust output.

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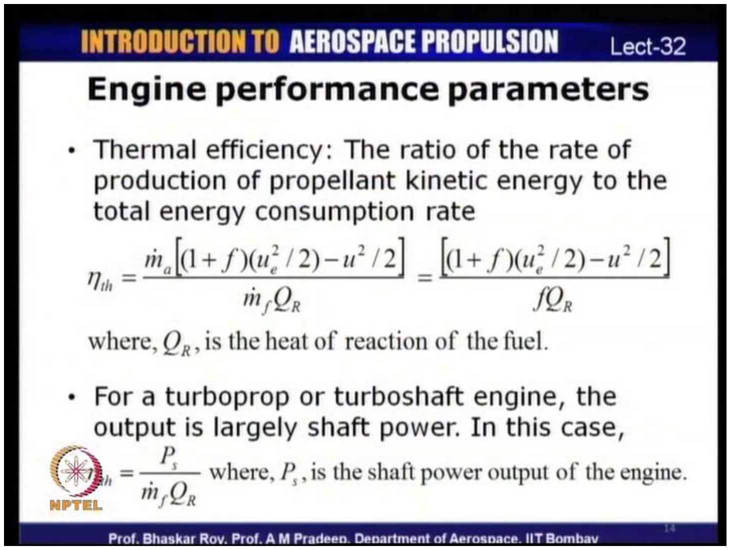

So, thermal efficiency is defined as the ratio of the rate of production of propellant kinetic energy to the total energy consumption rate which is equal to numerator, which we have already discussed, divided by the energy consumption rate which is mass per rate of fuel multiplied by Q R, which is the heat of reaction of the fuel. So, the thermal efficiency, this can again be simplified as 1 plus f into u e square by 2 minus u square by 2 divided by f into Q R - where Q R is a property of the fuel.

Now, in the case of this is primarily for turbojets and ramjets and if you look at engines which have which generate a large fraction of shaft power, where in the output is primarily shaft power like in turboprops or turbo shaft engines, then the thermal efficiency can be modified as the ratio of the shaft power to the rate of energy consumption. So, that is shaft power which is P s divided by m dot f into Q R.

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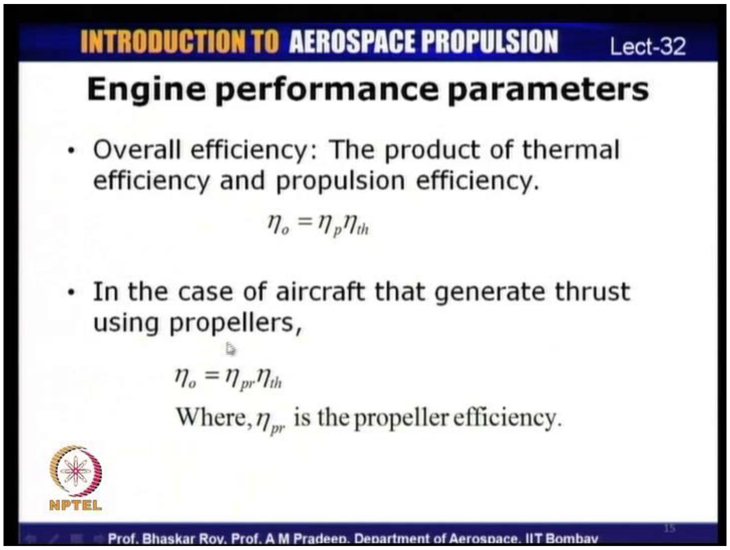

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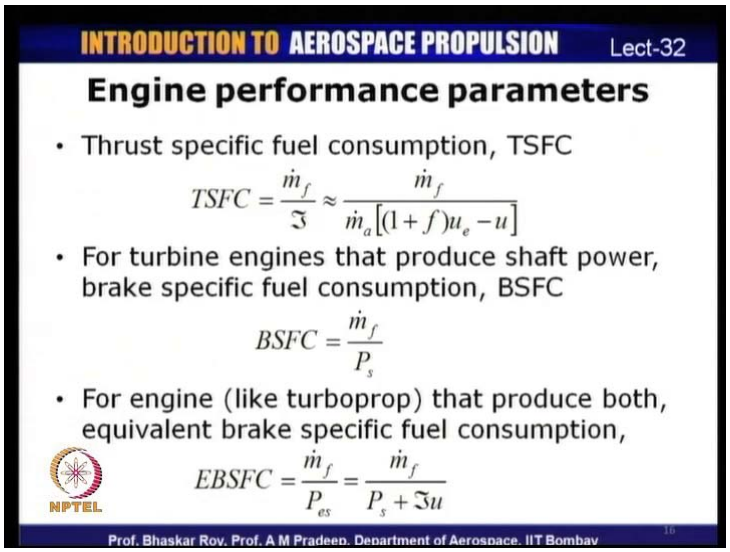

So, now, the overall efficiency is basically the product of the thermal efficiency and the propulsion efficiency. So, eta overall is equal to eta p which is propulsion efficiency and eta thermal which is the thermal efficiency, and in the case of engines, jet, that generate thrust using propellers like in thermal props for example or turbo shafts. Then the overall efficiency is the product of the propeller efficiency and the thermal efficiency. So, now that we have discussed about different types of efficiencies. Let us also look at the other important performance parameter which is the fuel consumption rate.

So, in the case of air breathing engines, we normally refer to the fuel consumption in the form of what is known as thrust specific fuel consumption denoted by TSFC. So, thrust specific fuel consumption - TSFC - is equal to m dot f divided by the thrust, which is equal to m dot f by m dot a into 1 plus into u e minus u.

Now, for engines, turbine engines which produce shaft power, we may want to define the fuel efficiency in the form of what is known as break specific fuel consumption, that is, BSFC, which is m dot f divided by the shaft power, and for engine like a turboprop which usually generates both that is shaft power as well as the nozzle thrust. Then we define equivalent brake specific fuel consumption which is equal to m dot f divided by equivalent power P e s, which is equal to m dot f by shaft power plus the thrust power.

So, these are used for engines like in turboprops which have both shaft power as well as it generates a thrust from the nozzle as well. So, these are different performance parameters that are used to qualify an engine in the sense that what is the thrust developed by the engine? What is the fuel consumption and what are the different efficiencies?

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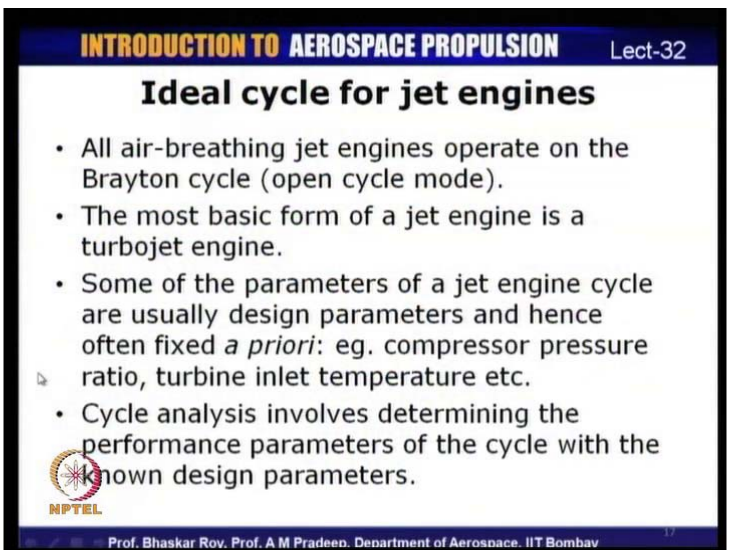

Now, with this background in mind, we shall now carry out an ideal cycle analysis for one of the basic forms of jet engines which is known as the turbo jet engine and we will be discussing about two types of turbo jet engines - one is without afterburning which is the basic turbojet, and then, we will also look at turbojet with after burning which is very similar to the Brayton cycle we had seen earlier the basic Brayton cycle and the Brayton cycle with reheating. So, afterburning which is used in turbojet engines is a reheating process. So, all jet engines, all air breathing engines basically operate on the open cycle mode of the Brayton cycle, and the basic form of the jet engine, we can consider is a turbojet engines, and in a turbojet engine, some of the parameters which we shall come across are so called design parameters and are often fixed a priori like the compressor pressure ratio or the turbine inlet temperature ratio, etcetera, and in cycle analysis, we shall be using some of these known parameters to determine some of the other parameters which are not necessarily known, and hence, find out the engine performance parameters like thrust, fuel consumption and the efficiencies.

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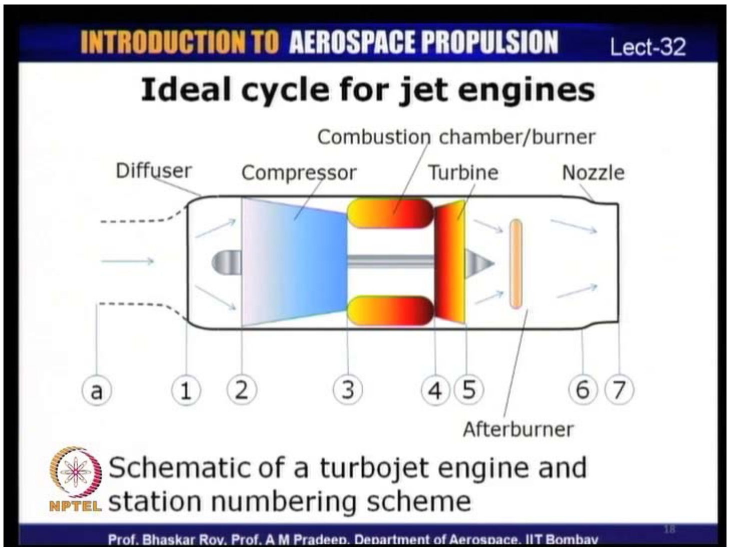

So, let us take a look at basic turbojet engine in its schematic form and we also have certain station numbering system which we shall be following in the next one or two lectures, that is, each component is designated by a certain number for its inlet and outlet. So, this is a turbojet engine and the components of the turbojet engine are indicated here. So, the first component in a turbojet engine we have is a diffuser. Diffuser is followed by a compressor, and so, the compression process begins from the diffuser and continues all the way up to the compressor exit.

In an ideal cycle, we are going to assume that the process from the diffuser inlet all the way up to compressor outlet is isentropic. So, compressed air from the compressor goes into a combustion chamber where fuel is added and the necessary heating addition to the cycle takes place here. From the combustion chamber, the hot gases are expanded in a turbine and turbine is indicated here. So, expansion process begins right at the turbine entry continues all the way up to the nozzle exit.

So, this process from turbine entry to nozzle exit is also assumed to be isentropic in an ideal cycle, and in the second modification, we will see a little later is what is known as afterburning which is shown here. Then afterburning system, it is very similar to that of a combustion chamber in the sense that heat addition, additional heat addition takes place in the afterburner to take the cycle temperature to a higher value, and therefore, generate higher thrust, and what are indicated here below are the different numbers associated with each of these components. The free stream or the ambient is indicated by a, and so, air is compressed all the way from point a up to point 3 which is the compressor exit 1 to 2 indicates the diffuser; 2 is the compressor entry; 3 is compressor exit, and therefore, the combustion chamber entry; 4 is combustion chamber exit and the turbine inlet; 5 is turbine exit; 6 is nozzle entry and 7 is nozzle exit.

So, it is, so, we are going to follow this numbering scheme in the sense that if we write temperature as t 3, it means static temperature at compressor exit, and if we write t 0 3, it means stagnation temperature at compressor exit. Similarly, we are going to use these numbering schemes for pressures and densities and so on.

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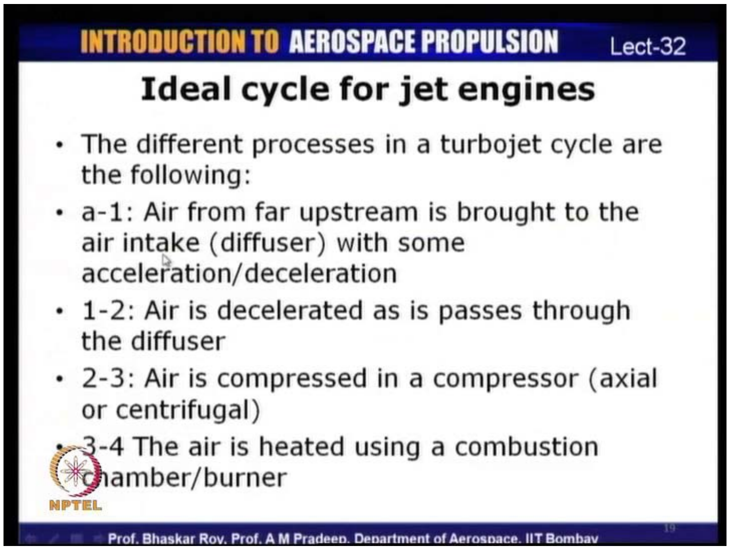

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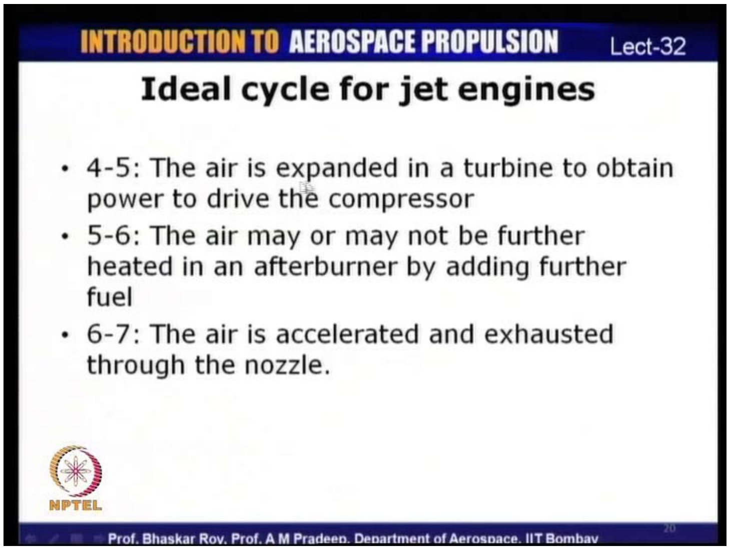

So, the different processes as I mentioned, the first process is a to 1 which air from upstream is brought to the diffuser or the entry with either some acceleration or sometimes even deceleration and process 1 to 2 is the intake or the diffuser where air is essentially decelerated as it passes through the diffuser. Process 2 3 is the compression process in the compressor. Air is compressed in a compressor which is either could be an axial compressor or a centrifugal compressor. Process 3 4 is the heat addition process air is heated during this combustion process in the combustion chamber. 4 5 is air is expanded in a turbine to obtain the power which is primarily used to drive the compressor and 5 6 is air may or may not be heated in an afterburner by adding further fuel.

So, if an afterburner is used, then there is heat addition during this process as well. Process 6 7 is the nozzle where the air is accelerated and exhausted through the nozzle. So, these are the different processes that are involved in a turbojet cycle starting from the free stream far upstream that is point a. It is initially, the compression process actually begins at a, and then, it continues in the diffuser and again in the compressor. Finally, it reaches the state 3 which is compressor exit.

So, isentropic expression all the way from a to 3; 3 to 4 is heat addition process in the combustion chamber; 4 to 5 is the turbine, that is, isentropic expansion in the turbine; 5 to 7 is expansion in the nozzle. So, 4 to 7 is an expansion process which is going to be assumed as isentropic in this ideal cycle analysis. So, let us take a look at these processes on Brayton cycle diagram or a T-s diagram.

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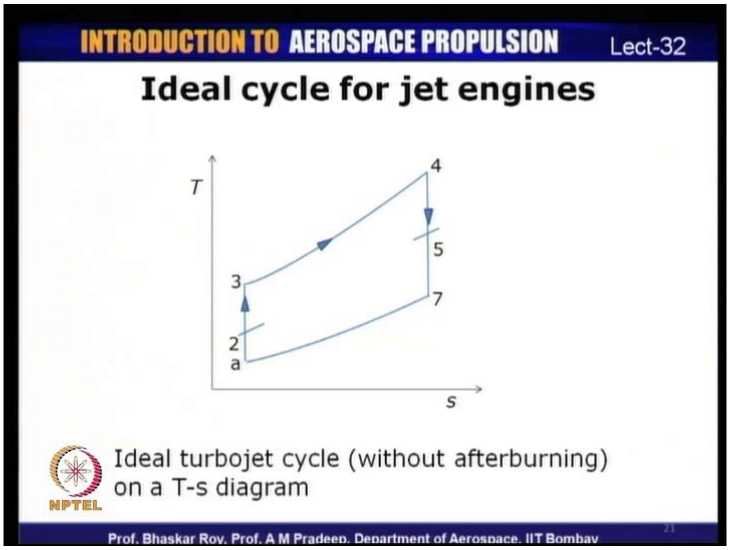

So, an ideal turbojet cycle without afterburning would look like this very similar to the Brayton cycle. Just that there are different processes which constitute the compression and expansion processes. So, process a all the way up to 2 is the compression in the intake. So, intake consists, compression consists of 2 compression 1 is external compression that is some a to 1 and 1 to 2 is the internal compression. Process 2 3 is compression in the compressor isentropic again; 3 4 is heat addition in the combustion chamber; 4 is the turbine entry or turbine inlet; 4 to 5 is expansion in the turbine; 5 to 7 is expansion in the nozzle, and so, you can see process 4 to 7 is isentropic, and so, is process 8 to 3.

So, we have isentropic processes here, constant pressure, heat addition, that is taking place between 3 and 4. So, this is an ideal cycle or ideal turbojet cycle without any afterburning. So, later on, we shall also see an ideal cycle for a turbojet with an afterburning. So, in the case of afterburning which is very similar to a Brayton cycle with reheating, we shall have an additional process taking place where heat is again added at the end of expansion process, and then, finally it is expanded in the nozzle. So, in the cycle analysis that we are going to do, what we shall primarily be doing is as that as I mentioned some of the parameters are fixed or design parameters like compression pressure ratio or turbine inlet temperature, etcetera.

So, these numbers are usually known, and based on this, we shall also be determining the other pressures and temperatures. Finally, arriving at an expression for the exhaust velocity that is u e, and so, once exhaust velocity is known, the fuel flow rate is known. We can calculate the thrust developed by the engine and also we can calculate the fuel consumption and the efficiencies, and so, in the cycle analysis which we are going to discuss today we shall be taking up each of these components 1 by 1, that is intake, then the compressor, then the combustion chamber, turbine and so on.

So, as we analyze each of these components, we shall be finding out the exit pressure and temperature of each component which will serve as the inlet pressure and temperature for the subsequent component. For example, at the intake exit, we shall be finding out what is the pressure and temperature, which will essentially be the compressor entry pressure and temperature, and similarly, for all other components.

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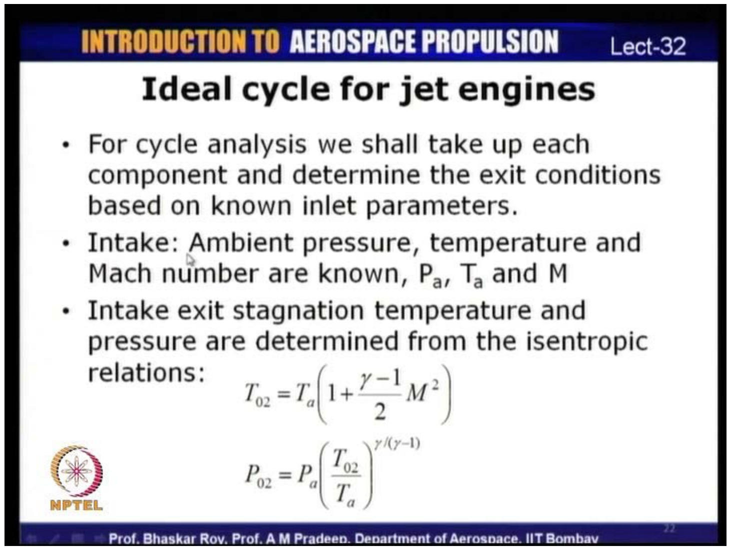

So, we shall first analyze the intake or the diffuser. So, in the case of intake or the diffuser, the ambient pressure, the temperature and mach number are usually known, that is, P a T a and mach number M are known a priori depending upon what altitude the aircraft is flying and at what speed it is supposed to fly.

So, the exit conditions, that is, exit stagnation temperature and pressure can be calculated from the isentropic relation, because we have assumed that this process is going to be isentropic. So, the exit static pressure and temperature – so, T 0 2 which is the exit stagnation temperature is equal to T A which is the inlet static temperature into 1 plus gamma minus 1 by 2 M square.

Similarly, the pressure can be determined from the isentropic relation P 0 2 is equal to P a into T 0 2 by T A raised to gamma by gamma minus 1. So, from these two isentropic relations, we calculate the intake exit stagnation temperature and stagnation pressure which will act as the compressor entry stagnation temperature and stagnation pressure.

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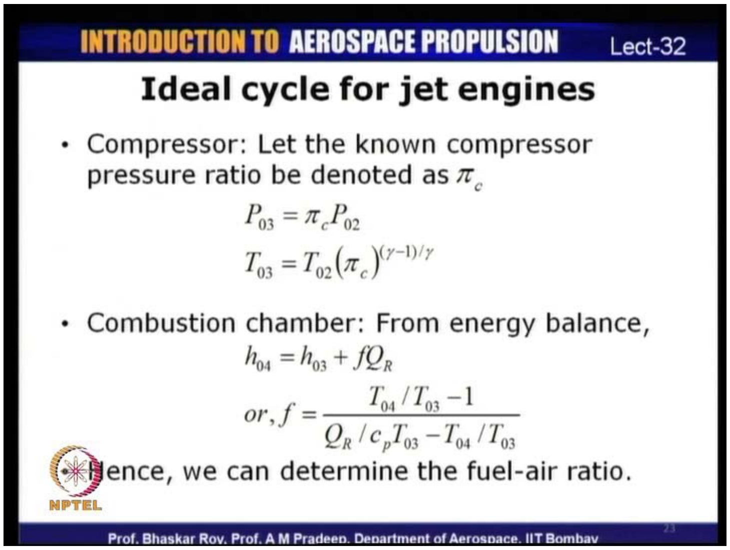

So, the next component is the compressor, and in the compressor, we have discussed that the compressor pressure ratio is a known parameter. We shall denote that by the symbol pi c - where c denotes the compressor; pi is for stagnation pressure ratios. So, if pi c is fixed, we have P 0 3 which is the compressor exit stagnation pressure equal to pi c times P 0 2. P 0 2 is known from the intake analysis. Similarly, T 0 3 is equal to T 0 in to pi c raised to gamma minus 1 by gamma which is again from the isentropic relation; compression process is isentropic.

So, we have now calculated the properties, that is, stagnation temperature and pressures all the way up to the compressor exit, and so, next component that we going to analyze is the combustion chamber, and for in the combustion chamber, we need to find out what is the fuel flow rate that is being added in the combustion chamber. So, we will basically carry out an energy balance.

So, energy balance across the combustion chamber gives us at the exit of the combustion chamber. We have h 0 4 stagnation enthalpy is equal to h 0 3, that is inlet stagnation enthalpy plus the fuel added, that is, f times Q R. Therefore, from this, we can from the ideal gas approximation of enthalpy which is equal to c p times the corresponding temperature. We have f is equal to T 0 4 by T 0 3 minus 1 divided by Q R by c p T 0 3 minus T 0 4 by T 0 3. So, from this, we can calculate the fuel to air ratio.

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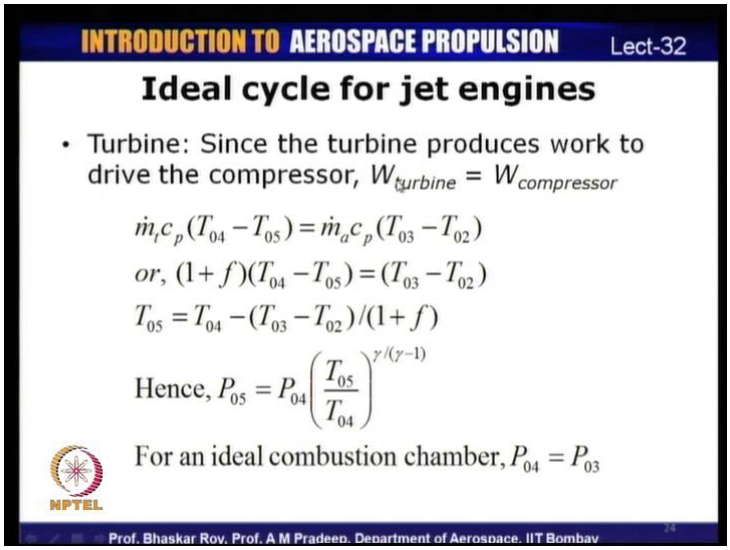

Now, in the turbine, the turbine is basically meant to drive the compressor. So, if we equate the work done by the turbine to that of the compressor, we have work done by turbine is equal to work done by compressor, that is, m dot t which is mass flow rate of turbine into c p multiplied by temperature difference T 0 4 minus t 0 5 is equal to m dot a into c p T 0 3 minus T 0 2. So, this can be simplified, and so, we get an expression for T 0 5 which is turbine, exit turbine. Inlet temperature is fixed; T 0 4 is always known. So, T 0 5 is T 0 4 minus T 0 3 minus T 0 2 divided by 1 plus f. Therefore, pressure P 0 5 is P 0 4 into the temperature ratio raised to gamma by gamma minus 1. So, in an ideal process, there is no pressure loss in the combustion chamber P 0 4 will be equal to P 0 3.

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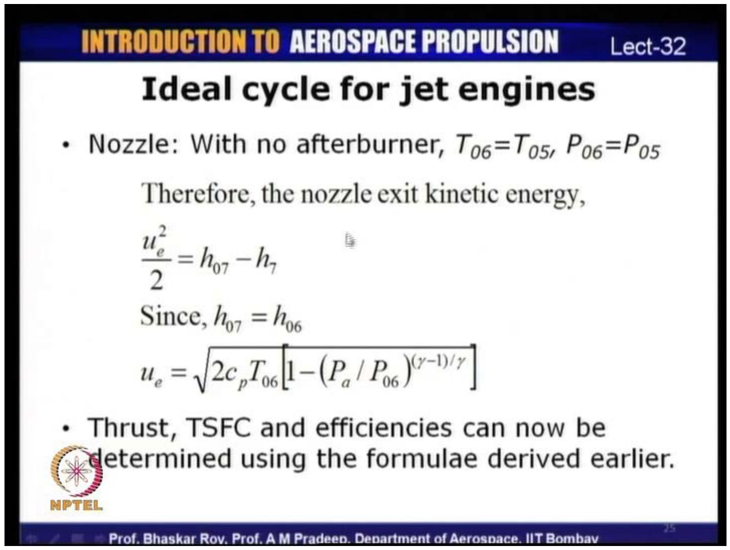

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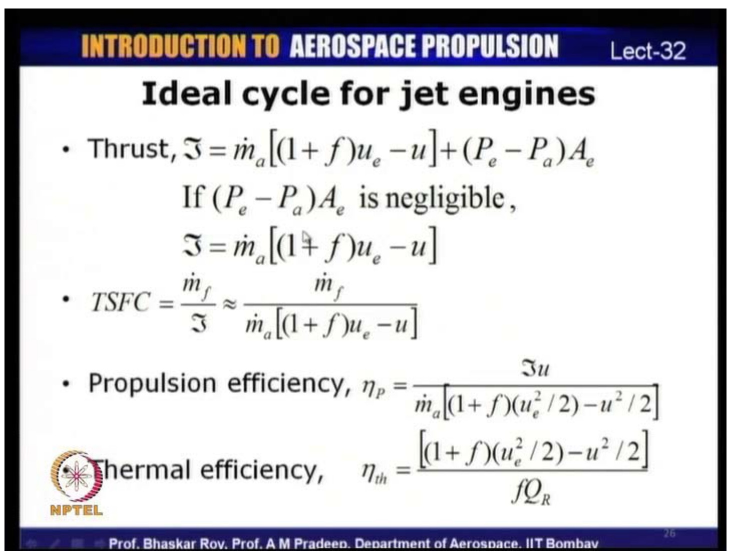

So, after the turbine, we have the nozzle. If there is no afterburner, we have T 0 6 is equal to T 0 5 P 0 6 is equal to P 0 5. So, the nozzle kinetic energy is equal to u e square by 2 which is h 0 7 minus h 7, and since there is no energy added in the before the nozzle, because there is no afterburning. We have h 0 7 is equal to h 0 6, and therefore, we can simplify an expression for u e which is equal to square root of 2 c p T 0 6 into 1 minus the pressure ratio P a by P 0 6 raised to gamma minus 1 by gamma. So, which means that we have u e the exhaust velocity, which is a function of its inlet temperature and the pressure ratio. So, now that u e is known. The thrust, the fuel consumption efficiencies, etcetera can be easily determined using what we have derived earlier the thrust will be equal to m dot a into 1 plus f into u e minus u plus the pressure thrust term, and if that is negligible, we have thrust is equal to m dot a into 1 plus f u e minus u.

TSFC is equal to m dot f by thrust, and similarly, the propulsion thermal and overall efficiencies, because we now have all these parameters which are known from the cycle analysis.

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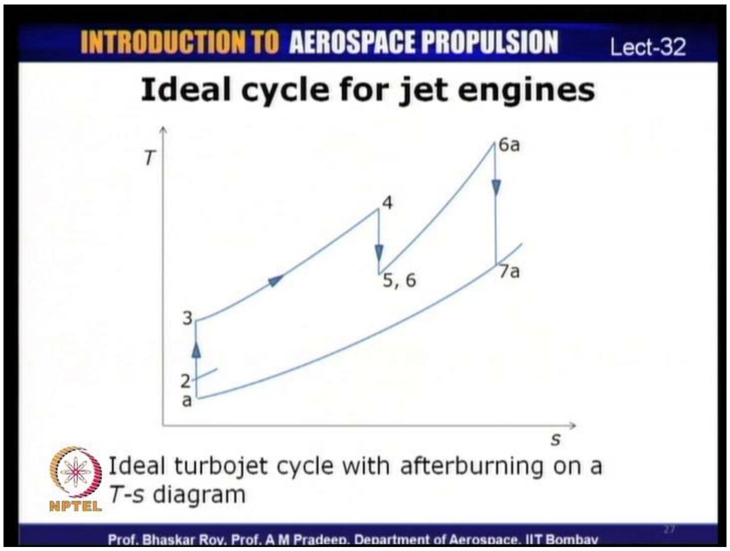

So, this is primarily the cycle analysis process for a turbojet engine without any afterburning. So, let us take a look at what happens if you have an afterburning process as well. That is there is reheating after the turbine stage 1. So, if there is an afterburning, then the ideal cycle gets modified like this 4 to 5 was the turbine, and in the previous case, we saw that 5 to 7 was again expanded in the nozzle. If there is afterburning, there is further heat addition taking place, and so, 5 to 6 a is the afterburning process and 6 a to 7 a is the expansion in the nozzle after the afterburning.

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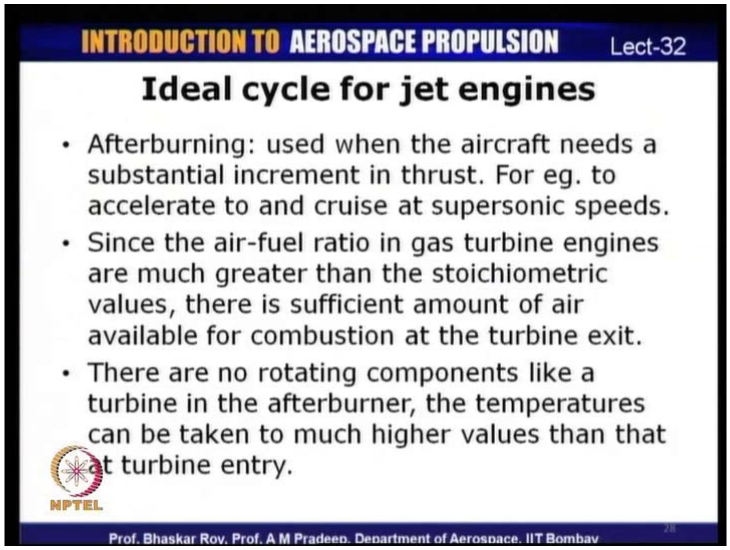

So, what basically happens is that afterburning is primarily used if an aircraft needs to have substantial increment in thrust like if it has to accelerate and cruise at supersonic speeds, and since the turbine exhaust has sufficient amount of air that is available for carrying out combustion, and the third point is that in an afterburner, there are no limits to temperature like in turbine entry, because there are no rotating components present in an afterburner. So, you can have higher temperatures than what is permitted for turbine entry.

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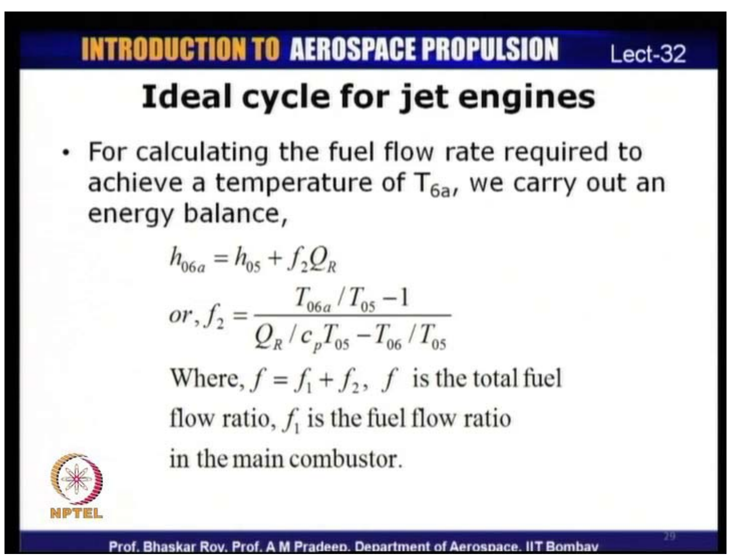

So, what is the difference between the afterburning cycle, and the other cycle, it is up to the turbine exit it is still the same, and in the afterburner, we have fuel is added additionally. So, you have to calculate the additional fuel that is added in the afterburner. We do it the same way. We did it. For the compression chamber, we carry out an energy balance and calculate f 2 which is the fuel added in the afterburner. So, f 2 we can calculate as this ratio T 0 6 a by T 0 5 minus 1 divided by Q R by c p times T 0 5 minus T 0 6 by T 0 5.

So, here, f the total fuel flow rate will now be equal to 2 components f 1 plus f 2 - where f 1 is the fuel added in the main combustor and f 2 is the fuel added in the afterburner. So, the basic cycle analysis remains the same as we did for the basic turbojet cycle. The difference is that in this afterburner, we have an additional fuel added, and therefore, we have the additional fuel that is added as well as the temperature. At the nozzle entry, the stagnation temperature at nozzle entry is going to be different.

So, based on that, we can calculate the exhaust velocity u e which in turn from in, we can calculate the thrust, the fuel flow rates and the efficiencies, and so, that is how we could calculate or carry out the cycle analysis for a turbojet engine with afterburning. So, up to the turbine entry, the cycle analysis is the same; it is only after the turbine exit that there is slight difference between the afterburning turbojet and the pure turbojet.

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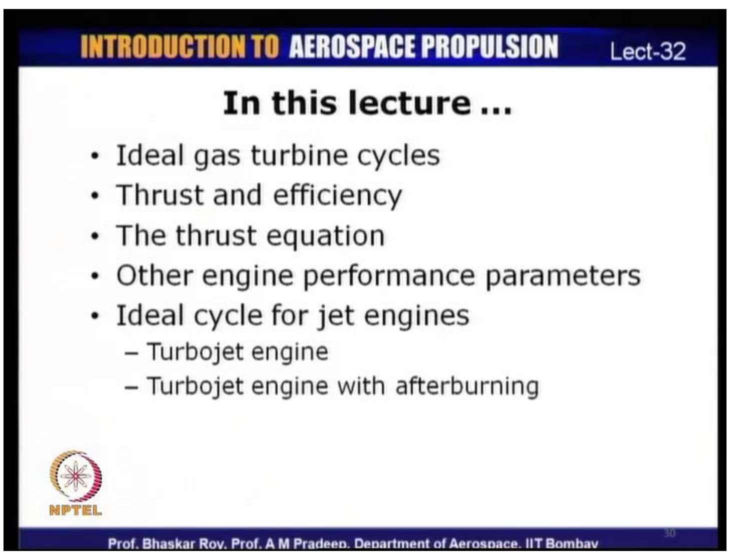

So, let us take a recap at what we had discussed in this lecture. We have been discussing about the ideal gas turbine cycles and we started our lecture with a discussion on the thrust and efficiency terms. We derived an equation for thrust of an air breathing engine, a generalized expression for thrust of an air breathing engine. We also defined different efficiencies like propulsion efficiency, thermal efficiency and overall efficiency, and we also defined what is meant by the fuel consumption, different forms of fuel consumption to be defined for different types of engines, and then, we discussed about the ideal cycle for a turbojet engine and how we can carry out a cycle analysis, an ideal cycle analysis for a turbojet engine without afterburning as well as with afterburning.

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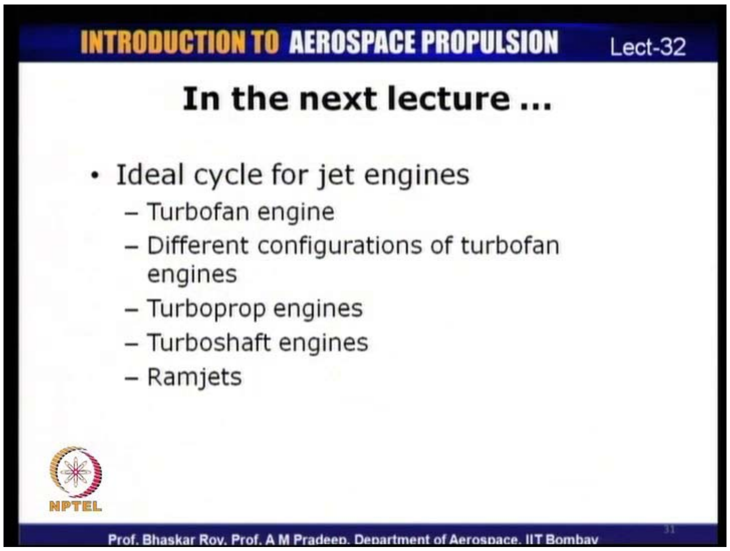

So, these were some of the topics that we had discussed during today's lecture. We shall continue this discussion in the next lecture as well, where we shall be talking about cycle analysis for different types of other types of engines like turbofan engine and different configurations of turbofan engine like mixed and unmixed. Then we shall be discussing about the turboprop engines and turbo shaft engines, and as well as the towards the end, we will discuss in brief about the cycle analysis for ramjet engines. So, these are some of the topics that we shall take up for our discussion during the next lecture.